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THE ART OF RAMON LLULL (1232–1350): FROM THEOLOGY TO MATHEMATICS

Abstract. In the present paper the roots of the Art of the Catalan philosopher Ramon Llull are examined. Moreover the impact of the Art on seventeenth mathematics is briefly discussed.

Introduction

Around 1272 Ramon Llull, a Catalan courtier turned theologian and philosopher, had an "illumination". While he was meditating on Mount Randa on Mallorca, God revealed to Llull "the form and method" ("forma i manera" in Catalan) that he needed to write a book that would convincingly prove the correctness of Christian dogmas to Muslims and Jews. The illumination led to the development of a formal universal art that was meant to enable the skilled practitioner to find the true answers to all important questions about God and his creation.

Llull's idea of such a universal art, partially based on a clever combination of characters, has been very influential. In the 17th century it was taken up by Athanasius Kircher, and in particular Gottfried Wilhelm Leibniz' work on logic and on calculus was arguably influenced by Llull's Art.

In the present paper we will concentrate on the illumination. What was this sudden insight? Suddenly several ideas fitted together. Where did these ideas come from? While we try to answer these questions, inevitably speculatively, we hope to get closer to understanding Llull's Art. The Art is rather inaccessible. Llull was a self-educated man and his Art is idiosyncratic. With the exception of his more popular work, Llull's own texts are difficult to read. The contrast with a modern text is huge and even in comparison with, for example, Thomas Aquinas' work, Llull's Art is not easy to understand. At the time there existed nothing like it. Papers introducing the Art or aspects of it often leave the reader puzzled. Readers who turn to

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books by specialists describing the Art, like (Platzeck, 1962), (Bonner, 2007) and (Fidora & Rubio, 2008), find themselves in a maze.

I will approach the Art in the following way. I will first discuss Llull's *Llibre de contemplació* (Book of Contemplation), which he wrote before he had the illumination. I will consider the first text Llull wrote after the illumination, the *Ars compendiosa inveniendi veritatem* (Compact art of finding the truth). Then I will argue that the illumination very probably consisted of four elements: *conceptual atomism*, a *specific grouping* of the atomic concepts, a *circular representation* of the groups, and an insight into the universal value of *combinatorics*. I will discuss the roots of these ideas and briefly review the ideas of some other scholars on the nature of the illumination that Llull experienced. Finally I will attempt to evaluate the Art. I will argue that it is not surprising that several authors have described the Art as worthless or as pseudoscience. Yet such judgments are unfair and they show a misunderstanding with respect to medieval and Renaissance philosophy. Moreover, llullism was so influential that it deserves to be studied. I will conclude the paper with the intriguing line of influence from Llull's theological considerations towards one of 17th century mathematics major leaps forward: differential and integral calculus.

God tells Llull to leave his worldly life

Towards 1263 Ramon Llull, the administrative head of the household of the King's son Prince James II, had several disturbing visions[1]. He saw the Lord Jesus Christ on the cross, as if suspended in midair. This happened five times in a period of a few weeks. Llull, 31 years old at the time, was terrified. Musing on the significance of these visions it became clear to him that God wanted him to leave his former worldly life, leave his wife and two daughters and dedicate himself totally to the service of Christ. More specifically he realized that God wanted him to prepare the best imaginable book against the errors of the unbelievers, in particular the Saracens, the Muslims.

Nine years of preparation followed. Llull set out on a pilgrimage to the shrine of Our Lady of Rocamadour in France and to the shrine of Saint James in Santiago de Compostela in Spain. After this long voyage he returned to Mallorca and devoted himself to studies, possibly in the Cistercian monastery of La Real near Palma[2]. Of course, he could have gone to Paris in order to get a formal scholastic education, but he did not. He realized that speaking the language of the Saracens and being familiar with their

Figure 1. Ramon Llull (1232–1350) Engraving published by Friedrich Roth-Scholtz

culture were necessary conditions in order to make his enterprise succeed. So he bought himself a Saracen slave from whom he learnt Arabic.

Mallorca had been ruled by Muslims for three centuries until 1229 when King James of Catalonia and Aragon, the father of Prince James II, conquered it again. In Llull's time Mallorca was still a melting pot of cultures inhabited by Christians, Muslims and Jews. It was the ideal environment to learn about the Jews and the Saracens. Llull used the opportunity well. It seems that in this period he participated under the patronage of Prince James II in disputations with Jews and Saracens who lived in Palma (Peers, pp. 44–45).

Llull's first attempt to write a book that should convert Jews and Muslims to the Christian faith was the *Llibre de contemplació* (Book of Contemplation). He wrote its first version in Arabic. Llull made a Catalan translation, and by doing so he made himself immortal in Catalonia. It is the first scholarly work written in Catalan. The Catalan text is very long. In Volume 2 of the *Obres Essencials* (llull, 1957–1960) the book covers more than 1000 pages in a very small letter font.

The *Llibre de contemplació*

The prologue of the *Book of Contemplation* and the 366 chapters (the numbers of days in a year plus a concluding chapter) all start with words like «Ah Jesucrist, senyor nostre» (Prologue), «Ah Déus senyor gran, meravellós!» (Chapter 10), «Ah Déus gloriós, vertuós, poderós, Senyor sobre tots senyors» (Chapter 281), «Déus vertuós, amorós, de gràcies e de benediccions abundós!» (Chapter 328), «Ah Déus, savi sobre tots sabers, acabat en tots acabaments!» (Chapter 366). All chapters consist of 30 paragraphs and many of the paragraphs begin in such a way too. The whole text praises, glorifies, and eulogizes God. The creation, Christian dogmas – they all show the greatness of God. In a sense the book is one long prayer.

In chapter 328 of the *Book of Contemplation* something remarkable happens. Llull starts to use letters to denote crucial notions. This continues until chapter 364. In chapter 335 for example we find the following table:

A	Secret de Déu (God's secret)	M	Secret intellectual (Intellectual secret)
B	Trinitat (Trinity)	N	Cara d'En Pere girada a llevant (Face of Peter towards the East)
C	Humanitat de Jesucrist (Humanity of Christ)	O	Cara d'En Pere girada a ponent (Face of Peter towards the West)
D	Predestinació (Predestination)	P	Déu (God)
E	Saviesa de Déu (God's wisdom)	Q	Justicia (Justica)
F	Poder de Déu (God's power)	R	Franc voler (Free will)
G	Voler de Déu (God's will)	S	Home precís (individual man)
H	Enteniment de Déu (Knowledge of God)	T	Infern (Hell)
I	Vera significació (True meaning)	V	Paradis (Paradise)
K	Falsa significació (False meaning)	X	Pecat (Sin)
L	Secret sensual (Secret of the sensual world)	Y	Bé (The good)

Figure 2. Table of fundamental notions, Chapter 335 (Llull, 1957–60), Vol. 2, p. 1098

The notions in the table, all denoted by means of letters, reveal Llull's intentions. It is about getting to know God's secret, denoted by A. This secret includes the Trinity B, the Human nature of Christ C, and Predestination D. The secret is not easily revealed, but contemplation of the

attributes of God, his Wisdom E, his Power F, and his Will R can lead to the desired knowledge H of God by a human being S. This requires that one distinguishes between false meanings K and true meanings I.

The predestination D of the individual human being S presents the human being with a problem. God's Power F implies Predestination D which seems to contradict the Free will R of S while God's Justice Q requires the Free will R so that man can choose between Sin X and Good Y and in this way influence his destiny: Hell T or Paradise V.

In order to solve this problem Llull compares the secret of the sensual world L and the intellectual secret M to mirrors. When we try to understand L or M we are looking in mirrors. We are like Peter looking to the east N and getting from the mirror the impression that he is looking to the West O. When we interpret predestination D, we are in a similar situation. God's Power F and his Wisdom E are perfect and imply predestination D. Yet we are not allowed to draw the conclusion that Free will does not exist, because God's Justice Q implies the existence of Free will R.

Llull often uses analogies in order to make a point. In my words: Peter is looking to the East in a mirror and sees John in front of him, while John is actually behind him. Contradiction! Yet, this is no real contradiction. The same holds for Free will. Why would I not be able to choose freely while God has decided and knows what I will choose? It seems a contradiction but it is not. God simply knows in advance what I will freely choose.

Llull uses the letters as abbreviations. Moreover, the same letters are used to denote different notions in different chapters. This makes life difficult for the reader. Look at a sentence like:

> the demonstration would give D to the H of A with F and G as it does with E, and yet K would not give significance to the H of any defect in F or G; but since diversity is shown in the demonstration that D makes of the E and the F and the G with the I and the K, therefore the H has certain scientific knowledge of Thy holy and glorious Trinity".

It is not my intention that the reader attempts to understand this, although he may try, of course. The sentence occurs in paragraph 17 of the same chapter 335 (See Fig. 2 above for the meaning of the letters[3]. It is hardly legible. A commentator, Carreras, wrote: "Reading becomes unbearable". Another smart commentator answered to Carreras: "the secret is to substitute in the text for the letters their meaning" (*Obres essencials*, Vol 2, p. 1268, footnote 264). Platzeck wrote with respect to the sudden use of letters that occurs in and after chapter 329:

> The reason for this change must have been very convincing; because Raimund puts away purely literary objections and pursues this new method obsessively (Platzeck, 1962 Band 1, p. 324).

I feel that Llull would have noted that in his arguments in the different chapters always a limited number of central notions could be distinguished. By denoting them by means of letters, and by forcing the reader to go back to the table again and again, these notions receive extra emphasis. In chapters 329–364 Llull was exploring this new possibility.

Llull also played with figures. For example, in chapter 335 we have a figure in the form of a U-shaped coat of arms with 11 letters in it.

Figure 3. Chapter 335, Llibre de contemplació, Obres essencials, Volume 2, p. 1101

The figure illustrates the problem of predestination that we briefly discussed above. E, F and G are the attributes the Wisdom, Power and Will of God. K refers to their false interpretation. D, S and R refer to the individual human being S between predestination D and free will R. X and Y are Sin and Good corresponding to T and U, Hell and Paradise. In Chapter 364 Llull uses 8 letters to denote different notions and 8 other letters to denote rectangles filled with combinations of the first 8 letters in a way similar to the U-shaped coat of arms above.

The Art is revealed to Llull

Llull was not fully satisfied with the *Book of Contemplation*. Maybe he felt that there was not enough unity, no explicit method and that the arguments were not convincing enough. In 1272 the slave that had taught Llull Arabic committed suicide after having been put in jail for attempting to kill Llull. The man tried to kill his master because of a beating he had received from Llull after blaspheming the name of Christ. As a result of the suicide Llull retired to Mount Randa for meditation. After eight days on the mountain while gazing heavenward the Lord suddenly illuminated his mind. God gave him "the form and method" for writing the book against the errors of the unbelievers. Llull came down from the mountain and in the abbey of La Real he wrote his *Ars compendiosa inveniendi veritatem* (ACIV: Compact Art of Finding the Truth), the first book in which he explained the *Art*. After having finished the book Llull allegedly went back to the spot where God had shown him the method. He had a hermitage built exactly there and he stayed there for four months without interruption, praying to God night and day that He would bring prosperity to the Art for the sake of the benefit of His church.

The subject of this paper is the illumination that Llull had in 1272 on Mount Randa. Concentrating on the illumination we hope to get closer to understanding Llull's Art. The Art is like a forest in which one easily gets lost. What are its essential elements? The illumination on Mount Randa might help us here.

Conceptual atomism, specific grouping, and circular representation

In order to speculate about the illumination we need to know the new insight that led to the writing of the *Ars compendiosa inveniendi veritatem* (ACIV). One expects it to be related to the *Llibre de contemplació*. The first striking aspect of the *Art* that is suggested by Llull's experiments with the use of letters in the last chapters of the *Llibre de contemplació* and is characteristic of the the *Ars compendiosa inveniendi veritatem* is *conceptual atomism*: understanding God and His creation is based upon a finite number of concepts[4]. This idea immediately generates the question: Which are these concepts? In the ACIV Llull gives his first answer to this question.

The core of the answer consists of five different sets of fundamental concepts that are put on the circumference of five circles. Next to the con-

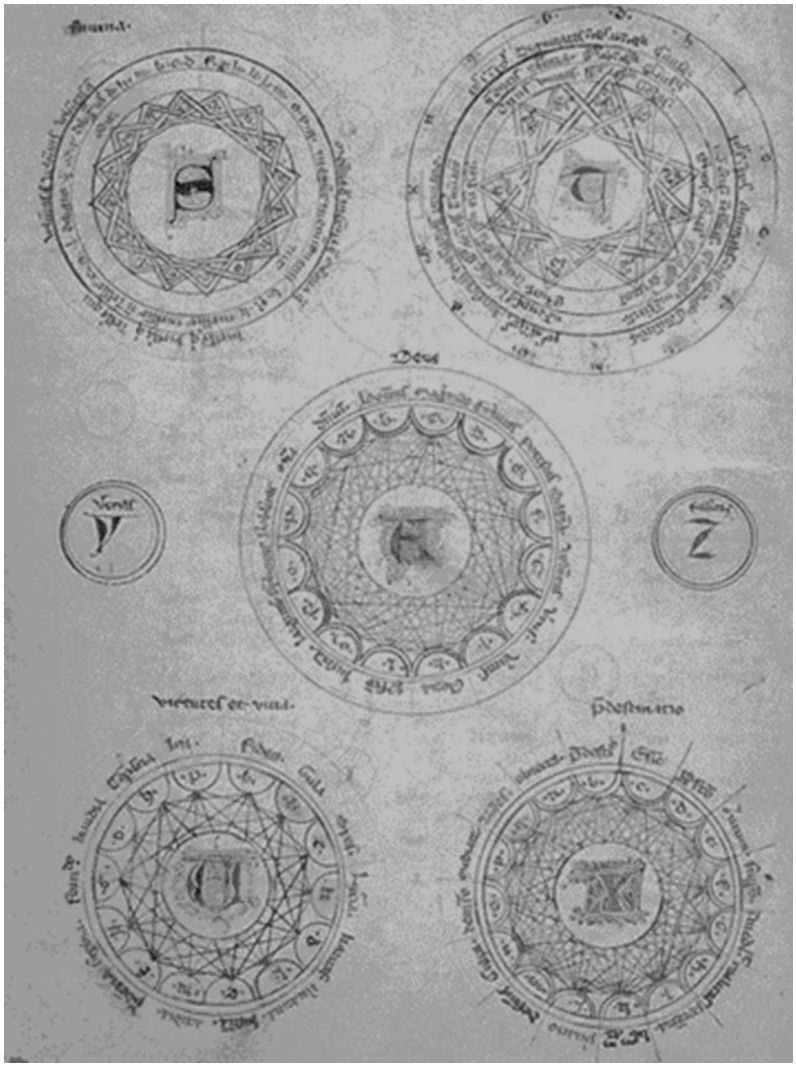

Figure 4. The machinery of the Art from the Ars demonstrativa. Source: manuscript VI 200, Biblioteca Marciana in Venice[5]

ceptual atomism this **specific grouping** and the **circular representation** are the second and third characteristics of the Art. See figure 4.

Nota bene, the picture in figure 4 is not from the ACIV but from the *Ars demonstrativa*, which was Llull's second attempt to describe the Art. Yet the same drawings are also included in the ACIV. The main difference with the ACIV is that in the *Ars demonstrativa* all notions are denoted by means of letters while in the ACIV the use of letters is still rather limited as we will see below.

The two small circles in figure 4 are Y for truth and Z for falsehood. The basic idea of the other circles is that the rational human mind, represented by the circle S in the top left corner, contemplates the circle A, in the center, and the circles V and X at the bottom of the page. Circle A represents God and has on its circumference his 16 attributes (or Divine virtues of dignities as Llull also calls them): Perfection, Justice, Power, Will, Wisdom, etc. Circle V has the seven virtues (Justice, Prudence, Charity, Faith, etc.) and the seven vices (Envy, Avarice, Temperance, etc.) on its circumference. Circle X has 16 other crucial notions on its circumference; several of them are oppositions or seeming oppositions: Being, for example, is opposite Non-being and Predestination is opposite Free will

The rational soul S contemplates the concepts from the circles A, V, and X using "logical" notions from circle T in the top right corner of figure 4. On the inner circumference of T we have 15 concepts. Examples are God, Difference, and Negation. Figure T is the only figure of which all of its concepts, written on the inner circle, have three secondary concepts that are written in outer circles. For example, God is one of the T's concepts and it has the secondary concepts Unity, Essence, and Dignities.

The circles in figure 4 do not contain all notions that we need in order to understand God and his creation fully. Other figures concerning the four elements, Fire Air, Water, and Earth, are needed for the principles of medicine. The principles of theology, natural philosophy and law too require three extra figures, each with 16 concepts concerning the discipline involved.

Combinatorics

The fourth striking aspect of the ACIV is that Llull is very much aware of the importance of combinations of fundamental notions. The ordering of sets of simple notions equidistantly on the circumference of a circle means that combinations of notions of the same kind correspond to geometrical figures. All notions together on a circle correspond to a regular polygon. Very important to Llull are the sides of the polygon and the diagonals, representing the combinations of two notions. Llull calls such combinations of a finite number of notions 'compartments' ('camere' in Catalan).

In the ACIV the circle S, corresponding to the rational soul, plays a central role. See figure 5. Here we find combinations of more than 2 notions and even combinations of combinations of notions. The functioning of the rational soul is based on the three Augustinian powers of the soul: memory, intellect, and will. The black square corresponds to the letter I, which is

the act of FGH and that is the combination of remembering, understanding, and hating. So the black square I represents the rational soul rejecting a statement. The blue square corresponds to the letter E, which is the act of BCD and that is the combination of remembering, understanding and loving. So the blue square E represents the rational soul accepting a statement. The red square N stands for the act of KLM, that is the act of forgetting, not knowing, and "loving or hating". The red square N stands for the rational soul not knowing. It is used when a supposition is being studied. The last square is the green square R. It corresponds to a state of total confusion.

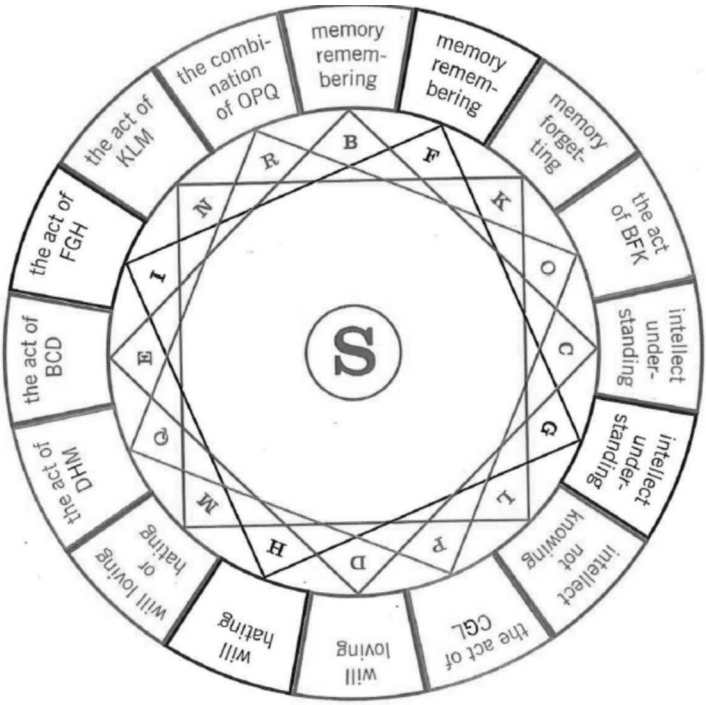

Figure 5. The acts of the rational soul (Bonner, 2007). Courtesy of Koninklijke Brill NV, Leiden The Netherlands

We are now ready to read a fragment from the ACIV. The fragment concerns the problem of predestination[6].

The affirmation: "S in EINR, with T in X enters the compartment of [**perfect wisdom**] [...]. As a result, when E contemplates A, the compartment of [**perfect predestination**] is formed by necessary reasons [...]. As a result of E remembering and understanding [**perfect wisdom**] in A, R falsely signifies [...] the compartment of [**free will non-being**]. And then N loves free will which it forgets and does not know, even when E in the First

Figure enters the compartment of [**perfect predestination**], and therefore I is interposed between E and N, with FGH attaching themselves to predestination, because of the fact that K forgets and L does not know free will for salvation or condemnation."

Let us try to understand the quotation. One notices that compartments consisting of two notions play an important role in the argument. In the original text they are enclosed in rectangles. The rational soul S in the possession of all its capabilities EINR enters the compartment of [**perfect wisdom**]. This is a contemplation of a compartment of God represented by the segment connecting two of the attributes of the circle A. It leads to the rational soul S to clearly understand a compartment of the circle X: [**perfect predestination**]. The role of T is not made explicit by Llull, but it is obviously needed to go from perfect wisdom to perfect predestination. Possibly notions like Beginning, Middle, End and Concordance, that all occur in T, could be used. After having understood perfect predestination confusion R takes over in the rational soul S. The reason is that [**perfect wisdom**] seems to imply another compartment of X: [**free will non-being**]. Yet the rational soul cannot deny free will. In Llull's words:

The denial: "[...] the contrary of the first in S, since as soon as E enters A by means of [**perfect justice**], B remembers and C understands free will [...]"

The opposition between the affirmation and the denial leads the rational soul to stance R: total confusion.

The Doubt: "Since S finds itself confused [...] with R joined to each of EIN, it therefore forms [...] the doubt. For just as fire strongly signifies dryness and water humidity, thus the wisdom of A strongly signifies predestination and the justice of A signifies free will equally strongly. Both C G cannot at one and the same time understand perfect predestination or perfect free will, and thus C G are in doubt, and don't dare to affirm or deny either predestination or free will, and all of S is perverted into R ..."

Fortunately we can resolve the doubt. Both predestination and free will exist:

The Resolution: "We place S, under the heading of E, in A, in the compartments of [**perfect power**] and [**perfect will**] [...] Now when C affirms these two compartments in A, it then affirms the compartment of [**predestination and free will**]."

One notices that the elements of the illumination that we mentioned do to a certain extent determine the form and the method of the argumentation, but hardly the content. And indeed form and method (forma I manera) were revealed to Llull on Mount Randa.

A new universal Art

We have seen how Llull uses the conceptual atoms and their combinations in the ACIV. Conceptual atomism, the possibility to systematically combine the fundamental notions and, from Llull's point of view, its successful application to major theological problems, convinced Llull of the fact that he had found something really new: a new Art applicable to any subject in which a systematic search for new arguments was in principle possible. I think that this was the Illumination: **Conceptual atomism, the specific grouping of the atoms, the circular representation, and the suggestion that systematic combination leads to all truths. This concerns both the form and the method. As for method the guiding idea, already present in the *Book of Contemplation*, is that all truths about reality and in particular the truth of the Christian dogmas follow necessarily from the Divine attributes of God.**

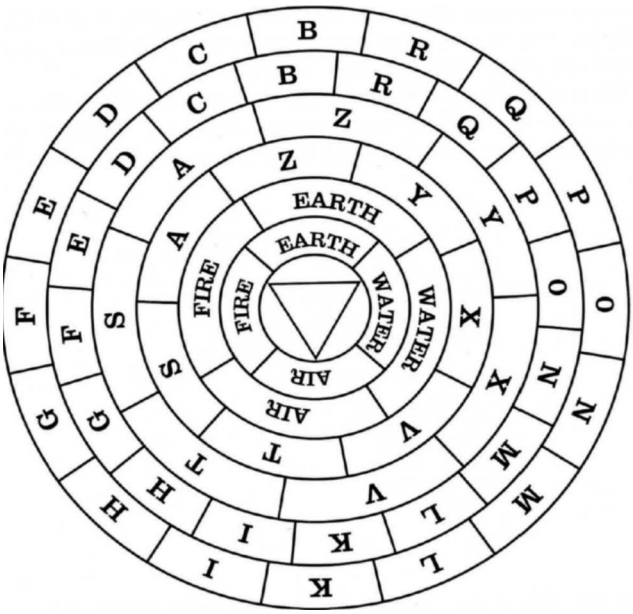

Figure 6. The demonstrative figure in the Ars Demonstrativa[7] Courtesy of Koninklijke Brill NV, Leiden The Netherlands

We find a very spectacular example of Llull's inclination to systematically combine notions in the demonstrative figure of the *Ars Demonstrativa* from about 1283 (See figure 6). In the *Ars Demonstrativa* Llull now denotes all the concepts on the circumferences of the circles of his Art by

means of letters[8]. The demonstrative figure is composed of six revolving circles each inside the other. In the medieval manuscripts the outer circle was drawn on the parchment and the other circles were drawn on separate pieces of parchment. A string held them together. The triangle in the center can rotate too. Let us describe the demonstrative figure from the inside to the outside. There are two inner circles each with the four elements earth, water, air, and fire. Then follow two circles each with the letters representing the core figures of the Art: A, S, T, X, Y, Z. The two outer circles are identical, each has 16 letters: B through R. Llull viewed the outer circles as multifunctional: the letters could be made to stand for the concepts of the circle A, the circle S, and for the circles of theology, philosophy, and medicine as well. By rotating the circles one can in principle systematically search all possible combinations.

Clearly there is quite a difference between Llull's art in practice and the ideal, and it is obvious that in no way is the ACIV a formal deductive system in the modern sense of the word. The same holds for the later versions of the Art. The intention of the Art is to deduce particulars from the universals of the Art. Llull would say that T must be introduced into the compartments of the Art, meaning that by means of the logical notions of T the particulars can be derived from the universals. Let us consider an example from the *Ars Demonstrativa*:

Question: Whether God can alter the end for which He has created a creature.

Answer: [TY | AA | EAY | IAZ]

At first F remembers the second compartment, and then the first, and G enters the third and fourth compartments, in which it understands the solution to this question; for by means of AA, F remembers Y in T, that is to say, how A has in itself goodness, wisdom, perfection, etc., and thus the end for which a creature has been created is unalterable, in order for the third compartment to be able to accord with the end, and not the fourth; for if the latter accorded with end, then the third would be in disaccord with the second, which is impossible[9].

The answer consists of a "figure" made up of four compartments: [TY | AA | EAY | IAZ]. According to Llull, the figure "signifies" the solution to the problem. This example comes from the first part of the *Ars Demonstrativa* where Llull is still kind enough to add an explanation to the solution. I remind the reader that the letters mean the following: F = memory remembering, G = intellect understanding, A is the figure God and his attributes, Y is truth, Z is falsehood, T is the figure of the logical instruments. E = the combined act of remembering, understanding and loving,

I = the combined act of remembering, understanding and hating. In my words the explanation of the answer runs as follows: If you remember God, then you will understand the solution by means of the third and fourth compartment. Because then you remember the truth about God: goodness, wisdom, etc. Compartment EAY merely means "understanding and loving the truth about God". Compartment [IAZ] merely means "understanding and hating the truth about God and believing falsehood about God". What Llull is basically saying is: The attributes of God imply that he cannot alter the end for which he has created a creature.

Obviously Llull's Art does not generate truths in any very precise way and certainly for a modern student of the Art it promises much more than it gives. It is remarkable that Bessarion, a close friend of Nicholas of Cusa, is said to have been present at a hunting party. When Bessarion during the hunt asked where a hare had gone his host told him: "Apply the rule CDK of the Ars Magna and you will find the hare"[10]. Although the host quoted exactly Llull's answer to the question Where? in the *Ars Brevis*, this was of course a joke and it is quite possible that it expresses a skeptical attitude with respect to the Art. Indeed the imperfections in Llull's work have led several authors to a negative judgment about the Art. In the 19th century Carl Prantl devoted more than 30 pages to Llull and his work. One cannot say that he did not seriously try to understand it. He concluded the chapter on Llull with the words: "that the whole art of Lullus is simply worthless now needs no further proof"[11] and in 1958 Martin Gardner classified Llull's work as pseudoscience, calling Llull a "remarkable tragicomic figure".

Yet, there are several reasons why we must reject this negative judgment. One should not judge a medieval author by means of 19th or 20th century criteria. Below, in the section on the impact of the Art, I will return to this point. Llull was an intelligent man. He, for example, invented an electoral procedure for the election of a new abbess by the nuns of a nunnery. See (Maclean, 1990) and (Hägele & Pukelsheim, 2001). Suppose we have nine candidates. Denote all of them with the letters a through i. Now form all 36 compartments of two different candidates. See figure 7 which is from Llull's *De Arte Eleccionis* (1299).

The voters now should express their preference with respect to all pairs. The candidate to be elected should be the candidate who wins most votes in the aggregate of the pairwise comparisons. This is what is usually called a Borda-procedure, because for a long time we thought Borda was the first to invent it in 1781. Now Llull seems to have been the first. Llull's ideas on electoral procedures nicely show that denoting crucial elements in a discourse by means of letters and considering combinations was an important

bc	cd	de	ef	fg	gh	hi	ik
bd	ce	df	eg	fh	gi	hk	
be	cf	dg	eh	fi	gk		
bf	cg	dh	ei	fk			
bg	ch	di	ek				
bh	ci	dk					
bi	ck						
bk							

Figure 7. Based on (McLean, 1990)

procedure in Llull's Art. It is understandable that he presented the electoral procedure as an application of the Art.

Some of Llull's arguments are fascinating. In order to show that the Christian dogma of the Trinity is a necessary truth Llull argued as follows in his *The Book of the Gentile and the Three Wise Men*[12]. He starts from the compartment of [Goodness Greatness] and argues, in my words, that Goodness and Greatness are in accordance with Perfection and Infinity. This Perfection implies that God cannot merely be a passive being whose power is only potential. His perfection implies that in God there must exist a great, perfect, and infinite good that necessarily begets in Him another great, perfect, and infinite begotten good. Moreover, the act of begetting in itself is great, perfect, and infinitely good as well. This proves that in God exists trinity[13]. When the gentile asks the Christian, who is one of the wise men (the others are a Jew and a Saracen), whether God would not be even more perfect if there existed four, five, or even more of these infinitely good things in him, the Christian obviously disagrees. The existence of more of these infinitely good things would be a sign of imperfection, which contradicts the perfection of God.

The roots of the Art

What are the roots of this conceptual atomism, the specific grouping of the atoms, the circular representation, and the suggestion that systematic combination leads to all truths? As for the specific choice of the atomic concepts, Josep Rubio in (Rubio, 1997) has systematically discussed their roots in the *Book of Contemplation*. Platzeck has suggested that Llull was familiar with Proklos' work on the symbolic meaning of geometrical figures. He argued that when Llull on Mount Randa was watching the stars in

their eternal circular movement it occurred to him that God's attributes necessarily had to be viewed on a circle[14]. Another suggestion by Platzeck concerns the kaballah. The *Sepher Yetzira*, the oldest known speculative treatise in Hebrew and certainly written before the tenth century, may have been a source. The most scholarly edition seems to be (Hayman, 2004). The text tells of the powers of the characters 0 through 9 and the 22 Hebrew letters. It is a text in which the emphasis is on combinations of letters. In one translation, paragraph 18 says:

> the twenty-two letters are the foundation. They are fixed on a wheel with two hundred and thirty-one gates. The wheel rotates backwards and forwards[15].

Platzeck argues that Llull may have gotten the basic idea of his demonstrative figure here[16]. When the 22 letters of the Hebrew alphabet are fixed on a wheel and the wheel rotates inside a circle with these same 22 letters we get 231 combinations of two different Hebrew letters. Llull may also have gotten the idea of the importance of binary combinations here.

Pring Mill has come up with an entirely different suggestion in which the major source of the illumination lies in the, at the time, generally accepted theories about how the four elements Fire, Air, Water, and Earth combine in physical reality. After having pointed out the combinatorial nature of the Art and the fact that Llull adhered to a neo-Platonic world picture, he thought it likely that Llull had seen on Mount Randa that the way in which the elements combine in the material world could also be applied to the higher levels of the ladder of being in which God takes the top position. This is an interesting suggestion. The existing theories of how the elements combine in the material world were of particular important in medicine. The following table characterizes some fundamental correspondences in such theories.

Element	**Sphere**	**Proper quality**	**Appropriated quality**	**Organ**	**Humor**	**Temper**	**Active or passive**	**Color in Llull's figures**
Fire	Lightest, Top	Hot	Dry from Earth	Gall bladder	Yellow bile	Choleric	Active	Red
Air	Light; Below top	Wet	Hot from Fire	Head	Blood	Sanguine	Passive	Blue
Water	Heavier; Above bottom	Cold	Wet from Air	Lungs	Phlegm	Phlegmatic	Active	Green
Earth	Heaviest. Bottom	Dry	Cold from Water	Spleen	Black bile	Melancholic	Passive	Black

Figure 8. The four elements and their correlates in Llull's work

In the physical theories of the Middle Ages each of the terrestrial elements was supposed to have two qualities: one proper quality and one appropriated quality coming from another element. Fire was hot and dry, Air was wet and hot, Water was cold and wet, Earth was dry and cold. In the human body the elements were thought to correspond to bodily liquids called humors. It was believed that pathological human moods, emotions, and behaviors were caused by an excess or lack of these body fluids: yellow bile, blood, phlegm and black bile. Llull:

> In a sickness the humors are mixed according to their action and passion. Indeed they function in one way in the humoral mixture which has an excess of choler, in another in a mixture with an excess of blood [...] (Bonner, 1985), Vol. 2, p. 1127.

In *The Principles of Medicine*, written by Llull in the same period in which he wrote the ACIV, the four elements, represented by their proper qualities, are denoted by the letters A (Fire-Heat), B (Earth-Dryness), C (Air-Wetness) and D (Water-Coldness). He combines these letters in order to define 16 characteristic herbs denoted by the letters E, F, G, H, K, L, M, N, O, P, Q, R, S, T, V and Y. They are characterized by the degree of presence of the elements A, B, C and D. For example, herb E is characterized by A in the 4th degree, B in the 3d degree, C in the 2nd degree and D in the 1st degree. Pepper is an example of such an herb[17]. These 16 letters and their defining mixtures correspond exactly to the 16 rows in what is called the *elemental figure* (See Figure 9). The elemental figure is also included in the ACIV. It consists of four squares that are each dominated by one particular element. All rows of the figure of fire are dominated by fire, which means that in the mixtures they define, fire is present in the highest degree.

This is not the place to discuss Llull's views on medicine. Important for our purpose is that the idea that the investigation of systematically generated combinations of fundamental notions forms an important guiding principle. Given Llull's approach to medicine it is understandable that he studies the mixing of two herbs out of the list E through Y. These mixtures correspond in Lull's Art to compartments consisting of two letters, like EF, ES etc. The *Principles of Medicine* is a complex book, but it is definitely an application of the Art. In this work also the triangles from Figure T (Figure 4, top right) play an important role: the triangle Beginning-Middle-End, the triangle Contrariety-Difference-Concordance and the triangle Minority-Equality-Majority.

Figure of Fire

Fire	Air	Water	Earth
Air	Fire	Earth	Water
Water	Earth	Fire	Air
Earth	Water	Air	Fire

Figure of Air

Air	Fire	Water	Earth
Fire	Air	Earth	Water
Water	Earth	Air	Fire
Earth	Water	Fire	Earth

Figure of Water

Water	Earth	Air	Fire
Earth	Water	Fire	Air
Air	Fire	Water	Earth
Fire	Air	Earth	Water

Figure of Earth

Earth	Water	Air	Fire
Water	Earth	Fire	Air
Air	Fire	Earth	Water
Fire	Air	Water	Earth

Figure 9. left The elemental figure as it occurs in the ACIV. Source: Biblioteca Virtual del Patrimonio Bibliográfico, Manuscript 1053(1)

Frances Yates has pointed out that Llull very often uses the elemental processes as he worked them out in *The Principles of Medicine* as a metaphor in ethics and theology. She called this "elemental exemplarism" and quoted, for example, Llull as follows:

> [...] the divine nature does not receive any alteration in the conjunction which it makes with the human nature: as fire does not receive any alteration in entering air through warmth but raises air to a nobler state through imparting to it its warmth through concordance, so the human nature is exalted in receiving the divine nature through concordance. Therefore the Saracens and Jews do wrong in denying the Incarnation of the Son of God, saying that God altered [...] in being joined to man [...] (Yates, 1954), p. 152.

She did this in a beautiful paper on Llull's astrology (Yates, 1954) in which she analyzed Llull's *Tractatus novus de astronomia* written in 1297. In this text Llull develops a very elegant astronomical theory in which the four elements play a central role. They are denoted by letters and the influence of a particular position of the planets is determined by combinations of these letters. The fact that the signs in the Zodiac and the planets all have special ties with the elements plays of course an important role.

Yates' paper could be used as an argument supporting Pring Mill's suggestion that the medieval theory of elements was of crucial importance in bringing about the illumination on Mount Randa. Platzeck basically rejected this suggestion. He argued that in the ACIV the elements play a secondary role[18]. I am inclined to agree with Platzeck. It seems to me probable that the Art was born more directly from the *Llibre de contemplació*.

In 2008 Ruiz and Soler wrote an interesting paper called *Ramon Llull in his historical context* in which they discuss Lllull's work and life against its wider cultural background. With respect to the illumination on Mount

Randa, they point out that Llull only started talking about it in the middle of the last decade of the 13th century. They suggest that Lllull, who was as we have seen operating without a formal scholastic education, had a permanent problem of legitimation. They point out that already in the *Book of Contemplation* Llull had expressed worries about his "slight authority" and that he was in a constant search for literary strategies to overcome this handicap. Yet when Llull started his enterprise he was extremely optimistic. In the 1290s he had come to realize how difficult what he had set out to do really was. By calling the illumination a "supposed enlightenment" Ruiz and Soler clearly suggest that the story about the sudden illumination was made up by Llull near the end of the 13th century. They may be right, but without further evidence I feel we must assume that Llull's own account is correct.

The impact of the Art

After writing *Ars compendiosa inveniendi veritatem* and its companion volumes on medicine, theology, philosophy, and law, Llull's Art underwent considerable development. The *Ars demonstrativa* (around 1283) is a much better organized presentation of the Art, although the basic features of the Art as it was presented in the ACIV are maintained. One difference is that almost all basic concepts are now denoted by means of letters and each figure is accompanied by a secondary figure showing all binary combinations of the fundamental concepts. (See figure 10.) In the course of time Llull realized that many students found the Art extremely difficult and he decided to simplify it. This culminated in the appearance of the *Ars brevis* in 1308.

In 1376 twenty of Llull's works were condemned in a Papal bull and in 1390 the Faculty of Theology in Paris condemned Llullist doctrines. He had apparently been influential; otherwise the authorities would not have bothered, although it is not so easy to precisely characterize his influence. In the 14th century alchemical works started appearing associated with his name, although he never wrote alchemical works at all[19]. This too points at his fame. His works continued to be copied, printed and read. Descartes refers to Llull, Newton had Llull's work in his library, and in particular Leibniz was influenced by him. There even existed a popular song in Spain: "Hubo tres sabios en el mundo: Adán, Salomón y Raymundo!" Sometime after the Scientific Revolution, Llull fell into obscurity in mainstream science[20]. Yet, this changed again in the second half of the 20th century. One reason is that some logicians and computer scientists see him as a precursor.

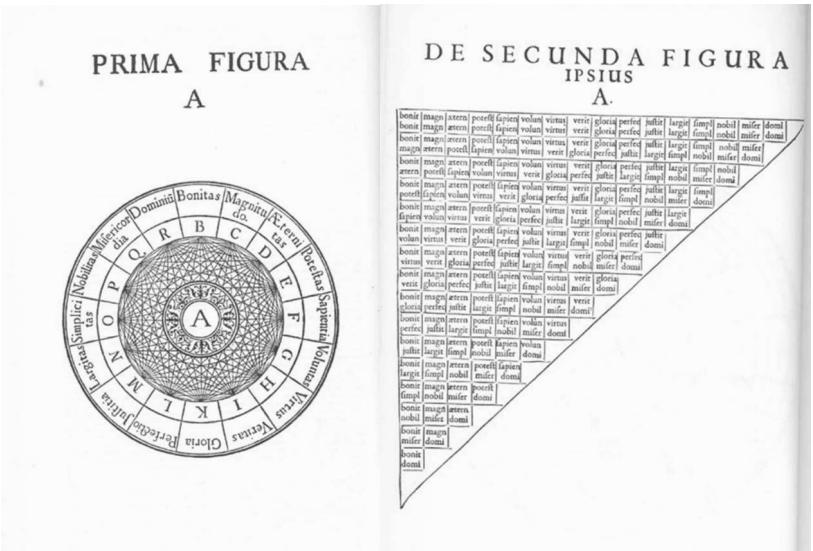

Figure 10. Figure A from the Ars demonstrativa, representing God and his dignities. Source: Mainz Latin edition (MOG), 1722, Vol. III[21]. The second figure shows all binary combinations

What was the real impact of Llull's Art? That is not so easy to say. He was widely read until the 18th century. However, which of his ideas in the end survived? The following quotation helps us judge Llull fairly:

> Between the mid-fifteenth century and the mid-sixteenth century, Nicolaus Cusanus, Cardinal Bessarion, Pico della Mirandola, Lefèvre d'Étaples, Charles de Bovelles and later Bernardo Lavinheta, Henricus Cornelius Agrippa and Giordano Bruno disseminated and commented on the works of Llull dealing with the **ars magna** and **combinatoria**, and initiated what was to become an intellectual obsession in European culture. The full significance of these figures and their adherence to a set of ideas which are completely alien to a post-Cartesian and post-Galilean mentality has been misunderstood both by interpreters who have seen the **ars magna** as a kind of historical precursor of symbolic logic, and by those who have preferred to dismiss, with facile irony, the 'strangeness' of many of the most significant thinkers of this important period of western culture[22].

Medieval and Renaissance authors had no idea about the future development of science. They were groping in the dark. In Lull's approach there is much that is indeed completely alien to the spirit of the scientific revolution. However, Rossi is right, Whig history is a very poor kind of history and Llull and Lullism absolutely deserve to be studied[23]. And there is

more. The lullist dreams of a universal art and a combinatorial approach to science were absorbed by 16th century culture and their influence guided several of the great 17th century minds. On 26 March 1629 Descartes wrote to Beeckman as follows:

> And to tell you quite openly what I intend to undertake, I do not want to propound an **Ars brevis** like that of Lullius, but a completely new science by which all questions in general may be solved that can be proposed about any kind of quantity, continuous as well as discrete[24].

This is the time when Descartes started his work on analytic geometry. Although Descartes rejected the *Ars brevis*, his dream of a completely new science was guided by Llull's concept of a universal art.

Even clearer is the influence of Llull's ideas on Leibniz. In 1666 as a young man Leibniz wrote his *Dissertatio de Arte Combinatoria*. He explicitly refers to Llull and to Llullists like Athanasius Kircher. Leibniz was fascinated by Llull's conceptual atomism, the view that concepts are compounds of a finite number of atomic primitive concepts. He differed from Llull in the sense that Llull according to Leibniz had not developed this idea in a sufficiently precise way. Leibniz proposed to give the system an arithmetical basis along, for example, the following lines: if 2 is rational and 3 is being alive, man as a rational living being is 2.3=6. Leibniz attempted to treat Aristotelian logic in this way, which turned out to be quite complicated[25]. Several of the mathematical problems that Leibniz touched in his dissertation continued to stimulate him[26]. Leibniz's project of a *characteristica universalis* (universal character in English) is obviously also a project entirely in the spirit of Llullism. This characteristica universalis is a recurring theme in Leibniz work. It undoubtedly guided him in his approach to the calculus.

Herbert Breger wrote the following about Leibniz:

> Rationality should be such as to allow for a mathematisation of our thought; just as mathematicians have introduced letters and other symbols to designate mathematical objects and rules for operating with them, Leibniz proposed to formalize a considerable part of our thought. Then two philosophers with different opinions on a philosophical topic would no longer need to quarrel; they could say to each other 'calculemus' (let's calculate). Therefore, Leibniz' invention of a calculating machine had a strong philosophical relevance. And besides, Leibniz tells us, this characteristica universalis will be an efficient means of converting pagans, because the true religion is the most rational religion and it is impossible to resist rational arguments.[27]

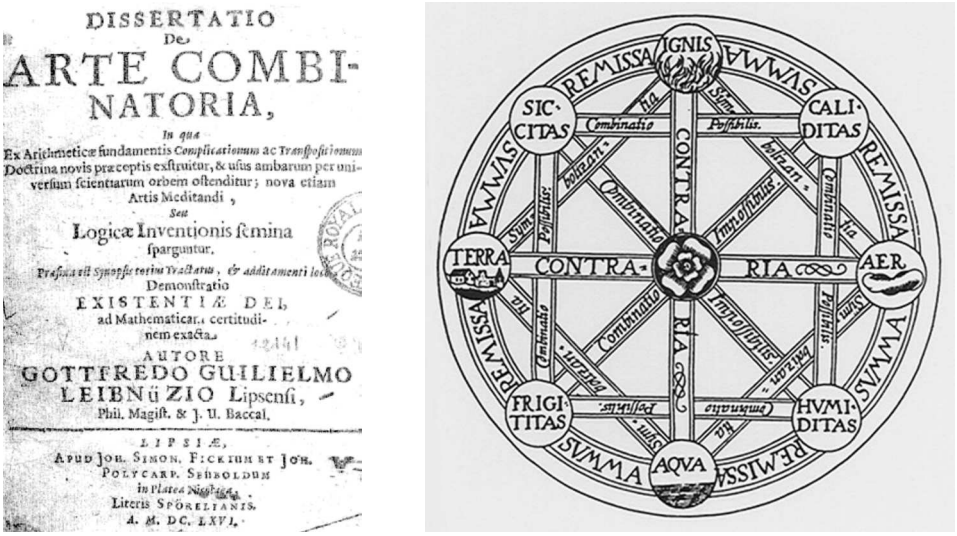
DISSERTATIO
De
ARTE COMBI-
NATORIA,
In qua
Ex Arithmeticæ fundamentis Complicationum ac Transpositionum Doctrina novis præceptis exstruitur, & usus ambarum per universum scientiarum orbem ostenditur; nova etiam Artis Meditandi,
Seu
Logicæ Inventionis semina spargunturs.
Præfatio ad Synopsin totius Tractatus, & additamenti loco Demonstratio
EXISTENTIÆ DEI,
ad Mathematicam certitudinem exacta.
AUTORE
GOTTFREDO GUILIELMO
LEIBNüZIO Lipsensi,
Phil. Magist. & J. U. Baccal.
LIPSIÆ,
Apud Joh. Simon. Fickium et Joh. Polycarp. Seboldum in Platea Nicolaitana.
Literis Sporeliianis.
A. M. DC. LXVI.

Figure 11. Leibniz' De Arte Combinatoria and the diagram of elements

Isn't it obvious from this quotation that Llull and Leibniz were two of a kind in several respects? They shared the dream to convert non-Christians by means of a formalization of thought. The diagram of elements that accompanies one edition of *De arte combinatoria* (see figure 11, right) also suggests that in some respects Leibniz was closer to Llull than to us with our modern scientific view of the world.

In Leibniz'diagram the four elements earth, water, air, and fire are the vertices of a square. The diagonals of the square say "contraria", meaning that earth and air and fire and water represent opposites. At the four corners of another square are the four qualities defining the elements. The edges of this square say "possible combination" or "impossible combination". In this way fire is formed from heat and dryness; air from wetness and heat; water from coldness and wetness; earth from dryness and coldness.

Concluding remark

Platzeck has argued that in 1270 the Jew Abraham Abulafia came to Barcelona and there studied the *Sepher Jezirah* under Baruch Togarmi. Abulafia made a lot of noise. Platzeck assumes that Jews on Mallorca heard about this and that Llull from them got the idea to use letters and combine them[28]. This is a tempting thought. It would mean that the kaballah is one of the sources of Leibniz' characteristica universalis.

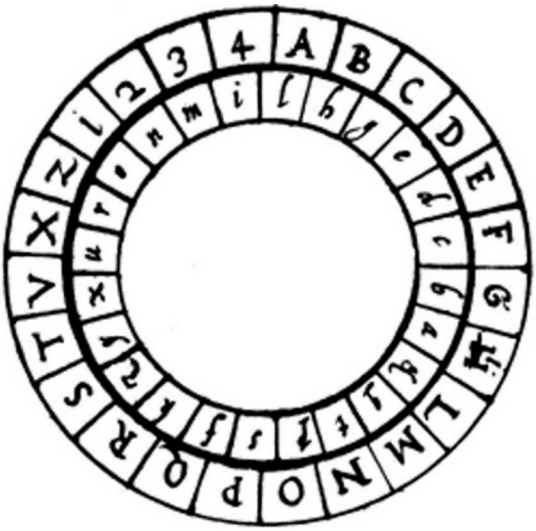

Figure 12. Alberti's cipher disc from 1466[29]

If the Sepher Jezirah is the source of Llull's rotating discs (see figure 6 above), David Kahn may have discovered another interesting line of influence. In 1466 Leon Battista Alberti (1404–1472) wrote what is possibly the first treatise on the poly-alphabetic substitution method in cryptography, "De Cifris". More complex versions of the poly-alphabetic method dominated cryptography until after the Second World War. In 1980 David Kahn suggested that Llull's rotating discs may have been the source[30]. See Albert's cipher disc in figure 12.

NOTES

[1] Llull left us an 'autobiography' dictated to monks around 1311. In the autobiography Llull said that he was major domo at the table of the Prince. See (Bonner, 1985), Vol. 1, p. 13). Given the fact that the autobiography was dictated when Llull was already an old man, some are skeptical with respect to its contents. See (Ruiz&Soler, 2008).

[2.] (Bonner, 1985), Vol. 1, p. 19.

[3] The translation is Peers' (Peers,1929, p. 64, footnote).

[4] Josep Enric Rubio has demonstrated that many of the atomic notions that we find in the Art have their roots in the *Llibre de contemplació*.

[5] http://quisestlullus.narpan.net/eng/82_figdemo_eng.html#.

[6] The translation is Bonner's. See (Bonner, 2007), pp. 48–51. The four parts of the text are called 'figures' in the version of the text that Bonner translated. I use the words affirmation, denial, doubt and resolution that are elsewhere used by Llull too. See (Bonner, 2007), p. 49, note 42.

[7] (Bonner, 2007), p. 61.

[8] With the exception of the secondary concepts in circle T.

[9] (Bonner, 1985), Vol. 1, pp. 450–451.

[10] (Bonner, 1985), Vol. 1, pp. 78–79, footnote 17.

[11] (Prantl, 1867), p. 177.

[12] (Bonner,1985), pp. 193–196.

[13] The argument rests on Llull's view of the attributes of God as active entities in combination with the Llullist correlatives, as they are called: agent, patient and action or potential, objectum and actus. See (Bonner, 1985), p. 35, note 128.

[14] (Platzeck,1962), Vol. 1, pp. 344–349.

[15] (Hayman,2004), p. 98. The text actually says 'two hundred and twenty-one" gates, but certainly the Hebrew character for 30 was read as the character for 20 by a copyist.

[16] Prantl too pointed at the cabala as a source of Llull. See (Prantl, 1867), p. 155.

[17] See (Dambergs, 2000).

[18] See (Platzeck, 1962), Vol. 1, p. 339.

[19] See (Bonner, 1985), Vol. 1, p. 74.

[20] María Rosa Menocal, Raymond P. Scheindlin, Michael Sells (eds), *The Literature of Al-Andalus*, Cambridge, 2000, p. 345.

[21] https://sites.google.com/site/zprime21/curriculumofchoice

[22] See (Rossi, 2000) p. 29.

[23] Actually Prantl, who was very critical, wrote many pages about Llull because according to him also logical nonsense deserves to be studied. See (Prantl, 1867), p. 145.

[24] See (Bos, 2001), p. 232.

[25] See, for example, (Marchall, 1977).

[26] See, for example, (Knobloch, 1974).

[27] (Breger,2005), pp. 487–488.

[28] (Platzeck, 1962), Band2, p. 328.

[29] Bibliotheca Apostolica Vaticana Ms Chigi M II 49, Vol. 35.

[30] (Kahn, 1980).

REFERENCES

Miquel Bertran and Teodor Rus (eds) (1997). *Transformation-Based Reactive Systems Development, 4th International AMAST Workshop on Real-Time Systems and Concurrent and Distributed Software*, ARTS'97 Palma, Mallorca, Spain, May 21–23, 1997 Proceedings, Lecture Notes in Computer Science, Volume 1231.

Anthony Bonner (1997). What was Llull up to?, In (Miquel Bertran and Teodor Rus, 1997), pp. 1–14.

Anthony Bonner (editor and translator) (1985). *Selected Works of Ramon Llull*, Vol. 1 and Vol. 2, Princeton University Press.

Anthony Bonner (2007). *The Art and Logic of Ramon Llull, A User's Guide*, Brill, Leiden–Boston.

Henk J. M. Bos (2001). *Redefining geometrical exactness, Descartes' transformation of the early modern concept of geometrical exactness*, Springer.

Herbert Breger (2005). God and Mathematics in Leibniz's Thought, In T. Koetsier and L. Bergmans, *Mathematics and the Divine: A Historical Study*, Elsevier, 2005, pp. 485–498.

J. N. Crossley (2005). *Raymond Llull's Contributions to Computer Science*, Clayton School of Information Technology, Monash University, Melbourne, 13pp. Technical report 2005/182.

Yanis Dambergs (2000). Elemental figure symmetry, *Studia Lulliana* 40, pp. 81–110.

Alexander Fidora & Josep E. Rubio (eds) (2008), *Raimundus Lullus, An Introduction to his Life, Works and Thought*, Turnhout, Brepols Publishers.

Martin Gardner (1958). McGraw Hill Book Company, New York–Toronto–London.

Harvey J. Hames (1999). Jewish Magic with a Christian Text, A Hebrew Translation of Ramon Llull's Ars Brevis, *Traditio* 54, 1999.

G. Hägele & F. Pukelsheim (2001), Llull's writings on electoral systems, *Studia Lulliana* 41, pp. 3–38.

Peter Hayman (2004). *Sefer Yesira: Edition, Translation and Text-Critical Commentary*, Tübingen: Mohr Siebeck.

J. H. Hillgarth, J.N.(1971). *Ramon Lull and Lulllism in Fourteenth-Century France*, Clarendon Press, Oxford.

David Kahn (1980). On the Origin of the Polyalphabetic Substitution, *Isis* 71, pp. 122–127.

Eberhard Knobloch (1974). The Mathematical Studies of G. W. Leibniz on Combinatorics, *Historia Mathematica* 1, pp. 409–430.

Ramon Llull (1957–1960). *Obres Essencials*, Vol. I and Vol. II, Editorial Selecta, Barcelona.

D. Marshall Jr. (1977). Lukasiewicz, Leibniz, and the arithmetization of the syllogism, *Notre Dame J. Form. Log.* 18, 235–242.

Ian McLean (1990). The Borda and Condorcet Principles: Three Medieval Applications, *Soc. Choice Welfare* 7, pp. 99–108.

Augustí Nieto i Galan (1998). Ramon Llull i la ciencia medieval, *Ars Brevis*, Número extraordinari, pp. 113–126.

E. Allison Peers (1929). *Ramon Llull, A Biography*, Society for Promoting Christian Knowledge, London.

Peter Pesic (1997). Secrets, Symbols, and Systems, Parallels between Cryptanalysis and algebra 1580–1700, *Isis*, 99, pp. 674–692.

Carl Prantl (1867). *Geschichte der Logik im Abendlande*, Band III, S. Hirzel Verlag, Leipzig.

Paolo Rossi (2000). *Logic and the Art of Memory, the Quest for a Universal Language*, Continuum, London–New York.

Erhard Wolfram Platzeck (1962). *Raimund Lull, Sein Leben-Seine Werke, Die Grundlagen seines Denkens (Prinzipienlehre)*, Band I und Band II, Verlag L. Schwann, Düsseldorf.

Robert D. F. Pring Mill (1973). Lull, Ramon. pp. 547–551 in: *Dictionary of Scientific Biography*, Vol. VIII; Charles Coulston Gillespie (Ed.), Charles Scribner's Sons Publishers, New York.

Robert D. F. Pring Mill (1973). Las relaciones entre el ars inveniendi veritatem y los cuatro libri principiorum, *Studia Lulliana* 17, pp. 1–25.

Josep Maria Ruiz and Albert Soler (2008). Ramon Llull in his Historical Context, *Catalan Historical Review*, 1, pp. 47–61.

Josep E. Rubio (1997). *Les Bases del Pensament de Ramon Llull*, Biblioteca Sanchis Guarner, Barcelona.

John R. Wlech (1990). Llull, Leibniz and the Logic of Discovery, *Catalan Review* IV, Nos. 1–2, 75–83.

Frances A. Yates (1954). The Art of Ramon Lull. An Approach to it through Lull's Theory of the Elements, pp. 115–173 in: *Journal of the Warburg and Courtauld Institute*, Vol. 17.