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USING DEFICIT FUNCTIONS FOR CREW PLANNING IN AVIATION

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We use deficit functions (DFs) to decompose an aviation schedule of aircraft flights into a minimal number of periodic and balanced chains (flight sequences). Each chain visits periodically a set S of airports and is served by several cockpit crews circulating along the airports of this set. We introduce the notion of "chunks" which are a sequence of flights serviced by a crew in one day according to contract regulations. These chunks are then used to provide crew schedules and rosters. The method provides a simplicity for the construction of aircraft schedules and crew pairings which is absent in other approaches to the problem.

Keywords: deficit functions, minimal chain decomposition, crew pairing and rosters

1. Introduction

The traditional practice of planning public transportation activities considers fleet size estimation and crew scheduling as two separate problems. Our approach in this paper is based on a combination of fleet optimization and crew scheduling. The generic object for applying our approach is work scheduling for cockpit crews (pilot, co-pilot, navigator, etc.). What we do in this paper can also be applied for long distance bus and ship transportation. However, we will adopt terminology that is typically used in the aviation industry.

The initial step in our approach is finding the minimal number of aircraft needed to carry out a set of aircraft flights departing and arriving at a set of airports. This is described in Section 2 using the fleet size theorem based on a deficit function (DF) representation of the flight schedule.

In further exposition, the crucial role will be played by so called chain decomposition of the schedule. This is a set of infinite periodic flight sequences, each of which can be carried out by one physical vehicle (aircraft). Section 3 will be devoted to this issue. We will state and prove some important properties of the DFs which allow obtaining the desired chain system.

Section 4 is the central in this paper. It is devoted to assigning the crews to carry out the flights of each chain in the chain decomposition. Crew assignment must take into account the maximal number of working hours per day, necessary night rests each day after a full day of work, maximal weekly load for any crew and one or two days off after 4 or 5 or 6 working days. We will present crew rosters for typical chains which have period 1, 2 and 3 days. Cockpit crews have probably the strictest limitations comparing to other crews, say of stewards, car operators, etc. On the one hand, it makes planning activities for these crews more difficult. On the other hand, since the cockpit crews are physically connected to the aircraft, the use of optimal chain decomposition makes the problem solution relatively simple. These and additional technical details, union of several crew rosters and the problem of crew operations for the off days are also discussed. A good review of crew rostering may be found in (Day and Ryan, 1997). Section 5 provides a discussion of the procedure for construction of weekly crew rosters including consideration of managing on and off work days. The paper ends with a conclusion and future work in Section 6.

2. Finding the Minimal Number of Aircraft (Fleet Size)

A fundamental characteristic of the schedule is the minimal number of aircraft needed to carry out all the flights in a periodic way. This number is called the fleet size (FS). A simple and efficient way to estimate the FS is to employ a set of deficit functions (DFs) for a flight schedule, see (Linis and Maksim, 1967).

We present here a brief overview of the main facts and notions related to the DFs. Without loss of generality, it can be assumed that all flight departure and arrival times lie within the interval of one period $[0, T]$. Arrival times are considered to include the preparation time Z for the next flight. Let S be the set of all terminals appearing in the schedule. Let us define for each departures from A on $[0, t]$ minus the number of arrivals to A during the same interval. Let M_A be the *maximum* of the DF $f_A(t)$. The following theorem (Gertsbakh and Gurevich, 1977; Gertsbakh and Gurevich, 1982) connects the FS with the maxima of the DFs:

Theorem 1.

$$FS = \sum_{A \in S} M_A. \quad (1)$$

Here, the minimal number of aircraft needed to carry out all flights in the schedule is equal to the sum of maxima of all DFs. As an example, Figure 1 presents the schedule of a Boeing-777 division of a small aviation company for one period of $T = 24$ hours. It has 17 daily flights numbered 1, 2, ..., 17. Each flight is represented by a line segment with capital letters on its ends designating the departure and arrival terminals. So, segment 1 represents a flight from A to B departing from A at 6:00 am and arriving at B at 9:00 am. The interval of 3 hours (flight duration) is the time in air + the time Z needed for preparing the aircraft for the next flight. There are 5 airports where the flights take-off and land: A, B, C, D, E . All arrivals take place inside the interval $[0, T]$. The schedule is balanced, i.e. the number of departures from each airport is equal to the number of arrivals at this airport. All 17 flights are repeated on each period $[T(k-1), Tk]$, $k = 2, 3, \dots$. The latest arrivals are at 8:00 pm allowing the flight crew to have at least 9-10 hours nights rest time before the earliest take-off for the next day.

Figure 1 (a) Flight Schedule (17 flights)

Figure 1(b) Deficit Functions (17 Flight Schedule)

Figure 1. (a) Flight schedule for 17 flights, (b) Deficit functions for terminals A-E

The deficit function (DF) for terminal X , $f_X(t)$ is defined for t in $[0, T]$ as the difference between the number of departures from X and the number of arrivals to X which take place on the interval $[0, t]$. In simple words, $f_A(t)$ starts from zero at $t = 0$, increases by 1 at any departure instant from A and decreases by 1 at any instant of an arrival to A , $f_A(0) = f_A(T) = 0$. Consider, for example, $f_A(t=10:20)$. There are three departures from A in the interval $[0, 10:20]$ and one arrival. Thus, $f_A(10:20) = 2$. Fs. The maximum of $f_A(t)$ is 3, maximum of $f_C(t)$, $f_D(t)$, $f_E(t)$ is 1, and a maximum of $f_B(t)$ equals zero. So, the sum of all maxima is equal 6, and by Theorem 1 the minimal FS = 6. So, six physical vehicles are needed each day to carry out all flights of this schedule. Often there are peaks in a deficit function, over a very small time interval, which if “smoothed” out can reduce the maximum and consequently the number of required vehicles. There are three main methods for removing such peaks; entropy informational smoothing (Gertsbakh and Stern, 1978) deadheading insertion (Ceder and Stern, 1981) and shifting or shortening flight times within limits (Ceder and Stern, 1985).

3. Minimal Chain Decomposition

A chain is a sequence of flights carried out by a single vehicle (aircraft). Formally, this chain is a sequence of type:

$$A(t_1; \delta_1) B \rightarrow B(t_2; \delta_2) C \rightarrow \dots \rightarrow D(t_n; \delta_n) E,$$

where the letters $A, B, C, \dots, D$ designate the arrival/departure airports visited by an aircraft, and δ_i, t_i are the duration and departure time of a flight i , respectively. For example, $B(t_2; \delta_2)C$ is the flight, which departs from B at time t_2 and arrives at C on time $t_2 + \delta_2$.

The aviation schedule is always assumed to be periodic, with period T , where $T = 24$ hours or $T = 1$ day. So, if there is a flight from A to B , departing from A at 18 : 00 PM, with duration $\delta = 3$ hours, then each next day there will be a similar flight from A to B departing at 18 : 00 and lasting 3 hrs. In addition, the schedule is assumed to be *balanced*, i.e. for each period $[0, T]$, the total number of arrivals to each airport equals the total number of departures from this airport. It is assumed that for any flight $W(t; \delta) Y$, δ includes the time in the air plus the recommended time Z which is necessary to prepare the aircraft for the next flight.

Continuing with the example in section 2, our goal is finding six chains for the flight schedule shown in Figure 1. Let us construct a so-called good chain (Gertsbakh and Gurevich, 1977). Before doing this we must define so called *hollow zones* of the DFs. A hollow zone is an interval between two neighbouring maximum areas of a DF. It is easy to see that $f_A(t)$ has three hollow zones shown on Figure 1 and denoted by $A1, A2$, and $A3$. So, $A1$ has three arrivals and three departures denoted by small arrows. When we construct a chain of flights, we must connect the arrival to the departure. The rule for obtaining a *good chain* is the following: each arrival should be connected by a departure from *the same hollow zone*. So, if a chain starts with flight 1, the arrival of flight 1 should be connected to the departure of flight 2, in the same hollow zone $B1$. So, the chain now has flights 1 and 2. The arrival of flight 2 in a hollow zone $D3$ must be joined to flight 3 departing from D and arriving at A . Let us call such a connection good. We stop the chain construction if there are *no more departures from A before T*. So, we have constructed a chain of three flights $1 \rightarrow 2 \rightarrow 3$, which by our definition is a good chain. Finally, a good chain always starts with the earliest departure from a terminal. (It starts with the earliest departure from A , ends with the latest arrival in a , and all connections are good). Now let us remove the flights of this chain from the schedule and construct the new system of DFs. We will now prove the following simple lemma.

Lemma 1

(I) If a good chain starts with a departure from A at t_d and ends with an arrival at A at t_a , then its removal from the schedule leads to decrease of $f_A(t)$ by one in the interval $[t_d, t_a]$ and leaves all other maxima of the DFs unchanged.

(II) If a good chain starts with a departure from A at t_d and ends with an arrival at terminal which is not A , then its removal from the schedule leads to the decrease of $f_A(t)$ by one in the interval $[t_d, T]$ and leaves all other maxima value of the DFs unchanged.

We will provide the proof with the aid of the schedule depicted on Figure 1. The change in the DF $f_A(t)$ follows directly from the definition of the DF. Indeed, removing one departure lowers the DF by 1. As to the other DFs, the changes will happen in the hollow-zones of those DFs which are related to terminals where arrivals are joined to the departures. These changes do not affect the maxima of these DFs. Indeed, removing flights 1, 2 and 3 leads to the decrease of $f_A(t)$ (see shaded area), and elimination

of hollow zones $B1$ and $D3$. Now we have the key to proving Theorem 1. Continue constructing and removing good chains and we will have the chain decomposition of all schedules consisting of FS chains. There is, however, one defect in this procedure and we will soon recognize it. Our chain $1 \rightarrow 2 \rightarrow 3$ is balanced, i.e. it starts at A and ends in A . One physical aircraft can perform all three flights indefinitely, on the second day, third, etc. Remove flights 1, 2, 3 from the schedule and let us continue constructing good chains, but we have a complication here.

Consider 5, 6 in the upper part of Figure 1. They constitute a good chain departing from D but the last arrival is not in D , but in A . Formally, it is a good chain, but it is *not balanced*. Removing it from the schedule will lead to another decrease by 1 of the sum of maxima of the DFs, this time by the decrease of $fD(t)$ in the interval $[tD = 6:30, 17:00]$. One physical aircraft can perform the sequence of flights $4 \rightarrow 5 \rightarrow 6$ for one day only because this chain is not balanced. To repeat the flights on the next day we need to have the last arrival at the departure airport D . Let us change our tactics and look for a continuation of our chain to the *next day*. By looking for an early available departure from A , we will discover that there is another suitable chain of three flights. That is chain $7 \rightarrow 8 \rightarrow 9$ departing from A and arriving to D . Thus, if we join all six flights together, we will obtain one long chain

$4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8 \rightarrow 9$,

which has a period of $2T$ and is balanced. This means that all the six flights can be carried out by one physical vehicle periodically, indefinitely. To provide that all six flights will be performed each day, we will need two physical vehicles, i.e., two aircraft. The first one flies the infinite chain:

$4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8 \rightarrow 9 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8 \rightarrow 9 \rightarrow \dots$

and the second the same sequence with a shift:

$7 \rightarrow 8 \rightarrow 9 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8 \rightarrow 9 \rightarrow \dots$

The principal question which arises here is the following: is it always possible to find an appropriate continuation of a good chain on the next day? This is another good chain starting in the last arrival terminal of the first chain (i.e. from A , for our chain). The answer is yes. If there will be no such chain, then the number of arrivals to A will not be equal to the number of departures from A , which contradicts the fact that the schedule is balanced. We can imagine another complication: the good chain joining process will never end by arriving at the departure terminal at the beginning of the chain. But it also cannot happen because it violates the balance property.

Let us finish the chain construction procedure. After removing the second chain of period 2, we will be left with 8 flights, starting with 10. Let us again start constructing a good chain: it will be $10 \rightarrow 11 \rightarrow 12$. It starts with E and ends in C . Continuing for the next day, we find a chain of three flights $13 \rightarrow 14 \rightarrow 15$ beginning in C and ending not in E . Finally, we must proceed to the third day, and we obtain a proper continuation from E adding flights 16 and 17. The whole chain now ends at E , thus forming a three day periodic *balanced* chain:

$10 \rightarrow 11 \rightarrow 12 \rightarrow 13 \rightarrow 14 \rightarrow 15 \rightarrow 16 \rightarrow 17$.

To carry out each day all the above flights, we need three aircraft, each of them performing the same sequence of flights with a shift of one day. We have obtained, therefore, a chain decomposition of the schedule into three different chain types each of different period lengths requiring 6 aircraft in order to insure balanced chains. In summary our chain decomposition is:

One chain type with period 1 day (1T) for a single aircraft.

$[1 \rightarrow 2 \rightarrow 3]$.

One chain type with period 2 days (2T) needing 2 aircraft each with a day's shift.

$[4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8 \rightarrow 9]$,

$[7 \rightarrow 8 \rightarrow 9 \rightarrow 4 \rightarrow 5 \rightarrow 6]$.

One chain type with period 3T needing 3 aircraft each with a day's shift

$[10 \rightarrow 11 \rightarrow 12 \rightarrow 13 \rightarrow 14 \rightarrow 15 \rightarrow 16 \rightarrow 17]$,

$[13 \rightarrow 14 \rightarrow 15 \rightarrow 16 \rightarrow 17 \rightarrow 10 \rightarrow 11 \rightarrow 12]$.

$[16 \rightarrow 17 \rightarrow 10 \rightarrow 11 \rightarrow 12 \rightarrow 13 \rightarrow 14 \rightarrow 15]$.

In total, we have obtained the optimal chain decomposition for 6 aircraft, in accordance with Theorem 1.

In the next section we show how to provide crews to serve the flights organized into these six chains.

4. Chunks, Crews and Rosters

Needless to say that operating a passenger aircraft is a difficult and extremely responsible work. Therefore, special attention should be paid to keeping strictly the norms of work and rest for cockpit crews. One aircraft can operate 10-15 hours during one day, and even more, allowing all three flights of the aircraft chain $1 \rightarrow 2 \rightarrow 3$ to be assigned to one physical vehicle. The work of crews is different and we shall see that two crews are needed to serve these three flights. This is because a cockpit crew has a strict limit of the number of daily hours in the air and a guaranteed rest interval of 8-9 hours in each day for a night's sleep. In addition, each crew must have a limited working load in a week (e.g. 25-30 hours) and a guaranteed day or two off after 4-6 days of work. The problem of assigning crews to aircraft flights is known as the crew pairing problem. Often this problem is solved using mathematical programming. See (Salazar-Gonzalez, 2014), where this problem is solved using a heuristic based on an integer programming mode, a time consuming approach. Our approach is simpler using the notion of a *chunk*.

A chunk is a set of flights served continuously by one crew during one day. A crew can do in one day only one chunk, and must have a nights rest after doing the chunk. For example, flights 1 and 2 (see Fig.1) constitute one chunk. Operation of these flights takes about $4=2+2$ hours in air, two departures and two landings, which together is close to maximal daily load of a single crew. Figure 1 shows that the interval between the departure and arrival times for flights 1, 2 are 3 hours, i.e. 1 extra hour is included in each flight time for passenger discharging and aircraft preparation for the next flight (e.g., for refuelling). We assume that about a half of this extra time needs presence and participation of the crews.

Flight 3 is also a separate chunk and thus the chain $1 \rightarrow 2 \rightarrow 3$ contains two chunks. We will denote a chunk by $U..V$, with two letters denoting the first departure and the last arrival terminals, respectively. So, the chain $1 \rightarrow 2 \rightarrow 3$ can be represented as sequence of two chunks:

$$A..D \rightarrow D..A (*)$$

Obviously, two crews are needed to serve this chain. Table 1 shows the crew roster for serving the flights of this chain during 7 consecutive days. Both chunks are not served by the crews on the seventh day.

Table 1. Crew roster for flights 1, 2, 3 for 7 consecutive days

Crew	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Crew1	$A..D$	$D..A$	$A..D$	$D..A$	$A..D$	$D..A$	<i>Off</i>
Crew2	$D..A$	$A..D$	$D..A$	$A..D$	$D..A$	$A..D$	<i>Off</i>

We see from Table 1 that both crews have a full load during six working days, and the location of each crew at the end of the working day (and the location of the crew night rest) is the departure airport for the next chunk. So, Crew 1 ends its first day at airport D , has a nights rest in the same location and starts the next day by operating the flight departing from D .

It can be seen that the crew assignment and their working load is optimal in the sense that after a nights rest all crews are already located in the airports in which the departure for the next day takes place. This is true except for the off days where the crew nomination is a special issue. We will discuss the crew assignment for the off days later when we will have several rosters with more crews operating in parallel.

Determining the crew assignment for the chain (*) above is almost trivial. Let us consider a less trivial example where the advantages of considering crews rostering in connection with minimal flight chains will be more impressive. Let us consider the second chain having period $2T$. It has flights 4, 5, 6, 7, 8, 9. We define two chunks for each operation day: flight 4 and 5 will be one chunk denoted as $D..B$, and flight 6 will be the second chunk denoted by $B..A$. For the second day, we define the third chunk $A..D$ for flight 7 and the fourth chunk will be $D..D$ for flights 8 and 9. For sake of convenience and for better distinction between the chunks, we replace the letter D in the flight 7 and the letter D on flight 8 by X , so now we will have four chunks $D..B$, $B..A$, $A..X$, $X..D$ which each of two aircrafts flies sequentially and periodically:

$$D..B \rightarrow B..A \rightarrow A..X \rightarrow X..D \rightarrow D..B \rightarrow B..A \rightarrow A..X \rightarrow X..D \rightarrow \dots$$

The second aircraft repeats the same sequence with one day shift, i.e. it does the sequence

$$A..X \rightarrow X..D \rightarrow D..B \rightarrow B..A \rightarrow A..X \rightarrow X..D \rightarrow \dots$$

To serve four chunks we need four crews. Table 2 shows the flight sequences for two aircraft $A1$ and $A2$ and the corresponding rosters for four crews $C1, C2, C3, C4$. The row $C1$ shows the roster for the Crew 1. On the first day it flies chunk $D..B$, arrives at airport B , and has a night rest (in a nearby hotel). The next morning Crew 1 is ready to pick up the chunk $B..A$ from the aircraft 2 (see the sequence for $A2$ in Table 2.) In a miraculous way the crew location always coincides with the aircraft available and therefore the rosters above provide 100% load for all four crews during the six day period.

Table 2. Crew roster for flights 4-9, for 6 consecutive days

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6
$A1$	$D..B; B..A$	$A..X; X..D$	$D..B; B..A$	$A..X; X..D$	$D..B; B..A$	$A..X; X..D$
$A2$	$A..X; X..D$	$D..B; B..A$	$A..X; X..D$	$D..B; B..A$	$A..X; X..D$	$D..B; B..A$
$C1$	$D..B$	$B..A$	$A..X$	$X..D$	$D..B$	$B..A$
$C2$	$B..A$	$A..X$	$X..D$	$D..B$	$B..A$	$A..X$
$C3$	$A..X$	$X..D$	$D..B$	$B..A$	$A..X$	$X..D$
$C4$	$X..D$	$D..B$	$B..A$	$A..X$	$X..D$	$D..B$

5. More on Rosters and Managing the Off Days

5.1. Rosters

Let us describe in general terms our procedure for constructing crew rosters. We will illustrate this by considering a periodic chain with period of $3T$ having 6 chunks. This is the set of flights 10 – 17 on Figure 1. We will define for this chain 6 chunks: $E..A$ for flights 10,11, $A..C$ for flight 12, $C..B$ for flights 13,14, $B..A$ for flight 15, $A..B$ for flight 16 and $B..E$ for flight 17.

Step 1.

Construct the sequence of flights for 6 days in a form of the chunk sequence for each of three aircraft. In our example, for the first aircraft we have the following sequence:

$$E..A, A..C \rightarrow C..B, B..A \rightarrow A..B, B..E \rightarrow E..A, A..C \rightarrow C..B, B..A \rightarrow A..B, B..E;$$

For the second aircraft we have the same sequence shifted by two chunks:

$$C..B, B..A \rightarrow A..B, B..E \rightarrow E..A, A..C \rightarrow C..B, B..A \rightarrow A..B, B..E \rightarrow E..A, A..C;$$

For the third aircraft we have:

$$A..B, B..E \rightarrow E..A, A..C \rightarrow C..B, B..A \rightarrow A..B, B..E \rightarrow E..A, A..C \rightarrow C..B, B..A.$$

The days are separated by arrows. We see therefore that each day all six chunks are carried out. Now we have to "equip" them with crews. In total, six crews are needed.

Step 2.

For each crew, pick up any chunk of the 6 available in the first day and pair it to this crew. Let it be chunk $X..Y$. Look up which chunks follows $X..Y$ in the flight sequence. Let it be chunk $Y..Z$ carried out on the next day. So, we have $X..Y \rightarrow Y..Z$. Join to it chunk $Z..W$ from the chunks available on the third day to obtain the chain $X..Y \rightarrow Y..Z \rightarrow Z..W$, and so on, until day 6 is attained. Repeat (1) for the next crew, and so on until all chunks will be manned. Table 3 shows how this works for our example.

Summing up, we have obtained a six crew schedule with full load for each crew for six working days, with a night sleep for each day and with each crew having its work continued on the next day at the same location of its arrival on the previous day. Looking at the rosters in Tables 1-4 we can formulate the following conjecture:

Conjecture: Given an aircraft schedule for a balanced chain containing M chunks, we need M crews. Each crew serves a periodic sequence of chunks with period M such that a balanced crew roster can be

constructed, where a balanced crew roster is defined as a crew roster in which the (time) average work load is distributed evenly among all crews.

Table 3. Weekly crew roster for flights 10-17

Crews	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
1	<i>E..A</i>	<i>A..C</i>	<i>C..B</i>	<i>B..A</i>	<i>A..B</i>	<i>B..E</i>	<i>Off</i>
2	<i>A..C</i>	<i>C..B</i>	<i>B..A</i>	<i>A..B</i>	<i>B..E</i>	<i>E..A</i>	<i>Off</i>
3	<i>C..B</i>	<i>B..A</i>	<i>A..B</i>	<i>B..E</i>	<i>E..A</i>	<i>A..C</i>	<i>Off</i>
4	<i>B..A</i>	<i>A..B</i>	<i>B..E</i>	<i>E..A</i>	<i>A..C</i>	<i>C..B</i>	<i>Off</i>
5	<i>A..B</i>	<i>B..E</i>	<i>E..A</i>	<i>A..C</i>	<i>C..B</i>	<i>B..A</i>	<i>Off</i>
6	<i>B..E</i>	<i>E..A</i>	<i>A..C</i>	<i>C..B</i>	<i>B..A</i>	<i>A..B</i>	<i>Off</i>

For example, suppose that one chain has chunks with duration's t_1 , t_2 , t_3 , equal to 3, 7, and 5 hours, respectively. Then 3 crews are needed to serve these chunks. Each crew is "circulating" along the chain, i.e., each particular crew is paired sequentially with a 3 hour chunk, afterward with 7 hour chunk and then with 5 hour chunk. So, all crews are equal with respect to their load. That is, each crew works 15 hours in 3 days, so each crew works 5 hours/day on average.

Note, that the above rosters can be trivially modified to a five-day week with two following off days and/or four-day week followed by two days off.

5.2 Managing the off days

There is no easy combinatorial solution to planning crew work during the pilot's off days. Let us go back to the schedule of 17 flights shown on Figure 1. We showed that we need 12 crews in total, each day because in total we have to serve 12 chunks each day. The simplest and quite admissible solution would be to have for the flights of each chain one or two weekly days off. It would be in accord with the national tradition to have one day off each week. If this is not acceptable, the difficulty in cockpit crew planning could be reduced by spreading the off days in time. For example, consider the flights for the third chain (flights 10, 11, 12,..., 17). Suppose that we plan four days' work, followed periodically by two days off. Table 4 shows a principal roster for organizing work for two groups of six crews on the basis of four days work +2 days off.

Table 4. Principal roster for 2 groups of six crews with 4 work/2 off days

Crews	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Mon
1 – 6	<i>W 11</i>	<i>W 12</i>	<i>W 13</i>	<i>W 14</i>	<i>O 11</i>	<i>O 12</i>	<i>W 11</i>	<i>W 12</i>
7 – 12	<i>Off</i>	<i>Off</i>	<i>W 21</i>	<i>W 22</i>	<i>W 23</i>	<i>W 24</i>	<i>O 21</i>	<i>O 22</i>

We see that in parallel to crews 1-6 we plan to have another 6 crews serving flights, which are not shown in the schedule on Figure 1. As it can be seen from Table 4, in the O11 off day, the departures take place from airports of the set $S_1 = (A, B, C, E)$. Note that all six off-day crews return, after serving their chunks, to the same set S_1 of airports. Crews 7-12, as seen from Table 4, have their off days shifted by two days, to on Sunday and Monday. It is expected that the 6 off-day crews working on Friday and Saturday will continue serving the flights of the second group, and work on Sunday and Monday. So, it seems that we found an ideal roster, which provides full four working days in a row for six off-day crews.

There might, however, be a complication. Suppose that the aircrafts on Sunday have departures from the airports of set $S_2 = (A, B, D, F)$. Therefore, two crews, arriving at C and E , should change their location and move to D and F , respectively, which might be a certain extra load for them. This would be the price paid for balancing the work during the off days. The solution to this problem will depend on local conditions. For example, the crews need to use trains or flights carried out by other aircraft types or companies to fly from C , E to D , F .

6. Conclusion and Future Work

In this paper we have addressed the problem of aircraft and crew scheduling. Our aim is to arrange a set of aircraft flights emanating from a set of airport terminals into sequences of flights, called chains,

each serviced by a single aircraft. For this we employ the theory of deficit functions which affords, using our methodology, to decompose the set of flights into chains, but also to insure that these chains are "balanced" in the sense that each aircraft starts and ends its flights at the same airport terminal. We also introduce the notion of "chunks" which are a sequence of flights serviced by a crew in one day according to contract regulations. These chunks are then used to provide crew schedules and rosters. The method provides a simplicity for the construction of aircraft schedules and crew pairings which is absent in other approaches to the problem. Future work includes addressing the problem of crews always being present at the proper aircraft terminal to continue their roster schedule without the need to relocate using other means of transportation.

References

1. Ceder, A. and Stern, H. (1981) Deficit function bus scheduling with deadheading trip insertions for fleet size reduction. *Transportation Science*, 15(4), 338-363.
2. Ceder, A. and Stern, H. (1985) The variable trip procedure used in the AUTOBUS vehicle scheduler. In: Rousseau, J.-M. (Ed.), *Computer Scheduling of Public Transport 2*. Elsevier Science, Publisher B.V., North-Holland, 371-390.
3. Day, P. R. and Ryan, D. M. (1997) Flight attendant rostering for short-haul operations. *Operations Research*, 45(5), 649-661.
4. Gertsbakh, I. and Gurevich, Y. (1977) Constructing an optimal fleet for a transportation schedule. *Transportation Science*, 11(1), 20-36.
5. Gertsbakh, I. and Gurevich, Y. (1982) Homogeneous optimal fleet. *Transportation Research Part B: Methodological*, 16(6), 459-470.
6. Gertsbakh, I. and Stern, H. (1978) Minimal resources for fixed and variable job schedules. *Operations Research*, 26 (1), 68-85.
7. Linis, V. K. and Maksim, M. S. (1967) On the problem of constructing routes. (in Russian). *Proceedings of Civil Aviation Institute, Riga*, 102, 36-45.
8. Salazar-Gonzalez, J. (2014) Approaches to solve the fleet-assignment, aircraft routing, crew-pairing and crew-rostering problems of a regional carrier. *Omega*, 42, 71-82.