A TRAVELING TIME MODEL AS FUNCTION OF WATER DENSITY AND VEGETABLE SIZE, SHAPE AND DENSITY

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Summary
In this research, the possibility of using the rising and dropping time of vegetables through water as a means of hydro-sorting of tomato and potato was studied. The dropping time of potato and rising time of tomato were theoretically formulated and then determined experimentally using water column. The best models for dropping time of potato (cv. Analytic) and rising time of tomato (cv. Rio grand s) as a function of water and vegetable densities, shape factor and vegetable volume were modelled with determination coefficients of 0.95 and 0.91, respectively. It was found that difference between vegetables and water densities had major effect on rising and dropping time. It can be concluded that in the sorting systems, difference in terminal velocities of vegetables can be used as a suitable factor for design the sorting system devices.

key words: tomato, potato, dropping time, rising time, density, volume, shape factor, size

INTRODUCTION
As world markets for fruit and produce become more sophisticated and technology continues to provide means to measure product quality, there is a corresponding market pull for produce with higher, or at least specified, quality levels. Fruit graders that employ near-infrared technologies are expensive and more importantly, the calibrations and maintenance they require tend to remain outside the skills of packing house staff (Jordan & Clerk 2004). Density, a good indicator of fruit dry matter (Richardson et al. 1997, Jordan et al. 2000) thus becomes an interesting tool for fruit quality sorting because of its inherently lower cost and simpler operation.

According to Jordan and Clerk (2004) an approach to fruit sorting is to use the terminal velocity of fruit moving in a fluid that has a density above or below the target density. Fruit with different terminal velocities will reach different depths after flowing a fixed distance in a flume and may be separated by suitably placed dividers. This approach could
use water as a sorting medium, which provides huge advantages in terms of the resulting low corrosion and disposal difficulties, and the fact that it does not need any density adjustment. Additionally, this approach allows purely mechanical setting of the separation threshold by adjusting the divider positions and no change in fluid density is required. Kheiralipour (2008) studied the terminal velocity and rising time of Redespar and Delbarstival apple varieties and reported that the apple was reached to its terminal velocity around 0.5 s after releasing and also most fruits showed little tendency to rotate and move in horizontal directions.

The authors embarked on a study to test dropping time of potato and rising time of tomato in water column to determine if there was potential for terminal velocity methods in sorting industry.

MATERIAL AND METHODS

The samples of 100 potatoes (cv. Analytic) and 100 tomatoes (cv. Rio grand s) were transferred to the laboratory in polyethylene bags to reduce water loss during transport, in May 2009. Samples were kept in cold storage at 4°C. All of the experiments were carried out at a room temperature. Each potato and tomato mass was determined with an electronic balance with 0.01 g sensitivity (GF3000, A&D, Japan). Projected area of the vegetables was determined from pictures of the samples taken by Area Measurement System-Delta Tengland. Volume and density of potatoes and tomatoes were determined by the water displacement method (Mohsenin 1986).

A glued Plexiglas column was used with a height of 1200 mm and a cross-section of 350 × 350 mm as shown in Fig. 1. The column was constructed with a diameter at least five times more that that of the samples (Vanoni 1975). The column was filled with water to a height of 1100 mm (Kheiralipour 2008).

![Fig. 1. Water column and camera setting to the right side](image)

The mean value of potato density was 1067.11 kg·m⁻³, hence, each potato was placed on the top of the column with hand and then released. While, the mean value of tomato density was 910.25 kg·m⁻³, therefore, each tomato was placed on the bottom of the column with a nondestructive instrument and then released. In all experiment, if any bubble appeared on samples, it was removed by rubbing the sample. In order to determine the rising time of tomato's samples and the dropping time of potato's samples, a digital camera, JVC (770) with 25 fps, recorded the moving of samples from releasing point in passing within column height, simultaneously. Subsequently, video to frame software were used to change video film to images in order to calculate rising and dropping time of samples by knowing the fact that each picture takes 0.04 s.
In other words, $T_d$ and $T_r$ were calculated as following equation:

$$T_r = T_d = 0.04 \times N$$  \hspace{1cm} (1)

where: $N$ is number of captured images for a sample in passing within column height (1100 mm). Three images of a potato were selected at the time of 0.0, 0.52 and 1.52 seconds as shown in Fig 2.

**Modelling of dropping and rising time**

The forces acting on potato in water will be a gravitational force, downward, buoyancy force, upward, and drag force, opposite to the direction of motion, upward. Buoyancy force and drag force were calculated using the following equations, respectively (Crowe et al. 2001):

$$F_b = \rho_w v g$$  \hspace{1cm} (2)

$$F_d = 0.5 \rho_w V^2 C_D A_p$$  \hspace{1cm} (3)

where: $F_b$ is buoyancy force (N), $\rho_w$ is water density (kg m$^{-3}$), $v$ is potato's volume (cm$^3$), $g$ is gravitational acceleration (m s$^{-2}$), $F_d$ is drag force (N), $V$ is potato's velocity (m s$^{-1}$), $A_p$ is cross-sectional area of potato (cm$^2$) which is perpendicular to the direction of motion, and $C_D$ is drag coefficient which is a function of vegetable velocity and can be modelled using Stokes law (Crowe et al. 2001):

$$C_D = \frac{24}{N_R} \text{ for } N_R < 1$$  \hspace{1cm} (4)

and

$$C_D = \frac{K}{N_R} \text{ for } N_R > 1$$

and

$$N_R = \frac{V D \rho_w}{\mu_w}$$  \hspace{1cm} (5)

Fig. 2. Actual images of potato positions in water column; A: at rest; B: after 0.52 s; C: after 1.52 s (A: Potato at rest; B: Potato 0.52 s after releasing; C: Potato 1.52 s after releasing).
where: \( \mu_w \) is the static viscosity of the water (Pa·s), \( N_R \) is the Reynolds number (dimensionless), \( K \) and \( n \) are constant factors, and \( D \) is the potato diameter (m). Potato has \( N_R > 1 \) in water column (with means of 6475.45), therefore:

\[
C_p = \frac{K \mu_w^n}{V^n D^n \rho_w^n} \quad (6)
\]

The combination of these forces (gravitational force, buoyancy force, and drag force) accelerates the potato proportional to its mass (Crowe et al. 2001):

\[
ma = F_w - F_d - F_b
\]

where: \( F_w \) is gravitational force (N). Replacing \( F_d \) and \( F_b \) from equations (2) and (3) into Eq. (7) gives:

\[
ma = mg - 0.5 \rho_w V^2 C_p A_p - \rho_w v g
\]

where: \( m \) is mass (g). On the other hand, dividing Eq. (7) by \( m = \rho v_p \), gives:

\[
a = g \left( 1 - \frac{\rho_p}{\rho_w} \right) - 0.5 \rho_w V^2 C_p A_p / (\rho v_p) \quad (9)
\]

For a spherical object, \( A/v \) can be computed directly as a function of the diameter. By separating \( A/v \) into two parts: a dimensionless shape factor (\( S_h \)), and size (\( S_z \)) (Jordan & Clerk 2004), the following relationship is obtained:

\[
\frac{A_p}{v} = \frac{S_h}{S_z} = \left( \frac{A_p}{V^\frac{1}{2}} \right) \sqrt{\frac{V}{3}} \quad (10)
\]

and with diameter represented as:

\[
D = \frac{6v}{\pi} \quad (11)
\]

where: \( S_z \) is shape factor (dimensionless) and \( e \) is constant factor. Substituting \( C_p, A/v \) and \( D \) from equations (6), (10) and (11) into Eq. (9), gives:

\[
a = g \left( 1 - \frac{\rho_p}{\rho_w} \right) - K_i \left( \frac{\rho_p (n+1) V^{(2-n)S_h}}{\rho_w v^{n+1}} \right) \quad (12)
\]

When the apparent weight of the particle due to gravitational force is equal to drag forces and the buoyancy force, a particle comes at rest, and its maximum velocity (terminal velocity) is reached (Crowe et al. 2001). Then, setting acceleration to zero in Eq. (12), the terminal velocity (\( V_t \)) of the sample becomes:

\[
V_t = K \left( \frac{\rho_p - \rho_w}{\rho_w} \right) \frac{1}{\left( \frac{n+1}{3(2-n)} \right)} \quad (13)
\]

Using the terminal velocity formula (Eq. 13), one can estimate the time (\( T_d \)) taken to depth \( X \) as:

\[
V_t = \frac{X}{T_d} \quad (14)
\]

and:

\[
T_d = BX \left( \frac{1}{S_z} \frac{(\rho_p - \rho_w)^{\frac{1}{2-n}} (3(2-n))^{\frac{n+1}{3(2-n)}}}{} \right) \quad (15)
\]

where: \( T_d \) is dropping time and \( B \) is constant factor. But for tomato, equation 7 will be as:

\[
ma = F_w + F_d - F_b \quad (16)
\]

Considering these steps for rising time of tomato and knowing that tomato has \( N_R > 1 \) in water column (with means of 8124.11) equation 15 will be as:

\[
T_r = BX \left( \frac{1}{S_z} \frac{(\rho_p - \rho_w)^{\frac{1}{2-n}} (3(2-n))^{\frac{n+1}{3(2-n)}}}{} \right) \quad (17)
\]

where: \( T_r \) is rising time. Theory of KHAT 4 is derived by generalizing equation 15 and 17 for dropping time of potato and rising time of tomato samples as following equations, respectively (Kheiralirop 2008):

\[
T_d = A (\rho_p - \rho_w)^{\gamma^\prime} S_h^n d + E \quad (18)
\]
where: parameters A, b, c, d and E are curve fitting parameters and take appropriate values and Parameter E is added to reducing errors.

Determined data were considered for modelling dropping and rising time using SPSS, 15, Software. The models (equations 18 and 19) were optimized by adjusting various combinations of the five parameters to maximize the determination coefficient (R²) and to minimize root mean square error (RMSE) and reduced chi-square (χ²).

\[
RMSE = \left[ \frac{1}{N} \sum_{i=1}^{N} (T_{\text{exp},i} - T_{\text{pre},i})^2 \right]^{0.5}
\]

\[
\chi^2 = \frac{1}{N - m} \sum_{i=1}^{N} (T_{\text{exp},i} - T_{\text{pre},i})^2
\]

In the above equations T_{pre,i} is the i\(^{th}\) predicted time, T_{exp,i} is the i\(^{th}\) experimental time, N is number of observations and m is number of constants.

**RESULTS AND DISCUSSION**

The two models were tested, and results are summarized in Tables 1 & 2 for potato and tomato, respectively.

### Table 1. Comparison of dropping time models developed with different parameters and corresponding correlation factors for potatoes samples

<table>
<thead>
<tr>
<th>χ²</th>
<th>RMSE</th>
<th>R²</th>
<th>E</th>
<th>d</th>
<th>c</th>
<th>b</th>
<th>A</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.83</td>
<td>1.79</td>
<td>0.95</td>
<td>6.681</td>
<td>-1.062</td>
<td>-0.925</td>
<td>-1.687</td>
<td>-0.207</td>
<td>1</td>
</tr>
<tr>
<td>6.81</td>
<td>17.27</td>
<td>0.21</td>
<td>-21.22</td>
<td>-0.207</td>
<td>0.185</td>
<td>0.000</td>
<td>11.85</td>
<td>2</td>
</tr>
</tbody>
</table>

### Table 2. Comparison of rising time models developed with different parameters and corresponding correlation factors for tomatoes samples

<table>
<thead>
<tr>
<th>χ²</th>
<th>RMSE</th>
<th>R²</th>
<th>E</th>
<th>d</th>
<th>c</th>
<th>b</th>
<th>A</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.91</td>
<td>2.11</td>
<td>0.91</td>
<td>48.38</td>
<td>-0.797</td>
<td>0.924</td>
<td>-0.836</td>
<td>-19.87</td>
<td>1</td>
</tr>
<tr>
<td>7.26</td>
<td>4.28</td>
<td>0.68</td>
<td>-84.81</td>
<td>0.684</td>
<td>-0.731</td>
<td>0.000</td>
<td>4.57</td>
<td>2</td>
</tr>
</tbody>
</table>

For potato samples, the effectiveness of all parameters including shape factor, volume, and water and potato densities for determining dropping time is shown in model 1 with R², RMSE and χ² of 0.95, 1.79 and 3.83, respectively.

\[
T_r = A(\rho_w - \rho_r)^{0.687} v^{0.925} S_h^{1.062} + 6.681
\]

R² = 0.95

Model 2 was studied to found effectiveness of density. With deleting differences between potato and water density in model 2, much change in R², RMSE and χ² was observed. From these models it can be seen that the most effective parameter on the dropping time of potato (cv. Analytic) is density.

For tomato samples, the effectiveness of all parameters including shape factor, volume, and water and tomato densities for determining rising time is shown in model 1 with R², RMSE and χ² of 0.91, 2.11 and 3.91, respectively.
Model 2 was studied to found effectiveness of density. With deleting differences between water and tomato density in model 2, much change in \( R^2 \), RMSE and \( \chi^2 \) was observed. From these models it can be seen that the most effective parameter on the rising time of tomato (cv. Rio grand s) is density.

According to Jordan and Clark (2004), Fruit density is a strong indicator of internal sugar status in kiwifruit, and this measurement minus the density of the supporting fluid has a major effect on drop velocity and thus on the transit time to reach the bottom of a fluid tank. Fruit shape also effect velocity but should not be of a magnitude to cause concern (Jordan & Clerk 2004). The similar finding was reported by Kheiralipour et al. (2010) for kiwifruit.

\[
T_r = -19.87 (\rho_w - \rho_t)^{-0.836} V^{0.924} S_h^{-0.797} + 48.38
\]

\( R^2 = 0.91 \)

CONCLUSIONS

In this study the best models for dropping and rising time of potato (cv. Analytic) and tomato (cv. Rio grand s) were found as a function of water and vegetables densities, shape factor and vegetables’ volume. The differences between water and vegetables densities were found to be the most effective parameter on their dropping and rising time while shape factor and volume of vegetables hadn’t important influence on traveling time. It can be concluded that in the sorting systems, difference in terminal velocities of vegetables can be used as a suitable factor for design the sorting system devices.

Acknowledgement
The authors acknowledge the University of Tehran for full support of this project. The authors are also grateful to Seyed Mohammad Taghi Gharibzahedi for his helps.

\[
\text{Table: Nomenclature}
\]

<table>
<thead>
<tr>
<th></th>
<th>Nomenclature</th>
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<tr>
<td>Bouncy force, N</td>
<td>( F_b )</td>
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<tr>
<td>Gravitational force, N</td>
<td>( F_w )</td>
</tr>
<tr>
<td>Drag coefficient, dimensionless</td>
<td>( C_D )</td>
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<tr>
<td>Reynolds number, dimensionless</td>
<td>( N_R )</td>
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<td>Acceleration, m s(^{-2})</td>
<td>( a )</td>
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<tr>
<td>Velocity, m s(^{-1})</td>
<td>( V )</td>
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<tr>
<td>Terminal velocity, m s(^{-1})</td>
<td>( V_t )</td>
</tr>
<tr>
<td>Dropping time, s</td>
<td>( T_d )</td>
</tr>
<tr>
<td>Rising time, s</td>
<td>( T_r )</td>
</tr>
<tr>
<td>Constant factor</td>
<td>( n, K, e, B)</td>
</tr>
<tr>
<td>Curve fitting parameter</td>
<td>( E, A, b, c, d)</td>
</tr>
</tbody>
</table>

\( \rho_w \) mass, g  
\( \rho_t \) tomato density, kg m\(^{-3}\) 
\( A_p \) Projected area, cm\(^2\) 
\( D \) tomato diameter, mm 
\( v \) Terminal velocity, m s\(^{-1}\) 
\( V \) Velocity, m s\(^{-1}\) 
\( m \) mass, g 
\( \rho_t \) tomato density, kg m\(^{-3}\) 
\( \rho_w \) Water density, kg m\(^{-3}\) 
\( S_h \) Shape factor of tomato 
\( \mu_w \) Static viscosity of water, Pa.s 
\( F_d \) Drag force, N 
\( Sz \) Size
REFERENCES


CZASOWY MODEL PRZEMIESZCZANIA SIĘ JAKO FUNKCJA MASY WŁAŚCIWEJ WODY ORAZ WIELKOŚCI, KSZTAŁTU I MASY WŁAŚCIWEJ WARZYW

Streszczenie

W niniejszej pracy badano mo¿liwoœæ wykorzystania czasu wznoszenia i opadania warzyw w wodzie, jako sposób sortowania wodnego (hydrosortowania) pomidorów i ziemniakiów. Czas opadania bulw ziemniaka i czas wznoszenia owoców pomidora zosta³ obliczone teoretycznie, a nast³pnie okreœlone eksperymentalnie przy u¿yciu s³upa wody. Najlepsze modele dla czasu opadania bulw ziemniaka (cv. Analytic) i czasu wznoszenia owoców pomidora (cv. Rio Grande S) jako funkcja masy właœciwej wody oraz masy właœciwej, wsp³óczyznika kszta³tu i objêtoœci tych warzyw zosta³y przedstawione przy wsp³óczyznikach determinacji odpowiednio 0,95 i 0,91. Stwierdzono, ¿e ró¿nica miêdzy mas¹ wzglêdnej wody a mas¹ wzglêd¹ warzyw ma znaczy³ wpływ na czas wznoszenia i opadania. Mog¹a wnioskowaæ, ¿e w systemach sortowania ró¿nicze w prêdkoœci koncowej (maksymalnej) miêdzy warzywami mog³y byæ wykorzystane jako odpowiedni parametr przy projektowaniu urz¹dœœw do sortowania.