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NEGATION AND PRESUPPOSITION, TRUTH AND FALSITY

Abstract. There are many kinds of negation and denial. Perhaps the most common is the Boolean negation *not* that applies to propositions-in-extension, i.e. *truth-values*. The others are, inter alia, the *property of propositions* of not being true which applies to propositions; the *complement function* which applies to *sets*; *privation* which applies to *properties*; *negation as failure* applied in logic programming; negation as *argumentation ad absurdum*, and many others. The goal of this paper is neither to provide a complete list, nor to analyse all of them. Rather, I am going to deal with negation of propositions that come attached with a presupposition that is entailed by the positive as well as negated form of a given proposition. However, there are two kinds of negation, namely internal and external negation. I am going to prove that while the former is presupposition-preserving, the latter is presupposition-denying. This issue has much in common with the difference between topic and focus articulation within a sentence. Whereas articulating the topic of a sentence activates a presupposition, articulating the focus frequently yields merely an entailment. The main contribution of this paper is the proof that the two kinds of negation are not equivalent. While the Russellian wide-scope (external) negation gets the truth-conditions of a sentence right for a subject occurring as a focus, Strawsonian narrow-scope (internal) negation is validly applicable for a subject occurring as the topic. I also deal with other kinds of presupposition triggers, in particular factive attitudes and prerequisites of a given property. My background theory is Transparent Intensional Logic (TIL).¹ TIL is an expressive logic apt for the analysis of sentences with presuppositions, because in TIL we work with partial functions, in particular with propositions with truth-value gaps. Moreover, the procedural semantics of TIL make it possible to uncover the hidden semantic features of sentences, make them explicit and logically tractable.

Keywords: negation of propositions, presupposition, wide-scope vs. narrow-scope negation, topic-focus articulation, truth-value gaps, requisites and prerequisites, Transparent Intensional Logic, if-then-else-fail function.

1. Introduction

There are many kinds of negation and denial. In this paper, I am not going to deal with the complement function, nor with negation as failure

or argumentation *ad absurdum*. Instead, I am going to deal with negation of propositions, or rather affirmative and denying forms of sentences denoting propositions that come attached with a presupposition.² Negation of propositions is often semantically restricted to contradictory opposition between propositions, in which *not A* is to be understood as “it is not the case that *A*”.³ However, negation of propositions often includes contrariety rather than contradiction:

As introduced in Aristotle’s *Categories* (11b17), the genus of opposition (*apophasis*) is divided into species that include contrariety and contradiction. Contradictory opposites, whether affirmative and negative counterparts of a singular predication (*Socrates is wise/Socrates isn’t wise*) or quantified expressions (*All pleasure is good/Some pleasure is not good*), are mutually exhaustive as well as mutually exclusive, while contrary opposites (*Socrates is wise/Socrates is unwise; All pleasure is good/No pleasure is good*) do not mutually exhaust their domain. Contraries cannot be simultaneously true, though they may be simultaneously false. Members of a contradictory pair cannot be true **or** false simultaneously; contradictories “divide the true and the false between them”. Contrary terms (*enantia*) come in two varieties (*Cat.* 11b38ff.). In immediate or logical contraries (*odd/even, sick/well*), a true middle—an entity satisfying the range of the two opposed terms but falling under neither of them—is excluded, e.g., an integer neither odd nor even. But mediate contrary pairs (*black/white, good/bad*) allow for a middle—a shirt between black and white, a man or an act neither good nor bad. Neither mediate nor immediate contraries fall under the Law of Excluded Middle [LEM] (*tertium non datur*).

Horn, Wansing (2017, §1.4)

Yet a similar phenomenon arises also in the case of *privative modifiers* of *properties*. The problem of privation of properties has been dealt with by Jespersen, Carrara and Duží (2017) where the authors show that modifiers privative with respect to a property *P* turn the root property *P* to the property [*M P*] that is *contrary* rather than *contradictory* with respect to *P*. To this end they apply the method of *intensional essentialism*, which operates on properties (intensions) rather than their extensions. The authors argue that each property *P* is necessarily associated with an essence, which is the set of the so-called *requisites* of *P* that jointly define *P*. Privation deprives *P* of *some but not all* of its requisites, replacing them by their contradictories. Thus, the standard rule of *single privative modification* that replaces privative modifiers by Boolean negation is valid, for sure, but also too simplistic. If an individual *a* instantiates the privatively modified property [*M P*] then it is true that *a* is *not* a *P*, but the rule fails to express the fact that the properties [*M P*] and *P* have something in common. For instance,

a *forged banknote* is not a banknote, yet it still has much in common with banknotes. A forged banknote is not just some object or other that fails to be a banknote, but rather it is an object that must have a host of properties in common with banknotes. Though forged banknotes and, say, horses and lions are not banknotes, there is a sense in which forged banknotes are much ‘closer’ to banknotes than are horses and lions. As mentioned above, the authors solve the problem by means of requisites assigned to each property P as its *essence* and define privation along these lines.

A modifier M is *privative* with respect to a property P iff the modified property $[M P]$ lacks at least one, but *not all*, of the requisites of the property P . Moreover, the essence of $[M P]$ contains at least one other requisite that does not belong to the essence of P , and *contradicts* at least one of the requisites of P . As a result, M is privative with respect to P iff the essence of $[M P]$ has a non-empty intersection with the essence of P , and this intersection is a *proper* subset of both the essences of P and of $[M P]$. (ibid.)

Thus, there is a difference between seemingly same propositions as “ a is *not* a banknote” and “ a is a *non*-banknote”, where ‘*non*’ stands for a privative modifier. While the law of excluded middle holds for the former, it fails in the case of the latter. For sure, a cannot be both a banknote and a *non*-banknote, but it can be the case that a is neither a banknote nor a *non*-banknote.

The law of excluded middle fails also in the case of sentences denoting propositions that come attached with a *presupposition*. A presupposition P of a proposition S is entailed both by the positive form of S and its negated form ‘*non*- S ’. If the presupposition P is not the case, the proposition S is neither true nor false. The problem I am going to deal with is the ambiguity of ‘*non*- S ’, to wit the external (wide-scope) and internal (narrow-scope) application of ‘*non*’, and the closely related issue of topic-focus articulation of sentences.

The rest of this paper is organised as follows. Section 2 summarises the issues connected with narrow-scope vs. wide-scope negation, and a long-term dispute on Russellian vs. Strawsonian analysis of sentences with the schematic form “The F is a G ”. The solution to this dilemma based on different topic-focus articulations of sentences is then proposed in Section 4, coming after Section 3, where the relevant foundations of my background theory TIL are introduced. In Section 4 the difference between narrow-scope and wide-scope negation is rigorously defined by applying TIL analysis. Using the difference between these two kinds of negation, I also differentiate presupposition from mere entailment. In Section 5 I deal with ambiguities

stemming from topic-focus articulation of a sentence in general and propose a general analytic schema for such sentences. Finally, in Section 6 I deal with other cases where a presupposition can be generated, for instance, by factive attitudes like ‘knowing’ or by prerequisites of a given property that is applied to the subject of a sentence. Finally, concluding remarks can be found in Section 7.

2. Negation, presupposition, and the excluded middle

The law of the excluded middle fails in the case of sentences that come attached with a *presupposition*. In his classic paper on sense and reference, Frege (1892) argues that both (a) and its contradictory (b) presuppose that the name *Kepler* has a denotation.⁴

- (a) Kepler died in misery.
- (b) Kepler did not die in misery.

Every affirmative or negative sentence with a singular description ‘*SD*’ presupposes the existence of a unique referent of ‘*SD*’. According to Frege, if the presupposition fails, no assertion is made, but this presupposition is not a part of the content of the sentence. Hence (a) does not entail existence, or the negation of (a) would not be (b) but “*Kepler did not die in misery or the name ‘Kepler’ has no reference*”, an outcome Frege seems to have taken as an absurdity. Yet this outcome foreshadows the need to deal with sentences that are neither true nor false due to the fact that their presupposition is not true.

Russell (1905) was not willing to tolerate the truth-value gaps incurred on Frege’s analysis, and reconsidered the status of contradictory negation with vacuous subjects:

By the law of the excluded middle, either “*A is B*” or “*A is not B*” must be true. Hence either “the present king of France is bald” or “the present king of France is not bald” must be true. Yet if we enumerated the things that are bald and the things that are not bald, we should not find the king of France on either list. Hegelians, who love a synthesis, will probably conclude that he wears a wig. (ibid. 485)

To resolve this ostensible paradox while preserving a classical analysis in which every meaningful sentence is true or false, Russell banishes singular terms like *the king of France* from logical form. He claims that we have to uncover the real logical form of such sentences that is otherwise hidden.

Though it seems that the form of (c) and (d) is the form of subject-predicate sentences, we have to unpack them as existentially quantified sentences.

(c) The king of France is bald.

(d) The king of France is not bald.

On Russell's theory of descriptions, the logical form of (c) is

(c') $\exists x(Kx \wedge \forall y(Ky \supset y = x) \wedge Bx)$

which is the (false) proposition that there is an entity with the property of being king of France, and this entity is unique, and it is bald.

However, the negated sentence (d) is ambiguous, depending on the scope of negation, which yields two non-equivalent logical forms of (d):

(d') $\exists x(Kx \wedge \forall y(Ky \supset y = x) \wedge \neg Bx)$

(d'') $\neg \exists x(Kx \wedge \forall y(Ky \supset y = x) \wedge Bx)$

The former, (d'), with *narrow-scope* (internal) negation, is the proposition that there is a unique and hairy king of France, which is simply false in the absence (or oversupply) of male French monarchs. Hence, for Russell (d') cannot be the right logical form of the negation of (c), because we would end up with the result that both (c) and its negation are false, which contradicts the principle of the excluded middle.

On the other hand, in (d'') the *wide-scope* (external) negation is applied. The description *the king of France* falls within the scope of the external negation and yields a true proposition. Indeed, the non-existence of a king of France guarantees the truth of (d''). Hence Russell in the effort of following the principle of the excluded middle must admit that (d'') is the right logical form of the negation of (c). Furthermore, (d''), unlike (d') fails to entail the existence of the king of France. Horn (1989, p. 107) clarifies this reading as

(d_r) The king of France isn't bald, because there isn't any king of France.

Though Russell's quantificational theory remains to this day a strong rival of referential theories, it has received its fair amount of criticism. Russell's opponents claim that he simply gets the truth-conditions wrong in important cases of using descriptions when there is no such thing as the unique *F*.⁵

This criticism was launched by Strawson who in (1950) objected that Russell's theory predicts the wrong truth-conditions for sentences like "The present King of France is bald". For Strawson, negation normally or invariably leaves the subject "unimpaired". Strawson tacitly lines up with Frege and against Russell (and Aristotle) in regarding negative statements

like (b) and (d) as unambiguous and necessarily *presuppositional*. For, if the sentence (c) were false then its negation, “The King of France is *not* bald”, would be true, which entails that there *is* a unique King of France, contrary to the assumption that there is none. Hence someone who utters (d) does not thereby assert (nor does their statement entail) that there is a king of France. Rather, (d)—along with its affirmative counterpart (c)—*presupposes* it. If this presupposition fails, neither (c) nor (d) can be judged true or false. A statement may be made but the question of its truth value fails to arise.⁶

Russell was very upset by Strawson’s criticism. In response to this criticism, he argues that, despite Strawson’s protests, the sentence is in fact false:⁷

Suppose, for example, that in some country there was a law that no person could hold public office if he considered it false that the Ruler of the Universe is wise. I think an avowed atheist who took advantage of Mr. Strawson’s doctrine to say that he did not hold this proposition false would be regarded as a somewhat shifty character. (Russell, 1957)

Donnellan (1966) observes that there is a sense in which Strawson and Russell are both right, and both wrong, about the proper analysis of definite descriptions, because definite descriptions can be used in two different ways. On a so-called *attributive use*, a sentence of the form “The *F* is a *G*” is used to express a proposition equivalent to “Whatever is uniquely *F* is a *G*”. Alternatively, on a *referential use*, a sentence of the form “The *F* is a *G*” is used to pick out a specific individual, *a*, and to say of *a* that it is a *G*. Donnellan suggests that Russell’s quantificational account of definite descriptions might capture attributive uses, but that it does not work for referential uses. Ludlow in (2007) interprets Donnellan as arguing that in some cases descriptions are Russellian and in other cases Strawsonian. Kripke (1977) responds to Donnellan by arguing that the Russellian account of definite descriptions can, by itself, account for both referential and attributive uses, and that the difference between the two cases is entirely a matter of pragmatics.

Neale (1990) supports Russell’s view by collecting a number of cases in which intuitions about truth conditions clearly do not support Strawson’s view. On the other hand, a number of linguists have recently come to Strawson’s defence on this matter. See Ludlow (2007) for a detailed survey of the arguments supporting Strawson’s view and arguments supporting Russell’s. Here it might suffice to point out that Strawson’s concerns have not delivered a knock-out blow to Russell’s theory of descriptions, and so this topic

remains very much alive. For instance, recently Von Fintel (2004) argues that every sentence containing a definite description ‘the *F*’ comes with the existential presupposition that there be a unique *F*.

In this paper I am disregarding Donnellan’s troublesome pragmatic notion of having somebody in mind. Instead, I will propose a *logical analysis* of sentences of the form “The *F* is a *G*”. What I want to show is this. First, definite descriptions are not deprived of a self-contained meaning and they denote one and the same entity in any context. Thus, they are never Russellian. Second, Russell’s insight that a definite description ‘the *F*’ does not denote a definite individual is spot-on. According to TIL, ‘the *F*’ denotes a *condition* to be contingently satisfied by the individual (if any) that happens to be the *F*. I will explicate such conditions in terms of possible-world intensions, viz. as *individual roles* or *offices* to be occupied by at most one individual per world/time pair. Third, I am going to show that Donnellan is right in holding that sentences of the form “The *F* is a *G*” are systematically ambiguous. However, their ambiguity does not concern a shift of meaning of the definite description ‘the *F*’, as Fregean or other theories maintain. Instead the ambiguity concerns different *topic-focus* articulations of these sentences.

There are two options. The description ‘the *F*’ may occur as the topic of a sentence and property *G* (the focus) is predicated of its referent. This case corresponds to Donnellan’s *referential* use. Using medieval terminology, I will say that ‘the *F*’ occurs with *de re supposition*. The other option is ‘*G*’ occurring as topic and ‘the *F*’ as focus. This reading corresponds to Donnellan’s *attributive* use of ‘the *F*’ and the description occurs with *de dicto* supposition. Consequently, and crucially, such sentences are ambiguous between a *de dicto* and a *de re* reading. On their *de re* reading they *presuppose* the existence of a unique *F*. Strawson’s analysis appears to be adequate for *de re* cases. On their *de dicto* reading they have the truth-conditions as specified by the Russellian analysis. They do not presuppose, but merely entail, the existence of a unique *F*. However, the Russellian analysis, though being equivalent to the one I am going to propose, is not an adequate *literal* analysis of *de dicto* readings.

Furthermore, I am going to bring out the *semantic* nature of the topic-focus difference by means of a literal logical analysis. As a result, I furnish sentences differing only as for their topic-focus articulation with different structured meanings producing different possible-world propositions.⁸ Since our logic is a hyperintensional logic of *partial functions*, I am able to analyse sentences with presuppositions in a both natural and principled manner. It means that I associate them with hyperpropositions, which in TIL are

abstract logical procedures that produce partial possible-world propositions, which occasionally yield truth-value gaps.⁹

We need to work with properly partial functions and propositions with truth-value gaps despite technical difficulties connected with this issue. On a Strawsonian reading, the sentence “The King of France is bald” talks about the office of the King of France (topic) ascribing to the individual (if any) that occupies this office the property of being bald (focus). Thus, it is presupposed that the King of France exists, i.e., that the office is occupied. If the office is vacant the proposition denoted by the sentence lacks a truth-value. On our approach this does not mean that the sentence is meaningless. The sentence has a sense, namely an instruction how in any possible world w at any time t to execute the *procedure* (hyperproposition) of evaluating its truth-conditions. Only if we evaluate these conditions in such a state-of-affairs where the office of the King of France goes vacant does the process of evaluation yield a truth-value gap.

This issue has much in common with the difference between topic and focus articulation within a sentence. As many linguists argue, whereas articulating the topic of a sentence activates a presupposition, articulating its focus frequently yields merely an entailment.¹⁰ The point of departure is that sentences of the form “The F is a G ” are ambiguous. Their ambiguity stems from the different topic-focus articulations of such sentences. The issue is this. If ‘the F ’ is the topic phrase, then this description occurs extensionally, i.e. with *de re* supposition, and Strawsonian analysis appears to be what is wanted. On this reading the sentence presupposes the existence of the descriptum of ‘the F ’, because the property G is ascribed to the object, if any, referred to by ‘the F ’. The other option is ‘ G ’ occurring as topic and ‘the F ’ as focus. This reading roughly corresponds to Donnellan’s attributive use of ‘the F ’ and the description occurs intensionally with *de dicto* supposition. On this reading the Russellian analysis gets the truth-conditions of the sentence right. The existence of a unique F is merely entailed.

Summarising, the two readings differ also in the way their respective negated form is obtained. Whereas the Strawsonian narrow-scope negated form is “The F is not a G ”, the Russellian wide-scope negated form would be “It is not true that the F is a G ”. Thus, in the former case the property of not being a G is ascribed to the object, if any, that is referred to by the *topic phrase* ‘the F ’. On the other hand, in the Russellian case the property of not being true is ascribed to the whole proposition that the F is a G .¹¹ I am going to prove that these two readings are not equivalent, because they denote different propositions (truth-conditions individuated up to logical equivalence). While “The F is not a G ” lacks a truth-value at those states

of affairs where the F does not exist, the wide-scope negation “It is not true that the F is a G ” is true at such states of affairs where there is no F .

3. Foundations of TIL

The terms of the TIL language denote abstract procedures (roughly, Church’s functions-in-intension) that produce set-theoretical mappings (functions-in-extension).¹² These procedures are rigorously defined as TIL *constructions*.

Definition 1 (*construction*)

- (i) Variables $x, y, \dots$ are *constructions* that *construct* objects (elements of their respective ranges) dependently on a valuation v ; they *v-construct*.
- (ii) Where X is any object whatsoever (even a *construction*), 0X is the *construction Trivialization* that *constructs* X without any change of X .
- (iii) Let $X, Y_1, \dots, Y_n$ be arbitrary *constructions*. Then *Composition* $[XY_1 \dots Y_n]$ is the following *construction*. For any v , the *Composition* $[XY_1 \dots Y_n]$ is *v-improper* if at least one of the *constructions* $X, Y_1, \dots, Y_n$ is *v-improper* by failing to *v-construct* anything, or if X does not *v-construct* a function that is defined at the n -tuple of objects *v-constructed* by $Y_1, \dots, Y_n$. If X does *v-construct* such a function, then $[XY_1 \dots Y_n]$ *v-constructs* the value of this function at the n -tuple.
- (iv) $(\lambda -)$ *Closure* $[\lambda x_1 \dots x_m Y]$ is the following *construction*. Let $x_1, x_2, \dots, x_m$ be pair-wise distinct variables and Y a *construction*. Then $[\lambda x_1 \dots x_m Y]$ *v-constructs* the function f that takes any members $B_1, \dots, B_m$ of the respective ranges of the variables $x_1, \dots, x_m$ into the object (if any) that is $v(B_1/x_1, \dots, B_m/x_m)$ -*constructed* by Y , where $v(B_1/x_1, \dots, B_m/x_m)$ is like v except for assigning B_1 to $x_1, \dots, B_m$ to x_m .
- (v) Where X is any object whatsoever, 1X is the *construction Single Execution* that *v-constructs* what X *v-constructs*. Thus, if X is a *v-improper construction* or not a *construction* as all, 1X is *v-improper*.
- (vi) Where X is any object whatsoever, 2X is the *construction Double Execution*. If X is not itself a *construction*, or if X does not *v-construct* a *construction*, or if X *v-constructs* a *v-improper construction*, then 2X is *v-improper*. Otherwise 2X *v-constructs* what is *v-constructed* by the *construction v-constructed* by X .
- (vii) Nothing is a *construction*, unless it so follows from (i) through (vi).

□

Comments. Being procedural objects, constructions can be executed in order to operate on input objects (of a lower-order type) and produce the object (if any) they are typed to produce, while non-procedural objects, i.e. non-constructions, cannot be executed. Hence the constituents of constructions cannot be non-procedural objects; non-procedural objects must be presented, or referred to, by atomic constructions. *Trivialization* and *Variables* are the two atomic constructions that present input objects (which can also be lower-order constructions) to be operated on. The operational sense of Trivialization is similar to that of constants in formal languages. A Trivialization presents an object X without the mediation of any other procedures. Using the terminology of programming languages, the Trivialization of X , ‘ 0X ’ in symbols, is just a *pointer* referring to X . Variables produce objects dependently on valuations; they *v-construct*. We adopt an objectual variant of the Tarskian conception of variables. To each type (see Def. 2) are assigned countably many variables that range over this particular type. Objects of each type can be arranged into infinitely many sequences. The valuation v selects one such sequence of objects of the respective type, and the first variable v -constructs the first object of the sequence, the second variable v -constructs the second object of the sequence, and so on. Hence the execution of a Trivialization or a variable never fails to produce an object; these constructions are not *v-improper* for any valuation v . The $(\lambda-)$ Closure $[\lambda x_1 \dots x_m Y]$ is also not *v-improper* for any v , as it always v -constructs a function. Even if the constituent Y is *v-improper* for every valuation v , the Closure is not *v-improper*. Yet in such a case the constructed function is a bizarre object; it is a degenerate function that lacks a value at any argument. However, the other molecular constructions, namely Composition, Single and Double Execution, can fail to present an object of the type they are typed to produce; they can be *v-improper*. The main source of improperness is an application of a function to an argument at which the function is not defined.¹³

With constructions of constructions, constructions of functions, functions, and functional values in our stratified ontology, we need to keep track of the traffic between multiple logical strata. The *ramified type hierarchy* does just that. The type of first-order objects includes all non-procedural objects. Therefore, it includes not only the standard objects of individuals, truth-values, sets, functions, etc., but also functions defined on possible worlds (i.e., the intensions germane to possible-world semantics). The type of second-order objects includes constructions of first-order objects and functions with such constructions in their domain or range. The type of third-order objects includes constructions of first- and second-order objects

and functions with such constructions in their domain or range; and so on, ad infinitum.

Definition 2 (*ramified hierarchy of types*).

Let B be a *base*, where a base is a collection of pair-wise disjoint, non-empty sets. Then:

$\mathbf{T}_1$ (*types of order 1*).

- i) Every member of B is an elementary *type of order 1 over B*.
- ii) Let $\alpha, \beta_1, \dots, \beta_m$ ($m > 0$) be types of order 1 over B . Then the collection $(\alpha\beta_1 \dots \beta_m)$ of all m -ary partial mappings from $\beta_1 \times \dots \times \beta_m$ into α is a functional *type of order 1 over B*.
- iii) Nothing is a *type of order 1 over B* unless it so follows from (i) and (ii).

$\mathbf{C}_n$ (*constructions of order n*)

- i) Let x be a variable ranging over a type of order n . Then x is a *construction of order n over B*.
- ii) Let X be a member of a type of order n . Then ${}^0X, {}^1X, {}^2X$ are *constructions of order n over B*.
- iii) Let $X, X_1, \dots, X_m$ ($m > 0$) be constructions of order n over B . Then $[XX_1 \dots X_m]$ is a *construction of order n over B*.
- iv) Let $x_1, \dots, x_m, X$ ($m > 0$) be constructions of order n over B . Then $[\lambda x_1 \dots x_m X]$ is a *construction of order n over B*.
- v) Nothing is a *construction of order n over B* unless it so follows from $\mathbf{C}_n$ (i)–(iv).

$\mathbf{T}_{n+1}$ (*types of order n + 1*)

Let $*_n$ be the collection of all constructions of order n over B . Then

- i) $*_n$ and every type of order n are *types of order n + 1*.
- ii) If $m > 0$ and $\alpha, \beta_1, \dots, \beta_m$ are types of order $n + 1$ over B , then $(\alpha\beta_1 \dots \beta_m)$ (see $\mathbf{T}_1$ (ii)) is a *type of order n + 1 over B*.
- iii) Nothing is a *type of order n + 1 over B* unless it so follows from (i) and (ii). □

For the purposes of natural language analysis, we are assuming the following base of ground types:

- o : the set of truth-values $\{\mathbf{T}, \mathbf{F}\}$;
- ι : the set of individuals (the universe of discourse);
- τ : the set of real numbers (doubling as times);
- ω : the set of logically possible worlds (the logical space).

We model sets and relations by their characteristic functions. Thus, for instance, $(o\iota)$ is the type of a set of individuals, while $(o\iota\iota)$ is the type

of a relation-in-extension between individuals. Empirical expressions denote *empirical conditions* that may or may not be satisfied at the particular world/time pair of evaluation. These empirical conditions are modelled as possible-world-semantic (*PWS*) *intensions*. PWS intensions are entities of type $(\beta\omega)$: mappings from possible worlds to an arbitrary type β . The type β is frequently the type of the *chronology* of α -objects, i.e., a mapping of type $(\alpha\tau)$. Thus α -intensions are frequently functions of type $((\alpha\tau)\omega)$, abbreviated as ' $\alpha_{\tau\omega}$ '. *Extensional entities* are entities of a type α where $\alpha \neq (\beta\omega)$ for any type β . Where w ranges over ω and t over τ , the following logical form essentially characterizes the logical syntax of empirical language: $\lambda w \lambda t [\dots w \dots t \dots]$.

Examples of frequently used PWS intensions are: *propositions* of type $o_{\tau\omega}$, *properties of individuals* of type $(oi)_{\tau\omega}$, *binary relations-in-intension* between individuals of type $(ou)_{\tau\omega}$, *individual offices* (or roles) of type $i_{\tau\omega}$, *attitudes to constructions* of type $(oi*_n)_{\tau\omega}$.

Logical objects like *truth-functions* are extensional: $\wedge$ (conjunction), $\vee$ (disjunction) and $\supset$ (implication) are of type (ooo) , and $\neg$ (negation) of type (oo) . Below, all type indications will be provided outside the formulae in order not to clutter the notation. The outermost brackets of the Closure will be omitted whenever no confusion arises. Furthermore, ' X/α ' means that an object X is (a member) of type α . ' $X \rightarrow_v \alpha$ ' means that X is typed to v -construct an object of type α , if any. We write ' $X \rightarrow \alpha$ ' if what is v -constructed does not depend on a valuation v . Throughout, it holds that the variables $w \rightarrow_v \omega$ and $t \rightarrow_v \tau$. If $C \rightarrow_v \alpha_{\tau\omega}$ then the frequently used Composition $[[Cw]t]$, which is the intensional descent (a.k.a. extensionalization) of the α -intension v -constructed by C , will be encoded as ' C_{wt} '. Whenever no confusion arises, we use traditional infix notation without Trivialisation for truth-functions and the identity relation, to make the terms denoting constructions easier to read.

Definition 3 (*quantifiers, singularizers*)

The *universal quantifier* $\forall^\alpha$ is a total polymorphic function of type $(o(o\alpha))$ that takes a class A of α -elements to $\mathbf{T}$ if A contains all elements of the type α , otherwise to $\mathbf{F}$. The *existential quantifier* $\exists^\alpha$ is a total polymorphic function of type $(o(o\alpha))$ that takes a class A of α -elements to $\mathbf{T}$ if A is a non-empty class, otherwise to $\mathbf{F}$. The *singularizer* ι^α is a partial polymorphic function of type $(\alpha(o\alpha))$ that takes a class A to its only α -element if A is a singleton, otherwise (if A is an empty or multiple-element class) is the singularizer undefined at A . $\square$

Again, to make the constructions easier to read, as a notational convention I will often write ‘ $\forall xB$ ’, ‘ $\exists xB$ ’ and ‘ ιxB ’, instead of the full TIL notation ‘ $[^0\forall^\alpha \lambda xB]$ ’, ‘ $[^0\exists^\alpha \lambda xB]$ ’ and ‘ $[^0\iota^\alpha \lambda xB]$ ’, whenever no confusion arises. The additional types are $x \rightarrow \alpha$, $B \rightarrow o$, $\lambda xB \rightarrow (o\alpha)$, $[^0\forall^\alpha \lambda xB] \rightarrow o$, $[^0\exists^\alpha \lambda xB] \rightarrow o$, $[^0\iota^\alpha \lambda xB] \rightarrow \alpha$.

We invariably furnish expressions with procedurally structured meanings, which are explicated as TIL constructions. Thus, TIL constructions are assigned to expressions as their context-invariant meanings, and the analysis of an unambiguous sentence (or generally any term) consists in discovering the logical construction encoded by a given sentence. To this end we have developed the *TIL method of analysis* that consists of three steps:

- 1) *Type-theoretical analysis*, i.e., assigning types to the objects that receive mention in the analysed expression.
- 2) *Type-theoretical synthesis*, i.e., combining the constructions of the objects obtained in step (1) in order to construct the object (if any) of the respective type denoted by the whole expression.
- 3) *Type-theoretical checking*, i.e. checking whether the proposed analysis is type-theoretically coherent.

To illustrate the method, I will use Jespersen’s exemplary sentence “Vulcan is a planet”. Here ‘Vulcan’ is not a proper name; rather it stands for the definite description ‘the small planet in an orbit between Mercury and the Sun’. Its existence was proposed by Le Verrier in an attempt to explain peculiarities of Mercury’s orbit. A number of reputable investigators became involved in the search for Vulcan, but no such planet was ever found, and the peculiarities in Mercury’s orbit have now been explained by Albert Einstein’s theory of general relativity. I am going to analyse the sentence *à la* Strawson, i.e. with ‘Vulcan’ as a topic referring to that hypothetical individual to which the property of being a planet is ascribed.

First, type-theoretical analysis. The sentence mentions the following objects. *Vulcan*/ $\iota_{\tau\omega}$ is an individual role; *Planet*/ $(oi)_{\tau\omega}$; the whole sentence denotes a proposition, that is, an object of type $o_{\tau\omega}$.

Second, synthesis. Since we intend to arrive at the *literal* analysis of the sentence, the objects denoted by semantically simple expressions are constructed by their Trivializations: 0Vulcan , 0Planet . Now we are to apply the property of being a planet to the holder of the Vulcan office. In other words, we want to express the hypothetical fact that the holder (if any) obtained by extensionalisation of the office with respect to world w and time t of evaluation, $^0Vulcan_{wt} \rightarrow \iota$, belongs to the population of planets in this world and time, i.e. $^0Planet_{wt} \rightarrow (oi)$. Since the population is a set

To define analytic entailment formally, we need the propositional property $True/(oo_{\tau\omega})_{\tau\omega}$ which is defined as follows. Let P be a propositional construction ($P/*_n \rightarrow o_{\tau\omega}$). Then

$[^0True_{wt}P]$ v -constructs $\mathbf{T}$ iff P_{wt} v -constructs $\mathbf{T}$, otherwise $\mathbf{F}$.

For completeness, there are two other properties of the same type, namely *False* and *Undefined*, defined as follows:

$[^0False_{wt}P]$ v -constructs $\mathbf{T}$ iff P_{wt} v -constructs $\mathbf{F}$, otherwise $\mathbf{F}$

$[^0Undefined_{wt}P]$ v -constructs $\mathbf{T}$ iff P_{wt} is v -improper, otherwise $\mathbf{F}$

Note that $[^0True_{wt}P]$ v -constructs $\mathbf{F}$ in two cases, namely if P_{wt} v -constructs $\mathbf{F}$ or if P_{wt} is v -improper. Hence, for instance, if a proposition v -constructed by P is not true at a given $\langle w, t \rangle$ -pair, it does not have to be false, because there is the third possibility of being undefined. Formally, we have these relations ($=/(ooo)$):

$$[^0True_{wt}P] = \neg[^0False_{wt}P] \wedge \neg[^0Undefined_{wt}P]$$

$$[^0False_{wt}P] = \neg[^0True_{wt}P] \wedge \neg[^0Undefined_{wt}P]$$

$$[^0Undefined_{wt}P] = \neg[^0True_{wt}P] \wedge \neg[^0False_{wt}P]$$

Analytical entailment is defined as follows ($P, S/*_n \rightarrow o_{\tau\omega}, \models /(oo_{\tau\omega}o_{\tau\omega})$).¹⁶

$$(S \models P) \text{ iff } \forall w \forall t [[^0True_{wt}S] \supset [^0True_{wt}P]]$$

The logical difference between a presupposition and mere entailment is this:

P is a *presupposition* of S iff $(S \models P)$ and $(non-S \models P)$

P is *merely entailed* by S iff $(S \models P)$ and neither $(non-S \models P)$ nor $(non-S \models non-P)$

Comments. If P is a presupposition of S and P is not true at a given $\langle w, t \rangle$ -pair, then *neither* S *nor* $non-S$ is true. Hence, S has no truth-value at such a $\langle w, t \rangle$ -pair at which its presupposition is not true. On the other hand, if P is merely entailed by S , then if S is not true we cannot deduce anything about the truth-value, or lack thereof, of P .

Using the properties of propositions *True* and *False*, we can rigorously define the difference between presupposition and mere entailment.

Definition 4 (*presupposition vs. mere entailment*)

Let P and S be propositional constructions ($P, S/*_n \rightarrow o_{\tau\omega}$). Then

P is *entailed* by S iff $\forall w \forall t [[^0True_{wt}S] \supset [^0True_{wt}P]]$

P is a *presupposition* of S iff $\forall w \forall t [[[^0True_{wt}S] \vee [^0False_{wt}S]] \supset [^0True_{wt}P]]$

□

Hence, I agree with Frege and Strawson that survival under negation is the most important test for a presupposition. However, in order to decide whether there is a presupposition of S , we have to take into account the two ways in which the negated form *non-S* can be obtained. To illustrate the situation, consider again the sentence “The King of France is bald”. If the royal office is occupied and we want to say that its holder is *not* bald, we would simply use the form “The King of France is *not* bald”. This is Strawsonian *narrow-scope negation*. The property of not being bald is ascribed to the holder of the royal office.¹⁷ Thus the analyses of the Strawsonian reading of the sentence and of its negation come down to these constructions:

$$(S) \quad \lambda w \lambda t [{}^0\text{Bald}_{wt} \lambda w \lambda t [{}^0\text{King-of}_{wt} {}^0\text{France}]_{wt}]$$

$$(non-S) \quad \lambda w \lambda t \neg [{}^0\text{Bald}_{wt} \lambda w \lambda t [{}^0\text{King-of}_{wt} {}^0\text{France}]_{wt}]$$

However, if the royal office is not occupied, one would protest, saying, “No, it is not true that the King of France is bald, for the King of France does not exist”. This is Russellian *wide-scope negation*. The property of not being true is ascribed to the whole proposition that the King of France is bald.¹⁸ And indeed, *it is not true* that the King of France is bald, because the proposition denoted by the sentence has a truth-value gap.

In order to analyse Russellian wide-scope negation, we apply the above defined propositional property $\text{True}/(oo_{\tau\omega})_{\tau\omega}$:

$$(non-R) \quad \lambda w \lambda t \neg [{}^0\text{True}_{wt} \lambda w \lambda t [{}^0\text{Bald}_{wt} \lambda w \lambda t [{}^0\text{King-of}_{wt} {}^0\text{France}]_{wt}]]$$

(*non-R*) is the wide-scope negation of the proposition that it is true that the King of France is bald constructed by

$$(R) \quad \lambda w \lambda t [{}^0\text{True}_{wt} \lambda w \lambda t [{}^0\text{Bald}_{wt} \lambda w \lambda t [{}^0\text{King-of}_{wt} {}^0\text{France}]_{wt}]]$$

This is not exactly Russell’s analysis. As mentioned above, the Russellian rephrasing of the sentence “The King of France is bald” is “There is a unique individual such that he is the King of France and he is bald”. The analysis of this sentence comes down to

$$(R^*) \quad \lambda w \lambda t [{}^0\exists \lambda x [[x = \lambda w \lambda t [{}^0\text{King-of}_{wt} {}^0\text{France}]_{wt}] \wedge [{}^0\text{Bald}_{wt} x]]].$$

Additional types. $\exists/(o(o\iota)); =/(ou); x/*_1 \rightarrow_v \iota$.¹⁹

Yet (R) gets the truth-conditions of the Russellian reading right, because (R) and (*R**) are equivalent in the sense of constructing the same proposition. This analysis *does not presuppose* the existence of the holder of the royal office, unlike the proposition constructed by (S). The analysis re-

veals that these two readings are not equivalent. Though (R) and (S) are *co-entailing* they denote *different propositions*, which I am going to prove now.

First, the equivalence of (R) and (R^*) .

- a) Let the royal office be occupied in a given world w and time t by an individual a . Then if this individual a is bald, the proposition

$$\lambda w \lambda t [{}^0\text{Bald}_{wt} \lambda w \lambda t [{}^0\text{King-of}_{wt} {}^0\text{France}]_{wt}]$$

takes the value **T** in such a world-time pair, otherwise **F**, and so does (R) according to the definition of the property *True*. By assumption

$$[{}^0a = [\lambda w \lambda t [{}^0\text{King-of}_{wt} {}^0\text{France}]_{wt}].$$

Hence the Composition

$$[{}^0a = [\lambda w \lambda t [{}^0\text{King-of}_{wt} {}^0\text{France}]_{wt}] \wedge [{}^0\text{Bald}_{wt} {}^0a]]$$

v -constructs **T** and so does

$$[\lambda x [x = [\lambda w \lambda t [{}^0\text{King-of}_{wt} {}^0\text{France}]_{wt}] \wedge [{}^0\text{Bald}_{wt} x]] {}^0a]$$

if a is bald. It means that the class of individuals

$$\lambda x [x = [\lambda w \lambda t [{}^0\text{King-of}_{wt} {}^0\text{France}]_{wt}] \wedge [{}^0\text{Bald}_{wt} x]]$$

is non-empty according as the individual a is bald or not. Thus according to the definition of the quantifier $\exists$, the proposition constructed by (R^*) takes the value **T** if a is bald, otherwise **F**, exactly as does the proposition constructed by (R) .

- b) Let the royal office be vacant in a given world w and time t . Then by Def. 1, iii) the following Compositions are v -improper:

$$[\lambda w \lambda t [{}^0\text{King-of}_{wt} {}^0\text{France}]_{wt}]$$

$$[{}^0\text{Bald}_{wt} \lambda w \lambda t [{}^0\text{King-of}_{wt} {}^0\text{France}]_{wt}]$$

$$[x = [\lambda w \lambda t [{}^0\text{King-of}_{wt} {}^0\text{France}]_{wt}]]$$

$$[[x = \lambda w \lambda t [{}^0\text{King-of}_{wt} {}^0\text{France}]_{wt}] \wedge [{}^0\text{Bald}_{wt} x]].$$

This in turn means that the proposition constructed by

$$\lambda w \lambda t [{}^0\text{Bald}_{wt} \lambda w \lambda t [{}^0\text{King-of}_{wt} {}^0\text{France}]_{wt}]$$

is undefined and by the definition of the property *True* the proposition constructed by (R) takes the value **F**. But so does (R^*) , because the class constructed by

$$\lambda x [[x = \lambda w \lambda t [{}^0\text{King-of}_{wt} {}^0\text{France}]_{wt}] \wedge [{}^0\text{Bald}_{wt} x]]$$

is empty and the application of the existential quantifier $\exists$ to an empty class results in **F**.

Note that from (b) it also follows that neither (R) nor (R^*) comes with the existential presupposition that the King of France exists. Non-trivial existence of empirical objects is in TIL explicated as a property of intensions to be instantiated at a given $\langle w, t \rangle$ -pair of evaluation.²⁰ Thus to say that unicorns do not exist is tantamount to saying that at the given world w and time t the property of being a unicorn has an empty class of individuals as its population. Similarly, that the King of France does not exist means that the office of the King of France is vacant at the $\langle w, t \rangle$ -pair of evaluation. Hence if there were an existential presupposition, the propositions constructed by (R) and (R^*) would have no truth-value in case of the royal office being vacant. Yet, these propositions take the value **F**. In other words, neither $(non-R)$ nor $(non-R^*)$ entails that the royal office is occupied.

Now I am going to prove that (S) , and thus also $(non-S)$, presupposes the existence of the King of France. To this end I must prove that the following arguments are valid (though not sound):

$$\frac{\text{The King of France is (not) bald}}{\text{The King of France exists}}$$

First, the analysis of the conclusion amounts to this construction:

$$\lambda w \lambda t [{}^0Exist_{wt} [\lambda w \lambda t [{}^0King_of_{wt} {}^0France]]]$$

where $Exist/(\iota_{\tau\omega})_{\tau\omega}$ is the property that an office has when it is occupied. This property is defined as follows:

$${}^0Exist =_{of} \lambda w \lambda t \lambda c [{}^0\exists \lambda x [x = c_{wt}]].$$

Types: $\exists/(o(\iota\iota))$; $c \rightarrow_v \iota_{\tau\omega}$; $x \rightarrow_v \iota$; $=_{of}/(o(o\iota_{\tau\omega})_{\tau\omega}(o\iota_{\tau\omega})_{\tau\omega})$: the identity of properties of individual offices; $=/(o\iota\iota)$: the identity of individuals, $x \rightarrow_v \iota$.

Now I am ready to prove the validity of the above arguments and thus the validity of the claim that the Strawsonian reading is associated with a presupposition of the royal office being occupied.

At any $\langle w, t \rangle$ -pair the following proof steps are truth-preserving:

1)	$(\neg)[{}^0Bald_{wt} \lambda w \lambda t [{}^0King_of_{wt} {}^0France]_{wt}]$	$\emptyset$
2)	$\neg[{}^0Improper_{wt} {}^0[\lambda w \lambda t [{}^0King_of_{wt} {}^0France]_{wt}]]$	1), by Def. 1, iii)
3)	$\neg[{}^0Empty \lambda x [x = [\lambda w \lambda t [{}^0King_of_{wt} {}^0France]_{wt}]]]$	2), by Def. 1, iv)
4)	$[{}^0\exists \lambda x [x = [\lambda w \lambda t [{}^0King_of_{wt} {}^0France]_{wt}]]]$	3), EG
5)	$[\lambda c [{}^0\exists \lambda x [x = c_{wt}]] \lambda w \lambda t [{}^0King_of_{wt} {}^0France]]$	4), λ -abstraction
6)	${}^0Exist_{wt} = \lambda c [{}^0\exists \lambda x [x = c_{wt}]]$	Def. of <i>Exist</i>
7)	$[{}^0Exist_{wt} [\lambda w \lambda t [{}^0King_of_{wt} {}^0France]]]$	5), 6), substitution of identicals

Remark. At step (2) the property of being *Improper* of type $(o*_1)_{\tau\omega}$ is applied to the *construction* $[\lambda w\lambda t [{}^0King_of_{wt} {}^0France]_{wt}]$ of type $*_1$ that is supplied here by its Trivialisation ${}^0[\lambda w\lambda t [{}^0King_of_{wt} {}^0France]_{wt}]$ belonging to type $*_2$. On the other hand, at step (3) the property of being *Empty* of type $(o(o\iota))$ is applied to the set of individuals constructed by $\lambda x [x = [\lambda w\lambda t [{}^0King_of_{wt} {}^0France]_{wt}]]_{wt}$. These two steps are necessary in order to existentially generalize at step (4). In a logic of partial functions such as TIL we cannot carelessly generalize before having proved that the set to which the existential quantifier is applied is non-empty.

The following Table 1 illustrates the truth conditions of the propositions constructed by (R) , (S) , $(non-R)$ and $(non-S)$ with respect to the occupancy of the office of the King of France (KF).²¹ Indeed, (R) and (S) are co-entailing. Whenever (R) is true (S) is true as well, and vice versa. Yet the propositions (R) and (S) are not identical, because $(non-R)$ and $(non-S)$ are not co-entailing. At those $\langle w, t \rangle$ -pairs where the King of France does not exist, both (S) and $(non-S)$ are undefined, the propositions having a truth-value gap, while (R) and $(non-R)$ are false and true, respectively.

Table 1
Russellian vs. Strawsonian analysis

	KF	(R)	(S)	$(non-R)$	$(non-S)$	
w_1, t_1	a	T	T	F	F	a is the KF and is bald
w_2, t_2	a	F	F	T	T	a is the KF and isn't bald
w_3, t_3	$\perp$	F	$\perp$	T	$\perp$	The KF does not exist
w_4, t_4	b	F	F	T	T	b is the KF and isn't bald
w_5, t_5	b	T	T	F	F	b is the KF and is bald
w_6, t_6	$\perp$	F	$\perp$	T	$\perp$	The KF does not exist
$\dots$						

5. Ambiguities in topic-focus articulation

5.1. Presuppositions generated by a singular term topic

The above analyses provide a solution to the almost hundred-year old dispute over Strawsonian versus Russellian definite descriptions that has been summarised in Section 2. The ambiguity of sentences of the form “The F is a G ” is not rooted in a shift of *meaning* of the definite description ‘the F ’. Rather the ambiguity stems from different *topic-focus*

articulations of such sentences. Whereas articulating the topic of a sentence activates a presupposition, articulating the focus frequently yields merely an entailment.²² If ‘the *F*’ is the topic phrase then this description occurs extensionally that is with *de re* supposition and Strawson’s analysis appears to be what is wanted. The sentence *presupposes* that the description ‘the *F*’ refers to an object of the proper type. The other option is ‘*G*’ occurring as topic and ‘the *F*’ as focus. On this reading the description ‘the *F*’ occurs intensionally that is with *de dicto* supposition, and the Russellian analysis gets the truth-conditions of the sentence right. The existence of a unique *F* is *merely entailed*.

The received view still tends to be that there is room for at most one of the two positions, since they are deemed incompatible. But there is no incompatibility between Strawson’s and Russell’s positions, because they simply do not talk about one and the same meaning of the sentence “The King of France is bald”. My novel *contribution* is to point out this *ambiguity* which yielded the false dilemma. Russell argued for the attributive use of ‘the King of France’ and Strawson for its referential use.

It is a matter of *pragmatics*, of course, which reading is the intended one on an occasion of use. Logic cannot decide which among multiple readings happens to be the intended one. Yet, I cannot agree with Kripke (1977) on two accounts. First, it is not *entirely* a matter of pragmatics which reading is the intended one; it is also a matter of semantics. Second, and more importantly, the Russellian analysis of sentences with definite descriptions cannot, by itself, account for both the referential and the attributive uses. Our fine-grained logical method of analysis as presented in this paper demonstrates that these readings are *not equivalent*, and that the Russellian reading does not take into account any presuppositions triggered by the topic of the sentence in question. Thus, though logic itself cannot decide between multiple readings, it can contribute to disambiguation of a sentence by making these different meanings *explicit*. In case the sentence is ambiguous, logic can bring out this ambiguity and, as a result, propose different constructions to be assigned as different meanings to (non-equivalent) readings. Choosing between them becomes also a matter of *semantics*.

5.2. Presuppositions generated by a general term topic

Up until now I have utilized the singularity of an individual office of type $\iota_{\tau\omega}$ when analysing sentences that come attached with the existential presupposition. If the office denoted by ‘the *F*’ goes vacant at a given world *w* and time *t* of evaluation, the extensionalization F_{wt} is *v-improper*, and if the description ‘the *F*’ occurs as the topic of a sentence (i.e. ref-

erentially or *de re*) the so constructed proposition has a truth-value gap. However, the regimentation of a presupposition can be more complicated. In particular, the topic term does not have to be a singular one; it can be also a plural term like ‘the popes of Rome and Avignon’ or a general one like ‘a penguin’. We encounter the phenomenon of topic-focus ambiguity and the associated *de dicto* – *de re* ambivalence also in sentences containing general terms. As an example, consider a seemingly unambiguous sentence

“All the students of my winter-term Logic course passed the exam test”

However, this sentence is ambiguous. If we take into account possible topic-focus articulations, we obtain a pair of non-equivalent sentences differing only in terms of this articulation (for clarity, I will mark the topic phrase in *italics*):

- (1) “All *the students* of my winter-term Logic course passed the exam test”
- (2) “All the students of my winter-term Logic course passed *the exam test*”

Now I am going to show that the two versions of the above sentence are not equivalent, because their truth-conditions are different.

Scenario (1).

Q.: What about the students who signed up for the Logic course, how did they do?

A.: Oh well, they all passed the final exam.

On this reading we are talking about the topic that is the students who signed up for the Logic course; hence, the sentence *presupposes* that there are some students enrolled in the Logic course in winter term; *if not* (for instance, because the course runs in summer term), then the proposition denoted by (1) has *no truth-value*. For, if there are no students in the course, the *negated* sentence cannot be true as well:

- (neg-1) “Some students of my winter-term Logic course *did not* pass the exam test”.

However, the sentence *only entails* that the final exam has taken place. Because, the sentence (1) can be false for two reasons: *Either* some of the students failed, *or* none of the students succeeded because the exam has yet to take place.

Scenario (2).

Q.: What about the final exam in Logic, what are the results?

A.: Oh well, all students passed.

Now the topic is the final exam in the Logic course. On this reading there is a *presupposition* that the final exam has already taken place; if it is *not the case*, then there is a truth-value gap, because the negated sentence cannot be true as well:

(neg-2) “The *final exam* has not been passed by all students of my Logic course”

Needless to say, that in these examples we apply an internal, narrow-scope negation. If we applied the wide-scope external negation, we would have:

“It is not true that all the students of my winter-term Logic course passed the exam test”

Perhaps because some of the students failed, or the exam test is still to take place, or perhaps no students attempted the test. No presupposition would be taken into account, which is not very informative.

Now if we want to analyse properly those readings that take into account the topic-focus articulation and the associated presuppositions, we need a general analytic schema for such sentences, which I am going to introduce now. To illustrate the schema, consider again a sentence S with a presupposition P . It encodes as its meaning the procedure the evaluation of which can be described as follows:

In any $\langle w, t \rangle$ -pair of evaluation,
 if P_{wt} is true
 then evaluate S_{wt} to produce a truth-value,
 else fail to produce a truth-value.

To formulate this schema rigorously, we need to define the *if-then-else-fail* function. Here is how. The procedure encoded by “If $P(\rightarrow o)$ then $C(\rightarrow \alpha)$, else $D(\rightarrow \alpha)$ ” behaves as follows:

- a) If P v -constructs **T** then execute C (and return the result of type α , that C is typed to produce, provided C is not v -improper).
- b) If P v -constructs **F** then execute D (and return the result of type α , that D is typed to produce, provided D is not v -improper).
- c) If P is v -improper then no result, gap.

Hence, *if-then-else* is seen to be a (strict) function of type $(\alpha o *_n *_n)$, and its definition decomposes into two phases.²³

First, select a construction to be executed on the basis of a specific condition P . The choice between C and D comes down to this Composition:

$$[{}^0_1 * \lambda c [[P \wedge [c = {}^0C]] \vee [\neg P \wedge [c = {}^0D]]]]$$

Types: $P \rightarrow_v o$ v -constructs the condition of the choice between the execution of C or D , $C/*_n$, $D/*_n \rightarrow_v \alpha$; $c \rightarrow_v *_n$; $1^*/(*_n(o*_n))$: the singularizer function that associates a singleton of constructions with the construction that is the only element of this singleton, and is otherwise (i.e. if the set is empty or many-valued) undefined.

If P v -constructs **T** then the variable c v -constructs the *construction* C , and if P v -constructs **F** then the variable c v -constructs the *construction* D . In either case, the set constructed by

$$\lambda c [[P \wedge [c = {}^0C]] \vee [\neg P \wedge [c = {}^0D]]]$$

is a singleton and the singularizer 1^* returns as its value either the construction C or the construction D .²⁴

Second, the selected construction is executed; therefore, Double Execution must be applied:

$${}^2[{}^01^*\lambda c [[P \wedge [c = {}^0C]] \vee [\neg P \wedge [c = {}^0D]]]]$$

As a special case of the *if-then-else-fail* function, *no* construction D is to be selected whenever P is not satisfied. Thus, the definition of the *if-then-else-fail* function of type $(\alpha o *_n)$ is this:

$${}^2[{}^01^*\lambda c [P \wedge [c = {}^0C]]]$$

Indeed, if P v -constructs **F**, then the class constructed by $\lambda c [P \wedge [c = {}^0C]]$ is empty so that the singularizer function does not supply as its value any construction. As a result, according to Def. 1, both the composition $[{}^01^*\lambda c [P \wedge [c = {}^0C]]]$ and its Double Execution are v -improper.

Now we are ready to apply this definition to the case of a presupposition. Let $P/*_n \rightarrow o_{\tau\omega}$ be a construction of a presupposition of $S/*_n \rightarrow o_{\tau\omega}$. Furthermore, let $c/*_{n+1} \rightarrow_v *_n$, ${}^2c \rightarrow_v o$. Then the type of the *if-then-else-fail* function is $(oo*_n)$ and its definition comes down to this construction:

$$\lambda w \lambda t [{}^0\text{if-then-else-fail } P_{wt} {}^0[S_{wt}]] = \lambda w \lambda t {}^2[{}^01^*\lambda c [P_{wt} \wedge [c = {}^0[S_{wt}]]]]$$

Gloss. In the first phase the construction S_{wt} is selected, provided P_{wt} v -constructs **T**. In the second phase S_{wt} is executed. In case P_{wt} does not v -construct **T**, no construction is selected and executed, hence

$${}^2[{}^01^*\lambda c [P_{wt} \wedge [c = {}^0[S_{wt}]]]]$$

is v -improper and the so-constructed proposition has a truth-value gap, as it should have.

In what follows, instead of the above definition I will use this abbreviated notation to make the *general analytic schema* easier to read:

$$\lambda w \lambda t [\text{if } P_{wt} \text{ then } S_{wt} \text{ else fail }].$$

For illustration, let us analyse Strawson's (1952, pp. 173ff) example

All *John's children* are asleep.

Here the topic of the sentence is 'John's children'. The sentence can be uttered for instance as an answer in the situation when we are talking about John's children (knowing that he has some children) and we just want to know what are they doing. Hence, there is a presupposition to the effect that John has children.²⁵ Hence the evaluation of truth-conditions of this reading can be formulated like this:

If John has any children
then check whether each and every one of them is asleep
else fail to produce a truth-value.

We have come down to this analysis:

$$\lambda w \lambda t [if [^0 \exists [^0 Children_of_{wt} ^0 John] then \\ [[^0 All [^0 Children_of_{wt} ^0 John]] ^0 Sleep_{wt}] else fail]$$

Types: $Children_of((o\iota)\iota)_{\tau\omega}$: the empirical function (attribute) that dependently on a state of affairs associates an individual with the set of those individuals who are his or her children; $John/\iota$; $Sleep/(o\iota)_{\tau\omega}$; $\exists/(o(o\iota))$; $All/((o(o\iota))(o\iota))$: a restricted quantifier that associates a set S of individuals with all the supersets of S .

Remark. Here I use the restricted quantifier *All*, because I want to arrive at the *literal* analysis of the sentence. Such an analysis follows Frege's principle (1884, p. 60): It is simply not possible to speak about an object without somehow denoting or naming it.²⁶ If the unrestricted general quantifier were used the resulting construction would be:

$$\lambda w \lambda t [if [^0 \exists [^0 Children_of_{wt} ^0 John] then \\ [^0 \forall \lambda x [[[^0 Children_of_{wt} ^0 John] x] \supset [^0 Sleep_{wt} x]]] else fail]$$

This is an equivalent construction producing the same proposition as the above one, yet it is not the literal analysis of our sentence, because the truth-function of implication is not mentioned in the sentence.²⁷

6. Other cases of presuppositions

Other cases of generating a presupposition are, for instance, *factive attitudes* like 'knowing', 'realizing', 'discovering', etc. For instance, if you say, "I didn't realize that he had left" then the conveyed message *presupposes*

that *it is true* that he had left. Notice that here we again apply narrow-scope negation. For sure, (using wide-scope negation) it cannot be true that I realized that he had left if he had not left. But it can neither be false, because then the proposition that I didn't realize that he had left could not be true either. In such a situation the proposition has a truth-value gap, and in compliance with Def. 4, the truth of the complement of the factive attitude is a presupposition of the attitude.

Similarly, the sentence

“John knows that arithmetic is recursively axiomatizable”

denotes the proposition with a truth-value gap. If somebody would claim it, you would protest. “No, it cannot be true, because Gödel proved in 1931 the two famous incompleteness theorems!”. Indeed, it is neither true nor false, because John can neither know nor not know that arithmetic is recursively axiomatizable. For these reasons we have to introduce special rules for factive attitudes.

Let $K \rightarrow (o \circ o_{\tau\omega})_{\tau\omega}$ be a factive attitude to a proposition, $x \rightarrow \iota$; $p \rightarrow o_{\tau\omega}$; $True/(o \circ o_{\tau\omega})_{\tau\omega}$. Then the rules are as follows:

$$\frac{[{}^0K_{wt} x p]}{[{}^0True_{wt} p]} \qquad \frac{\neg[{}^0K_{wt} x p]}{[{}^0True_{wt} p]}$$

This is the case of *intensional* attitudes; that is the agent x is related to a *proposition* of type $o_{\tau\omega}$. However, it is a well-known fact that every epistemic logic that takes into account only intensional attitudes faces the problem of *logical-mathematical omniscience* that can be restricted only up to equivalence. Yet, if it is possible that the agent's attitude is sensitive to the *way* in which a given proposition is presented (or constructed in TIL terminology), the more appropriate way of analysing such attitudinal sentences is a *hyperintensional attitude*, that is, a relation-in-intension of an agent to a *hyperproposition* (i.e. the construction of a proposition). For instance, John can know that arithmetic is *not* recursively axiomatizable, but he does not have to know that Peano arithmetic is incomplete.²⁸

The rules for hyperintensional factive attitudes are as follows. Let $K^* \rightarrow (o \circ *_{\tau\omega})_{\tau\omega}$ be a factive attitude to a construction of a proposition, $x \rightarrow_v \iota$; $c/*_{\tau\omega} \rightarrow_v o_{\tau\omega}$; $True/(o \circ *_{\tau\omega})_{\tau\omega}$. Then the rules are as follows:

$$\frac{[{}^0K^*_{wt} x c]}{[{}^0True_{wt} {}^2c]} \qquad \frac{\neg[{}^0K^*_{wt} x c]}{[{}^0True_{wt} {}^2c]}$$

The frequency of sentences that come attached with a presupposition in natural language is maybe much greater than one would suppose. Another example are sentences with verbs like ‘to stop doing something’. One cannot stop doing something if they have never done it. Imagine that somebody asks “Did you stop beating your wife? Answer *yes* or *no*.” What would the poor guy answer if he has never beaten his wife, or if he has not been married? Yes? “So, you did beat your wife.” No? “So, you still do beat your poor wife.”

The reason for this situation is due to what we call *intensional essentialism*. As mentioned at the outset, each intension $P/\alpha_{\tau\omega}$ (e.g. a property, relation-in-intension, etc.) comes with a collection of requisites that jointly define the intension P . We call this collection the *essence* of P . For instance, the property of having stopped smoking comes with a bulk of requisites including, not least, the property of being a former smoker. Thus, the predication of such a property P of a may fail, causing $[{}^0P_{wt} a]$ to be *v-improper*, if a is not a former smoker. And then the proposition $\lambda w \lambda t [{}^0P_{wt} a]$ comes with a truth-value gap for an a who has never smoked in the world w and time t of evaluation.

The requisite relations *Req* are a family of relations-in-extension between two intensions, so they are of the polymorphous type $(o\alpha_{\tau\omega}\beta_{\tau\omega})$, with the possibility that $\alpha = \beta$.²⁹ Infinitely many combinations of *Req* are possible, but for our purpose and for illustration it suffices to define this one:

$$Req/(o(ol)_{\tau\omega}(ol)_{\tau\omega})$$

Req is a relation between two properties of individuals, such that one is a requisite of the other defined as follows.

Definition 5 (*requisite relation*)

Let X, Y be constructions such that $X, Y/*_n \rightarrow (ol)_{\tau\omega}; x \rightarrow \iota$. Then $[{}^0Req Y X] = \forall w \forall t [\forall x [[{}^0True_{wt} \lambda w \lambda t [X_{wt} x]] \supset [{}^0True_{wt} \lambda w \lambda t [Y_{wt} x]]]]$.

□

Gloss *definiendum* as, “ Y is a requisite of X ”, and *definiens* as, “Necessarily, i.e. at every $\langle w, t \rangle$, any x that instantiates X at $\langle w, t \rangle$ also instantiates Y at $\langle w, t \rangle$.”

Note, however, that failing to satisfy some of the requisites does not suffice to generate a presupposition. For instance, the property of being a mammal is a requisite of the property of being a whale. It is a contingent matter whether this or that individual is a whale or say a lion or whatever.

Yet, necessarily, i.e. in all worlds and times, if an individual a happens to be a whale then it must be a mammal. Formally:

$$[{}^0\text{Req } {}^0\text{Mammal } {}^0\text{Whale}] = \\ \forall w \forall t [\forall x [({}^0\text{True}_{wt} \lambda w \lambda t [{}^0\text{Whale}_{wt} x]) \supset [{}^0\text{True}_{wt} \lambda w \lambda t [{}^0\text{Mammal}_{wt} x]]]]$$

Now assume that the individual a is *not* a mammal. Then the predication of the property of being a whale of a is simply false, $[{}^0\text{Whale}_{wt} a] = {}^0\mathbf{F}$, rather than v -improper, and the proposition that a is a whale, $\lambda w \lambda t [{}^0\text{Whale}_{wt} a]$, *does not presuppose* but *merely entails* that a is a mammal.

Similarly, finding something after a prior search does not presuppose the existence of the sought thing, it merely entails its existence. Recall the tragedy in Dallas on November 22, 1963. When police were seeking the murderer of JFK, the FBI and the Warren Commission finally officially concluded that Oswald was the lone assassin. Though many challenged the findings of the Warren Report and believed that Kennedy was the victim of a conspiracy, assume that their conclusion was right. Yet another scenario is possible. The police may have failed in identifying the only murderer of JFK, because there were more murderers or because the police simply did not work well. Then it would be *true* that they *did not find* the murderer of JFK. If the existence of the lone murderer were a presupposition of identifying the murderer, the proposition that they failed in finding the murderer could not be true; rather, it would have a truth-value gap.³⁰

In order to generate a presupposition, we need to define a stronger notion of requisite, to wit the notion of *prerequisite*, $\text{Prereq}/(o(o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$ defined as follows.

Definition 6 (*prerequisite relation*)

Let X, Y be constructions such that $X, Y/*_n \rightarrow (o\iota)_{\tau\omega}; x \rightarrow \iota$. Then

$$[{}^0\text{Prereq } X] = \\ \forall w \forall t [\forall x [([{}^0\text{True}_{wt} \lambda w \lambda t [X_{wt} x]) \vee [{}^0\text{False}_{wt} \lambda w \lambda t [X_{wt} x]]] \\ \supset [{}^0\text{True}_{wt} \lambda w \lambda t [Y_{wt} x]]]].$$

Gloss *definiendum* as, “ Y is a prerequisite of X ”, and *definiens* as, “Necessarily, any x for which it is true or false that x instantiates X at $\langle w, t \rangle$ then x also instantiates Y at $\langle w, t \rangle$.”

Corollary. If it is not true that x instantiates the prerequisite Y of the property X then the proposition that x instantiates X is neither true nor false, it has no truth-value. Hence, the proposition that x instantiates Y is a *presupposition* of the proposition that x instantiates X .

The property of being a previous smoker is not only a requisite of the property of having stopped smoking, it is a *prerequisite*. As we have seen above, necessarily, if *a* stopped smoking then *a* is an ex-smoker. Yet not only this. If *a* *did not stop smoking*, then *a* has been a (previous) smoker. Thus, the proposition that *a* stopped smoking has a *presupposition* that *a* used to be a smoker.

7. Conclusion

In this paper I have demonstrated and proved that narrow-scope and wide-scope negation are not equivalent. If a sentence comes with a presupposition, then narrow-scope negation is the relevant one, because wide-scope negation is presupposition-denying. I also dealt with the ambiguities in natural language stemming from different topic-focus articulations within a sentence. First, I dealt with the existential presupposition of sentences with a singular topic term like ‘the *F*’. I demonstrated that both the proponents of Russell’s quantificational analysis and of Strawson’s referential analysis of definite descriptions are partly right and partly wrong, because sentences of the form “The *F* is a *G*” are systematically ambiguous. Their ambivalence stems from different topic-focus articulation, and I brought out the *semantic*, as opposed to pragmatic, character of this ambivalence. I showed that a definite description occurring in the topic of a sentence with *de re* supposition corresponds to the Strawsonian analysis of definite descriptions, while a definite description occurring in the focus with *de dicto* supposition corresponds to the Russellian analysis. While the clause standing in topic position triggers a presupposition, a focus clause usually only entails rather than presupposes another proposition. The procedural semantics of TIL provides rigorous analyses such that sentences differing only in their topic-focus articulation are assigned different constructions producing different propositions (truth-conditions) and having different consequences. These analyses propose a solution to the old dispute about Russelian vs. Strawsonian analysis of the sentence “The King of France is bald”. In fact, there is no incompatibility between Strawson’s and Russell’s positions, because they simply do not talk about one and the same meaning of the sentence. I pointed out this *ambiguity* which yielded the false dilemma. Russell argued for the attributive use of ‘the King of France’ and Strawson for its referential use. I also generalized the method of analysing topic-focus ambiguities for sentences with a general term occurring in the topic position and provided a general analytic schema for sentences with a presupposition.

It is a matter of *pragmatics*, of course, which reading is the intended one on an occasion of use. Yet, our fine-grained logical method of analysis as presented in this paper demonstrates that sentences differing in point of topic-focus articulation are not equivalent, and thus choosing between particular readings becomes also a matter of *semantics*. Logic can contribute to the disambiguation of a sentence by making these hidden features explicit and logically tractable. In case there are more *non-equivalent* senses of a sentence we furnish the sentence with *different meanings*. Finally, I applied this general analytic schema to other cases of sentences that come attached with a presupposition, in particular presuppositions generated by factive attitudes or presuppositions generated by the prerequisites of a given property.

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N O T E S

¹ See Tichý (1988), Tichý (2004), Duží, Jespersen and Materna (2010).

² In this section I use terms like ‘proposition’, ‘presupposition’ and ‘property’ in their intuitive sense, with rigorous definition coming in Section 3.

³ In these paragraphs I partly draw on material from Horn (1989).

⁴ I refrain here from the analysis of *proper* proper names. In my background theory, which is Transparent Intensional Logic (TIL), proper names coin *something*, and thus if Kepler is the name of an individual, then Kepler trivially exists. TIL is based on a fixed universe of discourse, i.e. the fixed domain of bare individuals. Hence, there are no non-existing individuals.

⁵ Besides, many hold against Russell’s translation of atomic sentences like “The *F* is a *G*” into the molecular form “There is at least one *F* and at most one thing is an *F* and that thing is a *G*”, because Russell disregards the standard constraint that there must be a fair amount of structural similarity between analysandum and analysans.

⁶ Nevertheless, for Strawson, *sentences* are meaningful in and of themselves, independently of empirical facts like the contingent non-existence of the King of France.

⁷ Here I partly draw on and summarise material from Duží (2009, 2014).

⁸ For details on structured meanings, see Jespersen (2012) and Duží (2017).

⁹ For an introduction to the notion of hyperproposition, see Jespersen, (2010) and Jespersen, Duží (2015).

¹⁰ See, for instance, Hajičová (2008), Gundel (1999), Gundel and Fretheim (2004).

¹¹ Gahér (2001) makes a similar point on the two kinds of negation. The author talks about wide-scope negation as ‘weak’ (or ‘sentential’, ‘propositional’) negation and about narrow-scope negation as ‘strong’ negation. However, there is an issue on which our positions are not compatible. While Gahér argues that “[...] all forms of ‘strong’ (non-sentential) negation are substitutable by equivalent transparent paraphrases containing only sentential negation [...]”, I am going to prove below that these two forms are not equivalent.

¹² As an extreme case the produced function/mapping can be a nullary function, that is, an atomic object such as an individual, number, or a truth-value.

¹³ The other source can be a type-theoretically incoherent (‘nonsensical’) way of composing a construction, for instance, by composing the Earth with being a natural number.

¹⁴ To simplify the tree, I apply these rules: if $C \rightarrow \alpha_{\tau\omega}$ then $C_{wt} \rightarrow \alpha$, and if $D \rightarrow \alpha$ then $\lambda w \lambda t D \rightarrow \alpha_{\tau\omega}$. Indeed, unpacking the abbreviations ‘ $\alpha_{\tau\omega}$ ’ and ‘ C_{wt} ’, we have: $C \rightarrow ((\alpha\tau)\omega)$, $[Cw] \rightarrow (\alpha\tau)$, $[[Cw]t] \rightarrow \alpha$. Similarly the second rule: $D \rightarrow \alpha$, $\lambda t D \rightarrow (\alpha\tau)$, $\lambda w \lambda t D \rightarrow ((\alpha\tau)\omega)$.

¹⁵ For the slight difference between *analytical* and *logical* entailment see Duží (2010).

¹⁶ Again, I use the infix notation ‘ $(S \models P)$ ’ instead of the proper TIL notation ‘ $[^0 \models SP]$ ’ to make the formulae easier to read.

¹⁷ As explained at the outset, there is still a difference between being *non-bald* (property negation) and being an x such that *not: x is bald* (boolean negation). Here the latter is applied.

¹⁸ For the difference between narrow-scope and wide-scope negation with respect to a presupposition, see also Hajičová (2008).

¹⁹ Note that in TIL we do not need a construction to specify the uniqueness of the King of France, because it is inherent in the meaning of ‘the King of France’. The meaning of definite descriptions like ‘the King of France’ is a construction of an individual office of type $\iota_{\tau\omega}$ occupied at each $\langle w, t \rangle$ -pair by at most one individual.

²⁰ For details see Duží *et al.* (2010, § 2.3).

²¹ I use the symbol ‘ $\perp$ ’ to mark a truth-value *gap* rather than the truth-value F.

²² This assumption is based on Hajičová (2008), and supported by other linguists as well. See, for instance Gundel (1999), Gundel and Fretheim (2004) and Strawson (1952, esp. p. 173ff.).

²³ The definition introduced here is a slightly adjusted version of the definition presented in Duží (2010a).

²⁴ Note that in this phase C and D are not constituents to be executed; rather they are merely displayed as objects to be selected by the variable c . This is to say that, in TIL, constructions themselves can be objects to be operated on, and without this *hyper-intensional* approach we would not be able to define the *strict* function *if-then-else*. For the difference between constructions occurring in the *displayed mode* (*hyperintensionally*) and *executed mode* (as constituents of a super-construction), see, for instance, Duží (2017).

²⁵ The other option would be, for instance, the scenario of talking about those who are asleep, and the sentence would be offered as an answer, “Among those who are asleep are all of John’s children”. On this reading the sentence would only entail that John has children.

²⁶ The German original goes, “Überhaupt ist es nicht möglich von einem Gegenstand zu sprechen, ohne ihn irgendwie zu bezeichnen oder benennen.”

²⁷ For more details on the method of arriving at the best literal meaning of a sentence, see Duží *et al.* (2010, §2.1)

²⁸ For the sake of simplicity, I ignore here the fact that possible worlds and times are irrelevant for mathematical propositions. For details on mathematical attitudes, see Duží et al. (2010, Ch. 5).

²⁹ For comparison, Jespersen (2006) offers a detailed study of a requisite relation, of type $(\text{OL}_{\tau\omega}\text{L}_{\tau\omega})$, where one individual office is a requisite of another individual office, the way the office of Commander-in-Chief is a requisite of the office of President of the United States. The paper analyses “Superman is Clark Kent” as expressing that this particular requisite relation obtains between one office denoted by ‘Superman’ and another office denoted by ‘Clark Kent’. If you occupy the office of Superman you must co-occupy the office of Clark Kent, but you can occupy the Clark Kent office without occupying the Superman office. This goes to show that TIL offers an intensional analysis (based on intensional essentialism) of “Superman is Clark Kent”, contrary to the prevalent ‘Millian’ extensional analyses.

³⁰ More details on attitudes of seeking and finding can be found in Duží et al. (2010, §5.2.2).

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