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INDETERMINISTIC TEMPORAL LOGIC*

Abstract. The questions od determinism, causality, and freedom have been the main philosophical problems debated since the beginning of temporal logic. The issue of the logical value of sentences about the future was stated by Aristotle in the famous tomorrow sea-battle passage. The question has inspired Łukasiewicz's idea of many-valued logics and was a motive of A. N. Prior's considerations about the logic of tenses. In the scheme of temporal logic there are different solutions to the problem. In the paper we consider indeterministic temporal logic based on the idea of temporal worlds and the relation of accessibility between them.

Keywords: temporal logic, determinism, causality, freedom.

> I have declared a spiritual war upon all coercion that restricts man's free creative activity.
>
> Prof. Jan Łukasiewicz, March 7, 1918

1. Introduction

The logical questions that are raised when time is considered were already pointed by Aristotle (384 BC–322 BC) in his famous tomorrow's sea-battle passage of *De Interpretatione* 9 19^a 30. This consideration led Aristotle to rejection of the principle of bivalence for assertions concerning the future. Diodorus Cronus from the Megarian school of philosophy stated a version of the problem in his notorious Master Argument. Some achievement in this subject is due to Avicenna (980–1037). His work influenced the medieval logicians Albertus Magnus (1193/1206–1280) and William of Ockham (c. 1288–c. 1348). In the 19th century Charles Sanders Peirce (1839–1914) wrote that he did not share the common opinion that time is an extralogical matter. Jan Łukasiewicz (1878–1956) in debating questions of determinism considered arguments from the law of the excluded middle and from the principle of causality in that some logical problems of temporality were involved Łukasiewicz 1967. Arthur Norman Prior (1914–1969), the founder of

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temporal logic, was concerned with the philosophical matters of free will and predestination.

To use a formal logic to solve a philosophical problem, we have to have:

1. a formal language in that the problem can be formulated inan intuitively satisfactory way,
2. the logic of the language should be neutral with respect to this problem, i.e. the formulas that express the solution to the problem should not be theses of the logic (analytical truths of the language).

The thesis of determinism as consisting of two theses, the thesis of pre-determinism *PRE-DET* and the thesis of post-determinism *POST-DET*, can be formulated as follows:

[*DET*.] If at moment t it is true that α, then

[*PRE-DET*.] at any moment t_1 earlier than t, it was true that at t there would be α,

and

[*POST-DET*.] at any moment t_1 later than t, it will be true that at t there was α.

The theses *PRE-DET* and *POST-DET* are symmetrical with one another.

The principle of causality says:

[*PC*.] If α occurs at t, then at t_1, some moment earlier than t, and at any moment between t and t_1, it was true that at t there will be α.

The principle of effectivity, as symmetrical to *PC*, may be formulated as follows:

[*PE*.] If α occurs at t, then at t_1, some moment later than t, and at any moment between t and t_1 it will be true that at t there was α.

The principles *PC* and *PE* are mutually symmetrical to each other.

We are looking for a formal language such that:

– both the theses *PRE-DET* and *POST-DET* and both the principles *PC* and *PE* are expressible in it;
– the theses as formulated in the language are not truths of the logic of this language, i.e. they are not true in any model, especially irrespectively of properties of time (though they could be true in some models and some classes of time);

– the arguments from the principles PC and PE for PRE-DET and $POST$-DET could be avoided even if both principles are valid.

If all these conditions are fulfilled, we say that the logic is indeterministic.

Moreover, we are interested in such a solution that the most intuitive laws of temporal logic are preserved, i.e. laws that have some reason in the logic of natural language, e.g., are valid in the class of transitive times. In order to achieve these goals, not only certain properties of time will be supposed but also a relation of accessibility between different possible courses of events will be assumed.

2. Semantics

2.1. Temporal language

Let the language consist of:

- $p_1, p_2, \ldots$—propositional letters,
- a functionally complete set of classical propositional connectives,
- temporal operators (past tense and future tense operators).

Let AP (atomic propositions) be the set of propositional letters. Formulas are defined in the usual way and will be denoted by Greek letters: $\phi, \psi, \ldots$, if necessary with indices.

Let time be $\mathfrak{T} = \langle T, < \rangle$, where T is a non-empty set (of moments) and $<$ is a binary relation on T (earlier-later). No property of $<$ is supposed. $\mathfrak{W} = \langle T, <, V \rangle$, where $V : T \to 2^{AP}$, is a possible (temporal) world. V, valuation, is a function that to each point of T assigns a subset of AP, the set of propositional letters that are true at this point.

Prior defined temporal operators as follows:

Definition 2.1 (G)

$\langle T, <, V \rangle, t \models G\phi$ iff for any $t_1, t < t_1 : \langle T, <, V \rangle, t_1 \models \phi$.

Definition 2.2 (F)

$\langle T, <, V \rangle, t \models F\phi$ iff there is $t_1, t < t_1 : \langle T, <, V \rangle, t_1 \models \phi$.

Definition 2.3 (H)

$\langle T, <, V \rangle, t \models H\phi$ iff for any $t_1, t_1 < t : \langle T, <, V \rangle, t_1 \models \phi$.

Definition 2.4 (P)

$\langle T, <, V \rangle, t \models P\phi$ iff there is $t_1, t_1 < t : \langle T, <, V \rangle, t_1 \models \phi$.

Priorean temporal operators can be understood as a certain type of modal operators: G, H—as necessity operators, F, P—as possibility operators. The usual relations between necessity and possibility operators hold:

$$G\phi \leftrightarrow \neg F \neg \phi,$$

$$H\phi \leftrightarrow \neg P \neg \phi.$$

Let the Priorean language be denoted L_P.

Priorean language does not satisfy the conditions that are imposed on the language interesting us; namely, the thesis of determinism DET is true without any assumption about time: the formulas

$$\phi \rightarrow HF\phi,$$

$$\phi \rightarrow GP\phi$$

are satisfied in any case independently of properties of time ($<$). They are theses of the minimal tense logic K_t.

In the case of L_P—if time is without an end—at least one of the sentences $F\phi$ or $F\neg\phi$ is true at any t ($\in T$). We are interested in such a logic in which it could be that both $F\phi$ as well as $F\neg\phi$ would not be true at some t ($\in T$). $F\phi$, a proposition about the future, is true only if it is determined that ϕ will take place. Similarly, $F\phi$ is false only if it is determined that ϕ will not take the place. Such a situation is when ϕ will be true, or, respectively, false, independently of the course of future events. As Aristotle wrote in *De Interpretatione*, (http://ebooks.adelaide.edu.au/a/aristotle/interpretation/)

> A sea-fight must either take place to-morrow or not, but it is not necessary that it should take place to-morrow, neither is it necessary that it should not take place, yet it is necessary that it either should or should not take place to-morrow. Since propositions correspond with facts, it is evident that when in future events there is a real alternative, and a potentiality in contrary directions, the corresponding affirmation and denial have the same character. This is the case with regard to that which is not always existent or not always nonexistent. One of the two propositions in such instances must be true and the other false, but we cannot say determinately that this or that is false, but must leave the alternative undecided. One may indeed be more likely to be true than the other, but it cannot be either actually true or actually false. It is therefore plain that it is not necessary that of an affirmation and a denial one should be true and the other false. For in the case of that which exists potentially, but not actually, the rule which applies to that which exists actually does not hold good. The case is rather as we have indicated.

In branching time logic '*it will be* ϕ' is determined iff on each branch at some moment in the future *it will be* ϕ: on each course of affairs, sooner or later, it will be the case that ϕ. This solution omits the important fact that ϕ is determined if ϕ occurs at the same moment independently of the branch, independently of possible courses of affairs.

Let temporal operators be defined assuming that there could be more courses of events though branches do not differ in time (as a set of moments).

Let I be a non-empty set. $\langle T, < \rangle \times I$ is the class of times.

On the class $\langle T, < \rangle \times I$ a binary relation $\lhd$ of accessibility could be defined:

$$\lhd \subseteq (\langle T, < \rangle \times I) \times (\langle T, < \rangle \times I),$$

or formally equivalently:

$$\lhd \subseteq (T \times I) \times (T \times I).$$

It is reasonable to assume that between the relation of accessability, $\lhd$, and the relation "earlier-later", $<$, the following connections hold:

★ for any $t, t_1 \in T; i, j \in I$:

1. $(t, i) \lhd (t_1, i)$ iff $t \leq t_1$,
2. if $(t, i) \lhd (t, j)$, then $(t, j) \lhd (t, i)$.

By 1 $\lhd$ is reflexive. Condition 2, $SYMM_{\lhd}$, does not express symmetry of $\lhd$. It is weaker. E.g., it is not assumed that if $(t, i) \lhd (t_1, j)$, then $(t_1, j) \lhd (t, i)$.

Let $V_i : T \to 2^{AP}$, $i \in I$. $\langle T, <, \lhd, V_i \rangle$ is a possible (temporal) world.

In modal logic $V(t)$ is a possible world. Now a possible world is understood as $\{V_i(t) : t \in T\}$. In modal logic the accessibility of one world from another is a binary relation defined on T. Now $\lhd$, the relation of accessibility, takes into account not only a possible world (a course of events) but also a point of time, a constituent of the possible world.

Future and past tense operators will be defined with respect to a binary relation $\lhd$ of accessibility between possible worlds (possible courses of affairs). Let the same symbols of temporal operators as in the case of Priorean language will be used—in any case the context will determine the meaning of used symbol:

Definition 2.5 (G)

$\langle T, <, \lhd, V_i \rangle, t \models G\phi$ iff for any $t_1 \in T, j \in I$:

if $(t, i) \lhd (t, j)$ and $t < t_1$, then $\langle T, <, \lhd, V_j \rangle, t_1 \models \phi$.

Definition 2.6 (F)

$$\langle T,<,\lhd,V_i\rangle,t\models F\phi \quad \text{iff} \quad \text{there is } t_1\in T,\ t<t_1 \text{ such that:}$$
$$\text{for any } j\in I\text{: if } (t,i)\lhd(t,j),\text{ then } \langle T,<,\lhd,V_j\rangle,t_1\models\phi.$$

Definition 2.7 (H)

$$\langle T,<,\lhd,V_i\rangle,t\models H\phi \quad \text{iff} \quad \text{for any } t_1\in T, j\in I\text{:}$$
$$\text{if } (t,j)\lhd(t,i) \text{ and } t_1<t,\text{ then } \langle T,<,\lhd,V_j\rangle,t_1\models\phi.$$

Definition 2.8 (P)

$$\langle T,<,\lhd,V_i\rangle,t\models P\phi \quad \text{iff} \quad \text{there is } t_1\in T,\ t_1<t \text{ such that:}$$
$$\text{for any } j\in I\text{: if } (t,j)\lhd(t,i),\text{ then } \langle T,<,\lhd,V_j\rangle,t_1\models\phi.$$

Let L_D be the language with temporal operators as defined by 2.5–2.8.

Let us remark that in the case of one-element I, under the condition $\bigstar$ that for any $t,t_1\in T, i\in I : (t,i)\lhd(t_1,i)$ iff $t\le t_1$, all the definitions 2.5–2.8 collapse to 2.1–2.4, Priorean definitions of tense operators. It is also true if $\lhd$ is a relation such that: for any $i,j\in I$ if $i\neq j$, then for no $t_1,t_2 : (t_1,i)\lhd(t_2,j)$.

Modal operators could be defined in the usual way.

Definition 2.9

$$\langle T,<,\lhd,V_i\rangle,t\models\Box\phi \quad \text{iff}$$
$$\text{for any } j : \text{if } (t,i)\lhd(t,j),\ \text{then } \langle T,<,\lhd,V_j\rangle,t\models\phi,$$
$$\langle T,<,\lhd,V_i\rangle,t\models\Diamond\phi \quad \text{iff}$$
$$\text{there is } j \text{ such that } (t,i)\lhd(t,j) \text{ and } \langle T,<,\lhd,V_j\rangle,t\models\phi.$$

The usual connection between $\Box$ and $\Diamond$ holds:

$$\Box\phi\leftrightarrow\neg\Diamond\neg\phi.$$

Let us remark that there is a difference between Priorean temporal operators preceded by $\Box$ and temporal operators as defined in L_D. For example, $\Box F\phi$, as a formula of Priorean language, expresses inevitability: ϕ should be satisfied in a future moment of any accessible possible world, but $F\phi$—as the formula of L_D—is satisfied only if ϕ is satisfied at the same moment in the future in any accessible possible world. Hence $F\phi$ of L_D better expresses the idea of determinism. The new temporal operators could not be redefined using Priorean temporal operators and $\Box$. The language L_D is not reducible to Priorean language with a modal operator.

2.2. Satisfiability and definability

Definition 2.10

$$\langle T, <, \lhd, V_i\rangle \models \phi \quad \text{iff} \quad \text{for any } t \in T : \langle T, <, \lhd, V_i\rangle, t \models \phi.$$

$\langle T, <, \lhd, V_i\rangle \models \phi$ means that the formula ϕ is satisfied by $\langle T, <, \lhd, V_i\rangle$, or that $\langle T, <, \lhd, V_i\rangle$ is a model of ϕ. If $\langle T, <, \lhd, V_i\rangle \models \phi$, we say that ϕ is omnitemporally valid in $\langle T, <, \lhd, V_i\rangle$.

Definition 2.11

$$\langle T, <, \lhd\rangle \models \phi \quad \text{iff} \quad \text{for any } V_i,\ i \in I : \langle T, <, \lhd, V_i\rangle \models \phi.$$

$\langle T, <, \lhd\rangle \models \phi$ means that the formula ϕ is satisfied by $\langle T, <, \lhd\rangle$.

Let $\mathfrak{F}$ be a class of frames $\langle T, <, \lhd\rangle$.

Definition 2.12

$\mathfrak{F} \models \phi$, a formula ϕ is satisfied by $\mathfrak{F}$, iff ϕ is satisfied by any member of $\mathfrak{F}$, i.e.

$$\text{for any } \langle T, <, \lhd\rangle \in \mathfrak{F} : \langle T, <, \lhd\rangle \models \phi.$$

$\models \phi$ means that the formula ϕ is satisfied by any model that fulfills the condition $\bigstar$.

Definition 2.13

A formula ϕ defines (characterizes) $\mathfrak{F}$ iff:

$$\langle T, <, \lhd\rangle \models \phi \quad \text{iff} \quad \langle T, <, \lhd\rangle \in \mathfrak{F}.$$

2.3. Minimal logic

Let us consider some formulas of the language L_D that are satisfied by any model (if $\bigstar$ holds), i.e. all the formulas ϕ such that $\models \phi$. Let the set of these formulas will be denoted K_d, i.e. $K_d = \{\phi : \models \phi\}$.

In the case of Priorean language, the set of formulas satisfied in any model is axiomatized. It is proved that the system K_t, the system of minimal logic of the Priorean langauge, is valid and complete, i.e.:

$$\Sigma \vdash \phi \text{ iff } \Sigma \models \phi.$$

The system K_t is also decidable (McArthur, 1976).

Let formulas of the discussed logics be taken purely formally, i.e. the differences in meanings of the temporal operators of the languages L_P and the temporal operators of the language L_D not be taken into consideration. $K_d \subsetneq K_t$ because:

– in the case of one element set I all the definitions of temporal operators of the language L_D collapse to Priorean language,
– some formulas of language L_D are not true for all frames even though they Priorean language counterparts are.

$MI(\phi)$ is the mirror image of a formula ϕ iff it is the result of simultaneous replacing in ϕ all occurrences of G by H and F by P, and vice versa.

In the case of K_t, the rule of MI holds, i.e.:

$$K_t \vdash \phi \quad \text{iff} \quad K_t \vdash MI(\phi).$$

It is also true in the case of K_d (as the set of all valid formulas in any model).

Theorem 2.14

$\models \phi$ iff $\models MI(\phi)$.

Proof. Let $>$ is the converse of $<$ and $\trianglerighteq$ is the converse of $\triangleleft$. Let us prove by induction that for any $T, <, \triangleleft, V_i, t$:

A. $$\langle T, <, \triangleleft, V_i\rangle, t \models \phi \quad \text{iff} \quad \langle T, >, \trianglerighteq, V_i\rangle, t \models MI(\phi).$$

A is true if no temporal operator occurs in ϕ. Let us suppose that A holds for ψ and χ. Let us only prove that A holds also in the case of $G\psi$, i.e. that for any $T, <, \triangleleft, V_i, t$:

$$\langle T, <, \triangleleft, V_i\rangle, t \models G\psi \quad \text{iff} \quad \langle T, >, \trianglerighteq, V_i\rangle, t \models MI(G\psi).$$

Let for some $T, <, \triangleleft, V_i, t$ it does not hold, i.e.:

1. $\langle T, <, \triangleleft, V_i\rangle, t \models G\psi$ and $\langle T, >, \trianglerighteq, V_i\rangle, t \not\models MI(G\psi)$

or

2. $\langle T, <, \triangleleft, V_i\rangle, t \not\models G\psi$ and $\langle T, >, \trianglerighteq, V_i\rangle, t \models MI(G\psi)$.

Let us consider only case 1. By definition of MI: $MI(G\psi)$ is the formula $HMI(\psi)$. Since $\langle T, >, \trianglerighteq, V_i\rangle, t \not\models HMI(\psi)$ we have that for some $t_1, t_1 < t : \langle T, >, \trianglerighteq, V_i\rangle, t_1 \not\models MI(\psi)$. By inductive supposition:

$$\langle T, <, \triangleleft, V_i\rangle, t_1 \not\models \psi.$$

It means that also $\langle T, <, \triangleleft, V_i\rangle, t_1 \not\models G\psi$, which contradicts assumption 1. Hence $\models \phi$ iff $\models MI(\phi)$. □

The rule:

$MI.$ $\models \phi$ iff $\models MI(\phi)$

allows omitting the proof of $\models MI(\phi)$ if it is proved that $\models \phi$ and also if there is a counter-model for $\models \phi$, then there is a counter-model for

$\models MI(\phi)$. It means that if it is showed that $\not\models \phi$, then the argumentation that $\not\models MI(\phi)$ can be omitted.

In the case of Priorean language the operators F and G are dual and the same is true for P and H. It means that:

$F\phi \leftrightarrow \neg G\neg\phi, \quad G\phi \leftrightarrow \neg F\neg\phi,$

$P\phi \leftrightarrow \neg H\neg\phi, \quad H\phi \leftrightarrow \neg P\neg\phi$

In the language L_D temporal operators are not dual. The logical connections between these operators are established by theorems 2.15–2.18.

Theorem 2.15

$\models F\phi \to \neg G\neg\phi.$

Proof. Let for some $T, <, \lhd, V_i, t$ be such that:

$$\langle T, <, \lhd, V_i\rangle, t \models F\phi$$

and

$$\langle T, <, \lhd, V_i\rangle, t \not\models \neg G\neg\phi.$$

By definition of satisfiability

$$\langle T, <, \lhd, V_i\rangle, t \not\models \neg G\neg\phi$$

is equivalent to

$$\langle T, <, \lhd, V_i\rangle, t \models G\neg\phi.$$

But it contradicts the assumption that

$$\langle T, <, \lhd, V_i\rangle, t \models F\phi. \qquad \square$$

Theorem 2.16

$\not\models \neg G\neg p \to Fp.$

Proof. Let a counter-model be constructed.

$T = \{1, 2\},$
$< \; = \; \{(1, 2)\},$
$I = \{a, b\},$
$\lhd = \{\langle (1, a), (1, b)\rangle\},$
$p \in V_a(2),$
$p \notin V_b(2).$

$\langle T, <, \lhd, V_a\rangle, 1 \not\models G\neg p$. Thus $\langle T, <, \lhd, V_a\rangle, 1 \models \neg G\neg p$. But

$$\langle T, <, \lhd, V_a\rangle, 1 \not\models Fp. \qquad \square$$

Theorem 2.17

$\models P\phi \to \neg H \neg \phi.$

Theorem 2.18

$\not\models \neg H \neg p \to Pp.$

Theorem 2.19

$\models G(\phi \to \psi) \to (G\phi \to G\psi).$

Proof. Let:

$\langle T, <, \lhd, V_i \rangle, t \models G(\phi \to \psi)$

and

$\langle T, <, \lhd, V_i \rangle, t \not\models G\phi \to G\psi.$

By the definition of satisfiability

$$\langle T, <, \lhd, V_i \rangle, t \models G\phi$$

and

$$\langle T, <, \lhd, V_i \rangle, t \not\models G\psi.$$

Hence there exist $t_1, j : (t, i) \lhd (t, j)$ and $t < t_1$ such that

$$\langle T, <, \lhd, V_j \rangle, t_1 \not\models \psi.$$

Since

$$\langle T, <, \lhd, V_j \rangle, t_1 \models \phi \to \psi$$

and

$$\langle T, <, \lhd, V_j \rangle, t_1 \models \phi,$$

we have:

$$\langle T, <, \lhd, V_j \rangle, t_1 \models \psi.$$

This contradicts

$$\langle T, <, \lhd, V_j \rangle, t_1 \not\models \psi.$$ □

Theorem 2.20

$\models G(\phi \to \psi) \to (F\phi \to F\psi).$

Proof. Suppose that $\langle T, <, \lhd, V_i \rangle, t \not\models G(\phi \to \psi) \to (F\phi \to F\psi)$. Hence:

$$\langle T, <, \lhd, V_i \rangle, t \models G(\phi \to \psi),$$

$$\langle T, <, \lhd, V_i \rangle, t \models F\phi,$$

and

$$\langle T, <, \lhd, V_i \rangle, t \not\models F\psi.$$

$$\langle T, <, \lhd, V_i \rangle, t \models F\psi$$

is not true if there is no $t_1, t < t_1$, such that:

$$\text{for any } j \in I, \text{ if } (t,i) \lhd (t,j), \text{ then, } \langle T, <, \lhd, V_j \rangle, t_1 \models \psi.$$

Since

$$\langle T, <, \lhd, V_i \rangle, t \models F\phi,$$

there is t_2 such that:

$$\text{for any } j \in I, \text{ if } (t,i) \lhd (t,j), \text{ then } \langle T, <, \lhd, V_j \rangle, t_2 \models \phi.$$

Because

$$\langle T, <, \lhd, V_i \rangle, t \models G(\phi \to \psi)$$

and

$$(t,i) \lhd (t,j),$$

we have that

$$\langle T, <, \lhd, V_j \rangle, t_2 \models \phi \to \psi.$$

Hence

$$\langle T, <, \lhd, V_j \rangle, t_2 \models \psi.$$

Thus we get a contradiction. □

Theorem 2.21

$\models H(\phi \to \psi) \to (H\phi \to H\psi)$.

Theorem 2.22

$H(\phi \to \psi) \to (P\phi \to P\psi)$.

The following formulas are theses of minimal logic of Priorean language (McArthur, 1976, p. 22):

$(\phi \to HF\phi)$,

$(PG\phi \to \phi)$,

$(\phi \to GP\phi)$,

$(FH\phi \to \phi)$.

In the language L_D the logical connections between future and past temporal operators are established by theorems 2.23–2.26.

Theorem 2.23

$\not\models p \to HFp$.

Proof. Let us construct a counter-model.

$T = \{1, 2\}$,

$< \, = \{(1, 2)\}$,
$I = \{a, b\}$,
$\lhd = \{\langle(1, a)(1, b)\rangle\}$,
$p \in V_a(2)$,
$p \notin V_b(2)$.

p is satisfied at $(2, a)$, but HFp is not. □

Theorem 2.24

$\models PG\phi \to \phi$.

Proof. Let $\langle T, <, \lhd, V_i\rangle, t \models PG\phi$. Hence there is $t_1, t_1 < t$ such that

$$\text{for any } j \in I, \text{ if } (t, i)\lhd(t, j), \text{ then } \langle T, <, \lhd, V_j\rangle, t_1 \models G\phi.$$

Thus

$$\text{for any } t_2, t_1 < t_2 \text{ and any } l \text{ if } (t_1, l)\lhd(t_1, j), \text{ then } \langle T, <, \lhd, V_l\rangle, t_2 \models \phi.$$

Since $(t, j)\lhd(t, i)$ we get

$$\langle T, <, \lhd, V_i\rangle, t \models \phi.$$

□

Theorem 2.25

$\not\models p \to GPp$.

Theorem 2.26

$\models FH\phi \to \phi$.

In the next theorems, connections between $\neg G\neg$, $\neg H\neg$ of the language L_D and F, P of the Priorean language will be established.

Theorem 2.27

$$\langle T, <, \lhd, V_i\rangle, t \models \neg G\neg\phi$$

iff

$$\text{there are } t_1, t < t_1, j \in I, (t, i)\lhd(t, j) : \langle T, <, \lhd, V_j\rangle, t_1 \models \phi.$$

Proof.

$$\langle T, <, \lhd, V_i\rangle, t \models \neg G\neg\phi$$

means that:

$$\text{it is not true that } \langle T, <, \lhd, V_i\rangle, t \models G\neg\phi.$$

It means that it is not true that for any $t_2, t < t_2, l \in I$:

$$\text{if } (t, i)\lhd(t, l), \text{ then } \langle T, <, \lhd, V_l\rangle, t_2 \models \neg\phi.$$

Thus equivalently there are $t_1,\ t < t_1,\ j \in I,\ (t,i) \lhd (t,j)$ such that:

$$\langle T, <, \lhd, V_j \rangle, t_1 \models \phi. \qquad \Box$$

Theorem 2.28

$$\langle T, <, \lhd, V_i \rangle, t \models \neg H \neg \phi$$

iff

$$\text{there are } t_1, t_1 < t, j \in I, (t_1, j) \lhd (t, i) : \langle T, <, \lhd, V_j \rangle, t_1 \models \phi.$$

The theorems 2.27 and 2.28 say—roughly speaking—that the meanings of $\neg G \neg$ and $\neg H \neg$ of the language L_D are the same as the meanings of the operators, respectively, F and P of Priorean language.

Let ϕ^* be a formula obtained from ϕ by replacement of some its propositional letters by formulas of L_D in such a way that:

1. replacement is simultaneous in any place in which the letter occurs.
2. the same letter is replaced by the same formula.

The above results justify the following:

Corollary 2.29

If ϕ is a formula in that as temporal operators only G and H occur and the Priorean language counterpart of ϕ is a thesis of the minimal logic K_t, then: $\models \phi^*$, i.e. ϕ^* is valid in any model.

For example, because the Priorean language counterpart of $p \rightarrow H \neg G \neg p$ is a thesis of K_t, thus it is true that: $\models \psi \rightarrow H \neg G \neg \psi$, where ψ is a formula of L_D. But this could not be the case if F or P occur, e.g. though the Priorean language counterpart of $p \rightarrow HFp$ is a thesis of K_t, it is not the case that $\models p \rightarrow HFp$ (th. 2.23). The difference between G, F and H, P, temporal operators of K_d, is quite similar to that between intuitionistic general and existential quantifiers.

3. System K_d

Axioms

Axiom 1	ϕ, if ϕ is a tautology of the classical propositional logic of the language K_d
Axiom 2 $(G1)$	$G(\phi \rightarrow \psi) \rightarrow (G\phi \rightarrow G\psi)$
Axiom 3 $(H1)$	$H(\phi \rightarrow \psi) \rightarrow (H\phi \rightarrow H\psi)$
Axiom 4 $(G1')$	$G(\phi \rightarrow \psi) \rightarrow (F\phi \rightarrow F\psi)$

Axiom 5 $(H1')$ $H(\phi \rightarrow \psi) \rightarrow (P\phi \rightarrow P\psi)$

Axiom 6 $(G2)$ $\phi \rightarrow H\neg G\neg\phi$

Axiom 7 $(H2)$ $\phi \rightarrow G\neg H\neg\phi$

Axiom 8 $(G2')$ $PG\phi \rightarrow \phi$

Axiom 9 $(H2')$ $FH\phi \rightarrow \phi$

Rules

MP. $$\frac{\phi \rightarrow \psi \quad \phi}{\psi}$$

RG. $$\frac{\phi}{G\phi}$$

RH. $$\frac{\phi}{H\phi}$$

The formulas 2–5 are distribution axioms. In K_t the Priorean language counterparts of 4 and 5 are provable (McArthur, 1976, pp. 20–21). Formulas 6 and 7 as expressions of L_P are axioms of K_t. The Priorean language counterparts of axioms 8 and 9 are provable in K_t.

The rules **RG** and **RH** are temporal logic counterparts of the necessitation rule of modal logic.

The language L_D could be enriched with $\Box$, read: *it is a fact, that* (as defined earlier, def. 2.9). The language will be denoted $L_{\Box D}$:

$$\langle T, <, \lhd, V_i\rangle, t \models \Box\phi \quad \text{iff}$$
$$\text{for any } j\text{: if } (t,i)\lhd(t,j), \text{ then } \langle T, <, \lhd, V_j\rangle, t \models \phi.$$

We see the possibility of introducing $\Box$ as one of the main advantages of the considered model. "*It is (now) a fact, that*" is not redundant, i.e. ϕ and $\Box\phi$ do not mean the same (if set I has at least two elements and the relation $\lhd$ is not empty). The formal difference between $\langle T, \lhd, <, V\rangle, t \models \phi$ and $\langle T, \lhd, <, V\rangle, t \models \Box\phi$ can be interpreted as a difference between contingency and necessity. $\langle T, \lhd, <, V\rangle, t \models \phi$ says that at t ϕ is contingent. $\langle T, \lhd, <, V\rangle, t \models \Box\phi$ says that at t ϕ is necessary. In (Aquinas, 1905) we read:

> A contingent event differs from a necessary event in point of the way in which each is contained in its cause. A contingent event is so contained in its cause as that it either may not or may ensue there from: whereas a necessary event

> cannot but ensue from its cause. But as each of these events is in itself, the two do not differ in point of reality; and upon reality truth is founded. In a contingent event, considered as it is in itself, there is no question of being or not being, but only of being: although, looking to the future, a contingent event possibly may not come off.

This corresponds with what Aristotle wrote earlier in *De Interpretatione* 9, 19^a 23–26:

> What is, necessarily is, when it is; and what is not, necessarily is not, when it is not. But not everything that is, necessarily is; and not everything that is not, necessarily is not. For to say that everything that is, is of necessity, when it is, is not the same as saying unconditionally that it is of necessity. Similarly with what is not.

$\Sigma \vdash \phi$—means that ϕ is provable from the set of premises Σ. $\Sigma \vdash \phi$ iff there is a finite sequence of formulas such that any element of it is a member of Σ or an axiom or is obtained by application of a rule to some preceding elements and which last element is ϕ.

$K_d \vdash \phi$ means that in the system K_d ϕ is provable from the empty set of premisses.

The axiomatic system K_t consists of L_P language counterparts of axioms 1–3, 6–7 and all the rules **MP**, **RG** and **RH**. K_t is consistent, valid, complete and decidable. The L_P counterparts of additional axioms of K_d are theorems of K_t. Thus K_d is consistent. The system K_d is formally weaker than K_t, because counterparts of some theorems of K_t are not valid in the language L_D, e.g. $p \to GPp$, $p \to HFp$.

The facts that:

- all the axioms and rules of K_t could be expressed by formulas in which as temporal operators only G and H occur, and that
- all L_P counterparts of these axioms and rules are axioms and rules of K_d

justify the following theorem.

Theorem 3.1

If $K_t \vdash \phi$ and in ϕ, only G and H occur as temporal operators, then $K_d \vdash \phi^*$, where ϕ^* is a formula obtained from ϕ by replacement of some propositional letters by formulas of L_D in such a way that:

1. replacement is simultaneous in any place in which the letter occurs.
2. the same letter is replaced by the same formula.

Cf. p. 15.

Proof. All theses of K_t as expressed by formulas in which, G and H occur as only temporal operators, are theses of K_d. Any thesis of K_d in which propositional letters are replaced in the way described in the theorem is also a thesis of K_d. Thus if Priorean language counterpart of ϕ is a thesis of K_t, then ϕ^* is a thesis of K_d. $\square$

K_d is consistent and valid. The questions of decidability and completeness remain open.

The logic of language $L_{\square D}$ is based on logic K_d. All the axioms and rules of K_d as applied to language $L_{\square D}$ are valid. There are additional axioms of the form of axioms of system of modal logic, axioms expressing connections between $\square$ and temporal operators, and an additional rule:

RN. $\dfrac{\phi}{N\phi}$

The question of relations between $\square$ and temporal operators in general is omitted here.

Let $TRANS_{\triangleleft}$ be the class of frames such that:

$TRANS_{\triangleleft}$. For any t, i, j, k: if $(t,i)\triangleleft(t,j)$ and $(t,j)\triangleleft(t,k)$, then $(t,i)\triangleleft(t,k)$.

The condition $TRANS_{\triangleleft}$ does not express the transitivity of $\triangleleft$. The condition is weaker. Let us remember that about $\triangleleft$ it is supposed that ★:

$(t,i)\triangleleft(t,i)$—$REFL_{\triangleleft}$,

if $(t,i)\triangleleft(t,j)$, then $(t,j)\triangleleft(t,i)$—$SYMM_{\triangleleft}$.

Theorem 3.2

$\mathfrak{T} \models \square G\phi \leftrightarrow G\phi \quad \text{iff} \quad \mathfrak{T} \in TRANS_{\triangleleft}$.

It is also true for the other temporal operators.

Theorem 3.3

$\mathfrak{T} \models \square F\phi \leftrightarrow F\phi \quad \text{iff} \quad \mathfrak{T} \in TRANS_{\triangleleft}$.

Theorem 3.4

$\mathfrak{T} \models \square H\phi \leftrightarrow H\phi \quad \text{iff} \quad \mathfrak{T} \in TRANS_{\triangleleft}$.

Theorem 3.5

$\mathfrak{T} \models \square P\phi \leftrightarrow P\phi \quad \text{iff} \quad \mathfrak{T} \in TRANS_{\triangleleft}$.

4. Principles of causality and effectivity

PE. The principle of effecivity says that:

1. if at (t, i) occur $\Box\phi$, then at some $t_1, t < t_1$ and for any $j, (t, j) \lhd (t, i)$, at (t_1, j) there is an effect of ϕ,

and

2. for any $t_2, t_1 < t_2 < t$, at (t_2, j) there is an effect of ϕ.

PC. The principle of causality says that:

1. if at (t, i) occur $\Box\phi$, then at some $t_1, t_1 < t$ and for any $j, (t, j) \lhd (t, i)$, at (t_1, j) there is a cause of ϕ,

and

2. for any $t_2, t < t_2 < t_1$, at (t_2, j) there is a cause of ϕ.

Let us remark that

- $\Box\phi \to FP\phi$.

is valid only if time is endless, i.e.

$\mathfrak{T}^{\infty+}$. For any $t \in T$ there is $t_1, t_1 \in T : t < t_1$.

In Priorean language the endless time is characterized by: $Gp \to Fp$. The same is true about the L_D counterpart of this formula.

Theorem 4.1

$$\mathfrak{T} \models Gp \to Fp \quad \text{iff} \quad \mathfrak{T} \in \mathfrak{T}^{\infty+}.$$

It has been proved (th. 2.15) that: $F\phi \to \neg G \neg \phi$, thus by syllogism:

Corollary 4.2

$$\mathfrak{T} \models Gp \to \neg G \neg p \quad \text{iff} \quad \mathfrak{T} \in \mathfrak{T}^{\infty+}.$$

- $\Box\phi \to PF\phi$.

is valid only if time does not have a beginning, i.e.

$\mathfrak{T}^{\infty-}$. For any $t \in T$ there is $t_1, t_1 \in T : t_1 < t$.

In Priorean language time without a beginning is characterized by: $Hp \to Pp$. The same is true about L_D counterpart of this formula.

Theorem 4.3

$$\mathfrak{T} \models Hp \to Pp \quad \text{iff} \quad \mathfrak{T} \in \mathfrak{T}^{\infty-}.$$

It has been proved (th. 2.17) that: $P\phi \to \neg H \neg \phi$, thus by syllogism:

Corollary 4.4

$$\mathfrak{T} \models Hp \to \neg H \neg p \quad \text{iff} \quad \mathfrak{T} \in \mathfrak{T}^{\infty -}.$$

Let $\mathfrak{T}^E$ be a class of frames $\langle T, <, I, \lhd \rangle$ such that:

$\mathfrak{T}^E$. For any t and i there is $t_1,\ t < t_1$ such that for any j, k: if $(t, i) \lhd (t, j)$, then $(t_1, j) \lhd (t_1, k)$ iff $(t, i) \lhd (t, k)$.

Theorem 4.5

$$\mathfrak{T} \models \Box p \to FPp \quad \text{iff} \quad \mathfrak{T} \in \mathfrak{T}^E.$$

Proof. Let us prove only that $\mathfrak{T} \models \Box p \to FPp$, if $\mathfrak{T} \in \mathfrak{T}^E$.
Let us suppose that $\langle T, <, \lhd, V_i \rangle, t \not\models \Box p \to FPp$. Hence

$$\langle T, <, \lhd, V_i \rangle, t \models \Box p.$$

Since $\langle T, <, \lhd, V_i \rangle, t \models \Box p$ we have that for any $j, (t, i) \lhd (t, j)$:

$$\langle T, <, \lhd, V_j \rangle, t \models p.$$

By condition $\mathfrak{T}^E$ we have that there is $t_1, t < t_1$ such that

$$\langle T, <, \lhd, V_i \rangle, t_1 \models Pp.$$

Again by $\mathfrak{T}^E$: $\langle T, <, \lhd, V_i \rangle, t \models FPp$. Finally $\langle T, <, \lhd, V_i \rangle, t \models \Box p \to FPp$.
$\Box$

Let $\mathfrak{T}^C$ be a class of frames $\langle T, <, I, \lhd \rangle$ such that:

$\mathfrak{T}^C$. For any t and i there is $t_1,\ t_1 < t$ such that for any j, k: if $(t, i) \lhd (t, j)$, then $(t_1, j) \lhd (t_1, k)$ iff $(t, i) \lhd (t, k)$.

Theorem 4.6

$$\mathfrak{T} \models \Box p \to PFp \quad \text{iff} \quad \mathfrak{T} \in \mathfrak{T}^C.$$

Let us introduce Hans Kamp's (1968) binary temporal operators S (since) and U (until). Let us remark that there are various ways of defining S and U in our semantics. The definitions should be such that in the case of an empty I, they collapse to Kamp's definitions.

Definition 4.7 (S)

$\langle T, <, \lhd, V_i \rangle, t \models S(\phi, \psi)$ iff there is $t_1 < t$ such that for any j: $(t, j) \lhd (t, i)$:
$\langle T, <, \lhd, V_j \rangle, t_1 \models \phi$ and for any $t_2,\ t_1 < t_2 < t$: $\langle T, <, \lhd, V \rangle, t_2 \models \psi$.

Definition 4.8 (U)

$\langle T, <, \lhd, V_i \rangle, t \models U(\phi, \psi)$ iff there is $t_1 > t$ such that for any j: $(t, j) \lhd (t, i)$:
$\langle T, <, \lhd, V_j \rangle, t_1 \models \phi$ and for any t_2, $t < t_2 < t_1$: $\langle T, <, \lhd, V \rangle, t_2 \models \psi$.

$L_{\Box DK}$ denotes the language $L_{\Box D}$ enlarged by both the operators S and U.

In $L_{\Box DK}$ PE can be expressed as:

PE. $\quad \Box\phi \to U(P\phi, P\phi)$

PC can be expressed as:

PC. $\quad \Box\phi \to S(F\phi, F\phi)$

Let us remark that:

$\models U(\phi, \psi) \to F\phi$,

$\models S(\phi, \psi) \to P\phi$.

Theorem 4.9

$\not\models \Box p \to U(Pp, Pp)$.

Proof. Let $T = \{1, 2\}$, $I = \{a, b\}, 1 < 2, p \in V_a(1), p \notin V_b(1), (2, a) \lhd (2, b)$ $((1, a) \lhd (1, b)$ does not hold.) We have:

$\langle T, <, \lhd, V_a \rangle, 1 \models \Box p$

but

$\langle T, <, \lhd, V_a \rangle, a \not\models Pp$.

Thus

$\langle T, <, \lhd, V_a \rangle, 1 \not\models U(Pp, Pp)$. $\Box$

Theorem 4.10

$\not\models \Box p \to S(Fp, Fp)$.

Neither of the formulas PC and PE expresses the irreflexivity or the transitivity of time ($<$, relation "earlier later"), i.e. the validity of both does not depend on the irreflexivity or the transitivity of time.

No effect does take place at the same moment as its cause and no cause takes place at the same moment as its effect. This means that to better describe PE and PC a formula that characterizes irreflexivity should be used. It occurs that there is no such formula of L_P (Gabbay, 1981). Since in the case one element set I language L_D collapses to Priorean language, no formula of this language could do it, either.

Since any effect of ϕ is the cause of an effect of the effect of ϕ and any cause of ϕ is an effect of the cause of a cause of ϕ a formula characterizing the transitivity of $<$ should be added to description of both principles, effectivity and causality.

The transitivity of $<$ is expressible in Priorean language by formulas (McArthur, 1976, p. 26):

$$G\phi \to GG\phi,$$

$$H\phi \to HH\phi.$$

In K_t the formulas are mutually inferable.

The question of a formula of our language that characterizes the class of transitive time remains open.

Let $TRANS^{<}$ be a class of frames $\mathfrak{T}$ such that:

$$\text{if } t < t_1 \text{ and } t_1 < t_2, \text{ then } t < t_2.$$

Theorem 4.11

$$\mathfrak{T} \models Gp \to GGp \quad \text{iff} \quad \mathfrak{T} \in TRANS^{\lhd} \cap TRANS^{<}.$$

Proof. Let us prove only that if

$$\langle T, <, \lhd \rangle \notin TRANS^{\lhd},$$

then

$$\mathfrak{T} \not\models Gp \to GGp.$$

Let $T = \{1, 2, 3\}$ and $I = \{a, b\}$, $(2, a) \lhd (2, b)$, $p \in V_a(2, a)$. Thus

$$\langle T, <, \lhd, V_a \rangle, 1 \models Gp.$$

But

$$\langle T, <, \lhd, V_a \rangle, 1 \not\models GGp. \qquad \square$$

Theorem 4.12

$$\mathfrak{T} \models Hp \to HHp \quad \text{iff} \quad \mathfrak{T} \in TRANS^{\lhd} \cap TRANS^{<}.$$

Both the principles of causality and effectivity are independent of one another. PC and PE do not exclude one another.

5. The thesis of determinism

In the discussed language, the thesis of post-determinism is expressible as:

$POST\text{-}DET$. $\Box\phi \rightarrow GP\phi$.

The thesis of pre-determinism is expressible as:

$PRE\text{-}DET$. $\Box\phi \rightarrow HF\phi$.

$POST\text{-}DET$ and $PRE\text{-}DET$ are not valid (for all frames). The same argument as in the proof of theorem 4.9 can be used.

Theorem 5.1
$\not\models \Box p \rightarrow GPp$.

Theorem 5.2
$\not\models \Box p \rightarrow HFp$.

We ask, even if irreflexivity and transitivity is assumed, then
– it is possible that PC is valid but $PRE\text{-}DET$ is not,
and
– it is possible that PE is valid, but $POST\text{-}DET$ is not.

Let $\mathfrak{T}^{PE}$ be the class of frames that is characterized by PE, and $\mathfrak{T}^{PC}$ be the class of frames characterized by PC.

In order to show that $POST\text{-}DET$ is not a consequence of PE even if time is irreflexive and transitive we have to prove that $PRE\text{-}DET$ is not valid in the class of irreflexive and transitive frames characterized by PE, i.e.:

$$\mathfrak{T}^{PE} \cap IRREF \cap TRANS^{<} \cap TRANS^{\lhd} \not\models \Box p \rightarrow GPp.$$

Similarly, to prove that $PRE\text{-}DET$ is not a consequence of PC it should be showed that:

$$\mathfrak{T}^{PC} \cap IRREF \cap TRANS^{<} \cap TRANS^{\lhd} \not\models \Box p \rightarrow HFp.$$

The thesis of $POST\text{-}DET$ is usually accepted: everything that has been is unchangeable and eternal. For e.g. Augustine (2010, Book 26.5):

> Sententia quippe qua dicimus aliquid fuisse, ideo vera est, quia illud de quo dicimus, iam non est. Hanc sententiam Deus falsam facere non potest, quia non est contrarius veritati.

> The proposition asserting anything to be past is true when the thing no longer exists. God cannot make such a proposition false, because He cannot contradict the truth. (Augustine of Hippo, 2012)

According to Thomas Aquinas (Qu. 25, art. 4):

> Praeterita autem non fuisse, contradictionem implicat
>
> For the past not to have been implies a contradiction.

The fact that *POST-DET* is usually accepted justifies the fact that the principle of effectivity is not the subject of usual investigations as it is in the case of the principle of causality.

We ask if it is logically possible that the thesis *POST-DET* holds, but the thesis *PRE-DET* does not hold. In order to prove it we have to show that *PRE-DET* is not valid in the class of irreflexive and transitive frames characterized by *POST-DET*

The unchangeable past is questioned by Łukasiewicz. He points out a sort of symmetry between past and future: We should not treat the past differently from the future (1970b, p. 127). He rejects the Latin saying «facta infecta fieri non possunt» that is, what once has happened cannot become not happened. He observes (1970a, pp. 127–128):

> If the only part of the future that is now real is causally determined by the present instant, and if causal chains commencing in the future belong to the realm of possibility, then only those parts of the past are at present real which still continue to act by their effect today. Facts whose effects have disappeared altogether, and which even an omniscient mind could not infer from those now occurring, belong to the realm of possibility. One cannot say about them that they took place, but only that they were possible. It is well that it should be so. There are hard moments of suffering and still harder ones of guilt in everyone's life. We should be glad to be able to erase them not only from our memory but also from existence. We may believe that when all effects of those fateful moments are exhausted, even should that happen after our death, then their causes too will be effaced from the world of actuality and pass into the realm of possibility. Time calms our cares and brings us forgiveness.

Similarly Oscar Wilde (1998) believes that:

> Of course the sinner must repent. But why? Simply because otherwise he would be unable to realise what he had done. The moment of repentance is the moment of initiation. More than that: it is the means by which one alters one's past. The Greeks thought that impossible. They often say in their Gnomic aphorisms, 'Even the Gods cannot alter the past.' Christ showed that

> the commonest sinner could do it, that it was the one thing he could do. Christ, had he been asked, would have said—I feel quite certain about it—that the moment the prodigal son fell on his knees and wept, he made his having wasted his substance with harlots, his swine—herding and hungering for the husks they ate, beautiful and holy moments in his life. It is difficult for most people to grasp the idea. I dare say one has to go to prison to understand it. If so, it may be worth while going to prison.

To show the logical possibility that both the thesis PRE- and $POST$-DET do not hold though both the principles PC and PE are valid we have to prove that both the theses are not valid in the class of irreflexive and transitive frames characterized by both the principles PE and PC.

NOTE

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