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# TRAFFIC FLOW VARIABLES ESTIMATION: AN AUTOMATED PROCEDURE BASED ON MOVING OBSERVER METHOD. POTENTIAL APPLICATION FOR AUTONOMOUS VEHICLES

# Marco Guerrieri<sup>1</sup>, Giuseppe Parla<sup>2</sup>, Raffaele Mauro<sup>3</sup>

1,3 DICAM, University of Trento, Italy;
 1 marco.guerrieri@unipa.it, 3 raffaele.mauro@unitn.it2
 2 Polytechnic School, University of Palermo and ISMETT, Palermo, Italy giuseppe.parla@unipa.it

The estimation of traffic flow variables (flow, space mean speed and density) plays a fundamental role in highways planning and designing, as well as in traffic control strategies. Moving Observer Method (MOM) allows traffic surveys in a road, or in a road network. This paper proposes a novel automated procedure, called MOM-AP based on Moving Observer Method and Digital Image Processing (DIP) Technique able to automatically detect (without human observers) and calculate flow q, space mean speed  $v_s$  and density k in case of stationary and homogeneous traffic conditions.

In order to evaluate how reliable is the MOM-AP, an experiment has been carried out in a segment of one two-lane single carriageway road, in Italy. 30 datasets for the segment have been collected (in total 30 round trips). A comparative analysis between MOM-AP and traditional MOM has been carried out. First results show that the current MOM-AP algorithms underestimate the local mean flow variable values of around 10%. Nowadays MOM-AP may be implemented in smartphone apps. Instead, in the near future, it is realistic expecting the increase in the use of automated procedures for calculating the traffic flow variables (based on the "moving observer method"), due to the amount of sensors and digital cameras employed in the new autonomous vehicles (AVs). Considering such technical advances, the MOM-AP is a feasible model for real-time traffic analyses of road networks.

Keywords: traffic flow, moving observer method, digital image processing technique, autonomous vehicles

#### 1. Introduction

One of the most important objectives of traffic and transportation engineering is the evaluation of traffic flow. A key role in traffic theory is given by the Fundamental Equation (Eq. (1)) which represents the theoretical relationship between flow q, space mean speed  $v_s$  and density k in a given road section, in case of stationary and homogeneous traffic conditions (Gerlough, and Huber, 1975; Daganzo, 1997; Leutzbach, 1998):

$$q = v_s \cdot k . \tag{1}$$

Eq. (1) may be demonstrated in a very simple way only for the uniform flow, as explained in the Figure 1 (hydrodynamic analogy). Instead, the demonstration is more complex for the general cases of stationary and homogeneous traffic flow conditions, but not uniform.

Generally, the macroscopic variables q,  $v_s$  and k, are estimated by means of vehicle detection and classification at a single fixed location of the road (static count or fixed point measurement). For such purpose, different types of traffic counting equipment and devices are used, such as loop detectors, radar, pneumatic tubes, video cameras, and others. Another common technique used to analyse the flow on a road segment is the moving car observer method (MOM) developed by Wardrop and Charlesworth in 1954.

Moving Observer Method (MOM) is use to conduct traffic survey of road, or road network, even if it is conducted differently compared with the aforementioned static counting method. In fact, in the MOM, the observer travels along a section of road with its flow denoted as q. A series of runs of a test vehicle (observer vehicle), made traveling with and against a one way traffic flow q, are required. The basic traffic information (i.e. the number of vehicles and travel times) is collected at the same time by three human observers traveling inside the test vehicle.

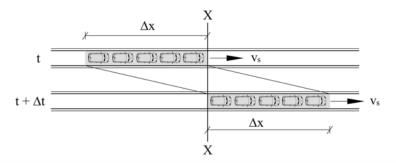


Figure 1. Hydrodynamic analogy in case of stationary and uniform traffic flow. In the gray area there are  $n=k\cdot\Delta x$  vehicles. In the time interval  $\Delta t=\Delta x/v_s$ , the gray area (and therefore all the vehicles) crosses the X section. The flow is  $q=n/\Delta t=k\cdot\Delta x/\Delta t=k\cdot v_s$ 

This research focuses on a novel automated traffic survey based on the moving observer method: the counting processes and the classification of vehicles are obtained by digital image processing (DIP) technique, suitable to provide the values of the macroscopic flow variables  $(q, v_s, k)$ . Differently from the traditional Wardrop method, the proposed one does not require human observers, thus reducing costs while increasing safety. Traffic flows are analysed via smartphone video cameras, these devices are attached to the windshield of test vehicles or onto their dashboards (Fig. 2).

The proposed technique has been verified, calibrated and validated. In addition, much traffic flow analysis has been conducted in order to investigate the macroscopic flow variables in a two-lane roadway. The first results, obtained using the proposed automatic method (MOM-AP), agree with the conventional moving observer method (MOM).





Figure 2. Typical simple device used for the macroscopic traffic variables estimation

#### 2. Theory Part A: Moving Observer Method (MOM)

This method involved a series of runs in a test vehicle made travelling 'with' and 'against' a one-way traffic stream. The observers in the test vehicles record the following information for each run (Barua *et al.*, 2015):

- the number of opposing vehicles met and the number of vehicles that the test vehicle overtook;
- the number of vehicles overtaking the test vehicle while it traveling;
- the average speed of the test vehicle;
- the distance of the run and the journey times of the test vehicle in each run.

Consider a stream with a space mean speed  $v_s$ , density k and flow q in stationary and homogeneous conditions along a segment of a lane of a two-lanes road, of length L.

If the test vehicle travels in the same direction of the flow q at speed  $v_w$ , the relative speed stream  $\Delta V$  and relative flow  $\Delta q$  with reference to the test are:

$$\Delta \mathbf{v} = \mathbf{v}_{\mathbf{S}} - \mathbf{v}_{\mathbf{W}}, \tag{2}$$

$$\Delta q = k \cdot \Delta v = k \cdot (v_s - v_w) = k \cdot v_s - k \cdot v_w. \tag{3}$$

From Eq.(1) and Eq. (3), it follows that:

$$\Delta q = q - k \cdot v_w \,. \tag{4}$$

As shown in Figure 3, the relative flow is  $\Delta q = q$  for  $v_w = 0$  and  $\Delta q = 0$  for  $v_w = v_s$ .

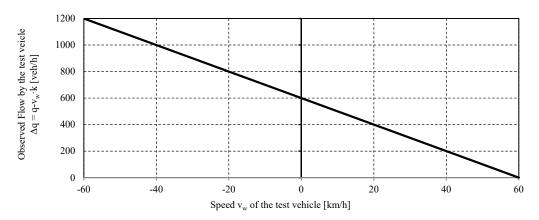


Figure 3. Relative flow  $\Delta q$  in function of the speed  $v_w$  of the test vehicle for q = 600 veh/h and  $v_s = 60$  km/h

If  $t_w$  denotes the journey time of the test vehicle with speed  $v_w$  ( $v_w = L/t_w$ ) and  $t_s$  the journey time corresponding to the mean space speed  $v_s$  of the flow ( $v_s = L/t_s$ ), the number "y" of vehicles with speed  $v_s$ , which overtaking the observer vehicle when it travels into the stream (minus the number it overtakes) is:

$$y = \Delta q \cdot t_w = k \cdot \Delta v \cdot t_w = k \cdot (v_s - v_w) \cdot t_w = \frac{L}{t_s} \cdot k \cdot (t_w - t_s) = v_s \cdot k \cdot (t_w - t_s).$$
 (5)

Therefore:

$$y = q \cdot (t_w - t_s). \tag{6}$$

Let x be the number of vehicles with speed  $v_s$  met by the observer vehicle when it travels the road segment L against the flow q and  $t_a$  the journey time corresponding to the test vehicle speed  $v_a$  ( $v_a = L/t_a$ ). Similarly to the previous demonstration, the x value is:

$$x = q \cdot (t_a + t_s). \tag{7}$$

By the equations (6) and (7), we obtain the values of the macroscopic variables q, v<sub>s</sub> and k:

$$q = \frac{x + y}{(t_a + t_s)}, \qquad (8)$$

$$t_{s} = t_{w} - \frac{y}{q}, \tag{9}$$

$$v_{s} = \frac{L}{t_{s}},\tag{10}$$

$$k = \frac{q}{v_s} \,. \tag{11}$$

As explained by Wright (Wright, 1972) the values of the macroscopic variables obtained with the conventional moving observer method are subject to a small degree of bias. Moreover, local variations in the speeds of the observer test vehicle and the observed vehicles have no effect on the tally count.

Comparisons between estimates of macroscopic traffic flow variables over a section against a fixed spot measurement were carried out by Abdulrahman (Abdulrahman *et al.*, 2017). It was found that the differences in the estimated values are not significant.

Wardrop (Wardrop and Charlesworth, 1954) demonstrated the validity of MOM, except for cases in which the test vehicle speed  $v_w$  is close to the flow mean speed  $v_s$ . As a matter of fact, if  $v_w \approx v_s$  then systematic errors occur. In addition when the MOM is used on one-way roads, it is usually necessary for the observer vehicle travelling the section at least at two different speeds (Bennet, 1977; Duncan, 1973; Hewitt *et al.*, 1974).

## 3. Theory Part B: an Automated Counting Approach (MOM-AP)

The proposed method, based on digital image processing technique, allows for automatically counting the observed vehicles "x" and "y" and categorizing them into prefixed types (motorcycles, cars, trucks, commercial vehicles, etc.). The procedure, here called "Moving Observer Method - Automated Procedure" (MOM-AP), is divided into the following steps:

- Vehicles Detection (VD): the pixels belonging to the objects in motion are found;
- Classification or recognition: only the objects of interest (vehicles) are identified;
- Labelling: a label is attached to each object (i.e. vehicle) of interest;
- Tracking: occurring when the same object (i.e. vehicle) is present in different frames which follow one another in the time;
- Counting: the numbers of vehicles "x" and "y" are estimated, t<sub>a</sub> and t<sub>w</sub> are measured and, finally, the macroscopic flow variables (q, v<sub>s</sub>, k) are calculated.

In the field of intelligent transportation systems (ITS) the major issue arises when vehicle detection is required (Kim *et al.*, 2015). Several current car detectors are vision-based detectors. Many methods for vehicle detection, using mono, stereo, and other vision-sensors are well-known (Sivaraman and Trivedi, 2013). This research focuses on vision-based car detectors using monocular information (Hu *et al.*, 2016).

Various researches adopt two different steps: the hypothesis generation (HG) and the hypothesis verification (HV). In the first phase, that is the hypothesis generation, the vehicle candidates are chosen in terms of their of symmetry (Broggi *et al.*, 2004; Bensrhair *et al.*, 2001), colour (Tsai *et al.*, 2007; Guo *et al.*, 2000), shadow (Tzomakas and Seelen, 1998; Feng and Xing, 2013) and edges (Sun *et al.*, 2002; Southall *et al.*, 2009).

In regard to the hypothesis verification (HV), the Histogram of oriented gradient (HOG) (Yuan and Ablavsky, 2011), Haar-like wavelet (Chang and Cho, 2010), Gabor feature (Sun *at al.*, 2005) and Aggregate Channel Features (ACF) (Dollar *et al.*, 2014) are largely used. In this study, the Aggregate Channel Features (ACF) has been used, in accordance with the research developed by Dollar (Dollar *et al.*, 2014), mainly for the pedestrian detection in urban context. The ACF detection framework is conceptually shown in Figure 4. Given an image I, it is necessary computing several channels  $C = \Omega(I)$ , summing every block of pixels in C, and smoothing the resulting lower resolution channels. The overall methodology is described in (Dollar *et al.*, 2010; Dollar *et al.*, 2014).



Figure 4. ACF detector (boosting is adopted to learn decision trees over these features (pixels) to distinguish object from background); adapted from Dollar et al., 2014

The tracking phase detects the same vehicle (object) in all its positions which may be shown in a series of successive video frames (*Matching operation*). A simplified schematization of the different operations employing the tracking algorithm is illustrated in Figure 5.

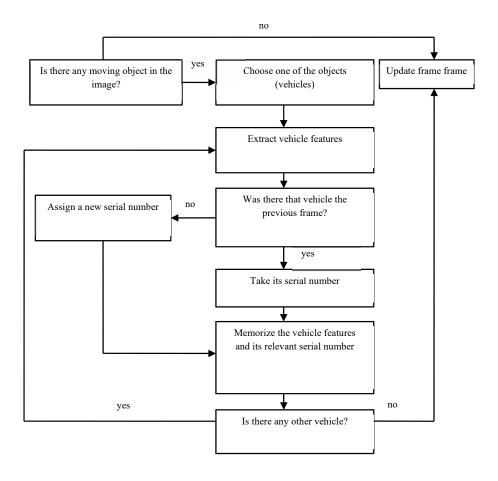


Figure 5. Tracking algorithm (simplified schematization)

The area of bounding box of each detected vehicle (i.e. pixels) changes in the sequence of video frames. The increase over the time of this area denotes a reduction in the space headways between the observed vehicle (leader vehicle) and the test vehicle (follower vehicle). After an overtaking manoeuvre, the bounding box disappears. Therefore, it is possible to estimate the number "y" of vehicles with mean speed  $v_s$ , which overtaking the observer vehicle when it travelling the segment of the road into the stream q. Similarly, the number "x" of vehicles met by the observer vehicle travelling the road segment L against flow q may be automatically measured.

The proposed method has been implemented in Matlab R2018b. The results, based on synthetic data, show that the automatic method is very reliable.

Nevertheless, at this stage of the research, numerous false positives (FP) and false negatives (FN) occur during the Vehicles Detection phase.

Nonetheless, the algorithmic aspects may be verified and checked using synthetic data, real-world datasets are essential to guarantee performance of algorithms in all the real situations (Janai *et al.*, 2017).

For instance, the proposed method employed in practice needs to handle complex environmental conditions; such as vibrations due to the road pavement surface, direct lighting, reflections from specular surfaces, rain, and more.

# 4. Empirical Results and Discussion

The road section taken into account for the study has been selected from a two-lane single carriageway SS640, in Italy. The selected straight segment, of length L (L = 1.1 km), has an uninterrupted flow (Fig. 6).

In order to ensure that the speed and flow present stable conditions, the selection of survey road segment should meet the following criteria (Lee and Brocklebank, 1993):

- Homogeneity: homogeneous in geometric characteristics (horizontal and vertical alignment, lanes and shoulders width, etc.) throughout the whole length L;
- Junctions: there is no at-grade junction within the road segment or within at least 250 m of its endpoints;
- Speed limits: no segment contain within it, or within 250 m of its endpoints, any restriction other than the national speed limit;
- Road works: no roadworks were taking place along the segment;
- Length: segments should preferably be more than 1 km and not greater than 5 km in length.
   The selected straight segment satisfies these criteria.

The test vehicle has been driven following and against the traffic stream (flow q) with a mean speed of around 60 km/h.

A total of 30 datasets for the segment of 1.1 km have been collected (30 round trips in total).

The videos have been recorded using a smartphone (IPhone 5) attached onto the windshield of the test vehicle (Fig. 2).

The recorded frames, used in the analysis, correspond to a run undertaken on November 7, 2018. A total number of 3 hours' traffic data was collected (9:00-10:00, 11:00-12:00, 15:30-16:30).

The data counted:

- the number y of vehicles which overtake the observer test vehicle when it travels the road segment L with the stream of flow q, from the section A to B (cfr. Fig. 6 and Fig. 7);
- the number x of vehicles met by the observer test vehicle when it travels the road segment L against the flow q, from the section B to A (cfr. Fig. 6 and Fig. 8);
- the journey times t<sub>a</sub> and t<sub>w</sub> required for crossing the road segment with the stream (A → B) and against the stream (B → A), respectively.

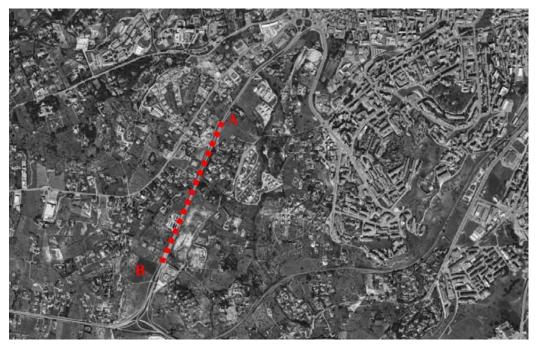


Figure 6. Segment analysed of the road SS 640 in Italy ( $L_{A-B}=1.1 \text{ km}$ )

Tables 1, 2 and 3 show the estimated values of the abovementioned parameters for the three hours' data acquired by the proposed automated counting approach (MOM-AP). In addition, in Tables 1, 2 and 3 the homologous data, carried out by means of the traditional moving observer method (MOM), the number of false positives (FP) and false negatives (FN) are given.



Figure 7. Detection and Tracking phases. Examples of two vehicles which overtake the observer test vehicle (counting of "y" vehicles from the section A to the section B)





Figure 8. Detection and Tracking phases. Examples of two light vehicles and two heavy vehicles met by the observer test vehicle when it travels the road against the flow q (counting of "x" vehicles from the section B to the section A)

**Table 1.** Results of counting processes for the first hour (9:00-10:00)

	y [vehicles]			x [vehicles]						
Trip n.	MOM-AP	MOM	FP	FN	MOM-AP	MOM	FP	FN	tw [s]	$t_a[s]$
1	0	0	0	0	11	13	2	0	61.0	60.7
2	2	2	0	0	10	12	2	0	57.5	65.2
3	1	1	0	0	12	14	2	0	63.3	61.8
4	1	0	0	1	13	14	1	0	69.0	67.4
5	0	0	0	0	12	12	0	0	47.2	62.9
6	1	2	1	0	8	9	1	0	65.6	61.8
7	0	0	0	0	10	12	2	0	64.4	61.8
8	1	1	0	0	12	11	0	1	64.4	62.9
9	0	1	1	0	12	10	0	2	65.6	58.4
10	0	0	0	0	14	16	2	0	62.1	62.9
TOT.	6	7	2	1	114	123	12	3	619.9	625.8
MEAN	0.6	0.7	0.2	0.1	11.4	12.3	1.2	0.3	62.0	62.6

**Table 2.** Results of counting processes for the second hour (11:00-12:00)

	y [vehicles]			x [vehicles]						
Trip n.	MOM-AP	MOM	FP	FN	MOM-AP	MOM	FP	FN	tw [s]	t <sub>a</sub> [s]
1	1	2	1	0	14	15	1	0	57.8	65.5
2	0	1	1	0	8	10	2	0	60.2	66.6
3	3	2	0	1	8	9	1	0	63.7	67.8
4	2	4	2	0	15	13	0	2	68.4	67.8
5	0	1	1	0	14	12	0	2	56.6	72.6
6	0	0	0	0	9	12	3	0	68.4	75.0
7	0	0	0	0	7	7	0	0	63.7	64.3
8	0	0	0	0	7	7	0	0	64.9	67.8
9	1	3	2	0	7	8	1	0	66.1	64.3
10	1	1	0	0	9	11	2	0	68.4	70.2
TOT.	8	14	7	1	98	104	10	4	638.4	681.9
MEAN	0.8	1.4	0.7	0.1	9.8	10.4	1	0.4	63.8	68.2

Table 3. Results of counting processes for the third hour (15:30-16:30)

	y [vehicles]			x [vehicles]						
Trip n.	MOM-AP	MOM	FP	FN	MOM-AP	MOM	FP	FN	t <sub>w</sub> [s]	t <sub>a</sub> [s]
1	2	3	1	0	14	16	2	0	60.0	66.1
2	4	5	1	0	14	14	0	0	65.0	68.4
3	2	2	0	0	10	14	4	0	66.3	64.9
4	3	4	1	0	15	14	0	1	73.8	70.8
5	1	1	0	0	11	12	1	0	68.8	66.1
6	0	0	0	0	12	9	0	3	72.5	64.9
7	0	1	1	0	11	12	1	0	73.8	64.9
8	0	1	1	0	8	11	3	0	71.3	66.1
9	0	1	1	0	9	9	0	0	68.8	61.4
10	1	1	0	0	13	14	1	0	66.3	66.1
TOT.	13	19	6	0	117	125	12	4	686.3	659.6
MEAN	1.3	1.9	0.6	0	11.7	12.5	1.2	0.4	68.6	66.0

With the values of the Tables 1, 2 and 3, the flow q, the space mean speed  $v_s$  and the density k have been calculated (cfr. Eqs. (8), (10), (11)).

The results are given in Tab. 4. Significant is the reduction of time spent for the counting process via the automated method (MOM-AP). Nevertheless, the method of vehicle detection requires improvement, in terms of DIP algorithms and technology of devices. In fact, in terms of flow q, the error between automated (MOM-AP) and traditional (MOM) counting methods exceeds 10%. Instead, the maximum error for the density k is around 7%.

Table 4: Values of the Traffic flow variables (MOM-AP and MOM)

	Flow q [veh/h]			Space me	an speed v <sub>s</sub> []	km/h]	Density k [veh/km/lane]			
Hour	MOM-AP	MOM	Δ [%]	MOM-AP	MOM	Δ [%]	MOM-AP	MOM	Δ [%]	
9:00-10:00	346.8	375.7	-7.7	71.0	71.6	-0.9	4.9	5.2	-6.9	
11:00-12:00	289.0	321.8	-10.2	73.5	82.2	-10.6	3.9	3.9	0.5	
15:30-16:30	347.7	385.2	-9.7	71.8	77.9	-7.8	4.8	4.9	-2.1	

Furthermore, counting errors may rise when flow increases and the speed of the test vehicle is higher than 60 km/h, as it was verified in some tests. In this regard, most detection or tracking errors are due to a variety of reasons, including the camera viewing angle, occlusions owing to other vehicles and objects, oscillations, and others. Other errors occur for the complexity of the 3D motion fields, widely termed "scene flow" (Menze and Geiger, 2015).

#### 5. Conclusions

In this paper, we present a technique to estimate traffic flow variables using the autonomous traffic survey technique called MOM-AP, based on the "Moving Observer Method (MOM)", which is characterized by autonomous vehicle counting processes. The vehicle detection, tracking and classification are obtained by Digital Image Processing (DIP) technique. MOM-AP is suitable to measure the values of the macroscopic flow variables (q, vs, k) for stationary and homogeneous traffic flow conditions. The traffic flow may be analyzed by means of videos, recorded by smartphones attached on windshields (or onto dashboards) of test vehicles. Therefore, the suggested method does not require human observers (as in the MOM), with benefits under the aspect of safety and costs. MOM-AP has been verified, calibrated and validated by synthetic data. In addition, the algorithmic calculations were validated and verified by comparison between real-world datasets and our simulation results. A segment of the two-lane single carriageway road SS640, in Italy, was the sample chosen for this study. The selected straight segment, of length of 1.1 km, has an uninterrupted flow. The test vehicle was driven along and against the traffic stream, with a speed of around 60 km/h. 30 datasets for the segment were collected (in total 30 round trips) with a total of 3 hours' traffic data (9:00-10:00, 11:00-12:00, 15:30-16:30). MOM-AP allows to obtain the following data:

- y number of vehicles overtaking the observer test vehicle when traveling along the road segment L within the stream of flow q;
- x number of vehicles met by the observer test vehicle when traveling the road segment L against the flow q;
- the journey times t<sub>a</sub> and t<sub>w</sub> required for traveling the road segment along stream and against the stream respectively.

The MOM-AP uses the above parameters to calculate the macroscopic flow variables: flow q, space mean speed  $v_s$ , and density k. Empirical data was used for the comparative analysis between MOM-AP and traditional MOM. The current MOM-AP algorithms underestimate the local mean flow variables of around 10%. Furthermore, vehicles detection errors may occur when flow increases and the speed of the test vehicle is higher than 60 km/h. Therefore, the vehicle detection method requires improvement in terms of DIP algorithms and technology of devices. Nowadays, the proposed method may be implemented using smartphone apps. It is worth underlining that in the near future it is realistic expecting the increase in the use of automated procedures for calculating the traffic flow variables based on the "moving observer method". In fact, sensors and digital cameras employed in the autonomous vehicles (AVs) allow the detection and the tracking of other vehicles traveling along selected stream flows; as it is required by the MOM-AP. Therefore, it may be inferred that by means of MOM-AP (or by similar methods), the new AVs will enable real-time monitoring of traffic macroscopic flow variables (q,  $v_s$ , k) taken from road networks throughout the world.

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