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Optimizing Firm Inventory Costs as a Fuzzy Problem*

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Abstract. The fixed order quantity model of inventory management system is used in the deterministic part. Several elements of inventory cost, such as ordering cost, transportation and storing costs, frozen capital cost, as well as extra rebates, are taken into account in the model. Then the fuzzy optimization problem for the total cost function is formulated within the space of Ordered Fuzzy Numbers when all variables of the model are fuzzy. After the choice of a particular defuzzification functional an appropriate theorem is formulated which gives the solution of the problem.

Keywords: Ordered Fuzzy Numbers, defuzzification funcionals, management of supply

1. Introduction

The primary objective of a good inventory management system is to keep the inventory costs to the minimum. There are several elements of inventory cost, such as ordering cost, transportation cost, frozen capital cost, cost of loss (i.e. aging), cost of lost sales due to inventory shortages, and others.

Several inventory models have been built based on the above. There are two most commonly used inventory models: replenishment system and fixed order quantity system.

Under the first system the quantity to be ordered is not fixed, the next order is decided based on the lead time of the material, maximum stock level, i.e. the ordered level changes with time.

In the second system the quantity to be ordered is fixed and re-orders are made once the stock reaches a certain pre-determined level called 're-order level' or safety stock. It means that the next order is typically fixed

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and based on the average consumption during the lead time plus some buffer stock. In our calculation the buffer stock is equivalent to one day inventory consumption.

In the paper we stay at the fixed order quantity model. In this case, one of the most known determination method of the optimal level is the Economic Order Quantity (EOQ). It is an inventory-related evaluation to determine the optimum order quantity which a firm should use to ensure that inventory is not overstocked whilst at the same time it maintains sufficient stock to prevent a stock-out. The objective therefore is to minimize the combined costs of acquiring and carrying inventory (Jaruga, Nowak, and Szychta, 2001).

The EOQ results from solving some optimization problem, in which the aim (gain) function is the total cost of inventory. Notice that in solving the EOQ problem we are faced with the competition of more than two components in the gain function. Namely, the greater the number of orders placed per year would contain fewer items per order which results in lower inventory costs but would incur a larger overall ordering cost, which could contain the frozen capital cost, as well. Conversely, the fewer orders that are placed per year would contain larger quantities per order but have lower overall ordering costs. This however, results in inventory being held in stock for longer periods resulting in increased inventory (holding) costs.

For many years the only tool representing imprecise and vague notions was the probability theory. There is another tool which can represent the vagueness. In the present paper we will focus our fuzzy approach on applications to economical problems, for which modelling the influence of imprecise quantities and preferences on decision maker's opinions is important, in a number of administrating accounting problems.

With the help of a fuzzy number it is possible to express incomplete knowledge about a quantity giving the possible intervals of its realization, and writing it in the form of a (subjective) function of the information, representing the capability degree of this realization.

The organization of the paper is as follows: A generalization of the classical concept of fuzzy numbers and the definition of Ordered Fuzzy Numbers (OFN) are given in Section 2. In Section 3 problem of management of supply and determining the optimal size of a delivery from outside, which minimizes total costs, when unit costs of purchase, transportation and storage are considered. Section 4 brings an example and Section 5 some conclusions.

2. Ordered Fuzzy Number

Proposed recently by the second author and his two coworkers: P. Prokopowicz and D. Ślęzak (Kosiński, Prokopowicz, and Ślęzak, 2002; Kosiński, Prokopowicz, and Ślęzak, 2003; Kosiński, 2006) an extended model of convex fuzzy numbers – CFN (Nguyen, 1978), called Ordered Fuzzy Numbers – OFN, does not require any existence of membership functions. In this model we can see an extension of CFN – model, when one takes a parametric representation of convex fuzzy numbers known since 1986 (Goetschel and Voxman, 1986).

Definition 1

An Ordered Fuzzy Number A (OFN) is an ordered pair (f, g) of continuous functions $f, g : [0, 1] \to \mathbb{R}$.

The set of all OFN we denote by $\boldsymbol{R}$. The functions f and g are called the branches of fuzzy number A. Notice that in our definition we do not require that the two continuous functions f and g are inverse functions of some membership function. This means that, referring to classical fuzzy numbers defined by membership functions, corresponding membership function needs not exist for OFN. The above definition of Ordered Fuzzy Numbers has been recently generalized (Kosiński, 2006) by allowing the pair (f, g) to be functions of bounded variation.

To be in agreement with further and classical notations of fuzzy sets (numbers), the independent variable of both functions f and g is denoted by y, and the values of them by x. The continuity of both parts implies that their images are bounded intervals, say UP and $DOWN$, respectively (Fig. 1a).

We could use the following symbols to mark the boundaries for $UP_A = [l_A, 1_A^-] = [f(0), f(1)]$ and for $DOWN_A = [1_A^+, p_A] = [g(1), g(0)]$ in Figure 1. In general, these intervals need not be proper. If we assume, additionally, that

1) f is increasing, and g is decreasing, such that
2) $f \leq g$ (pointwise)

we may define the membership function:

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \notin [l_A, p_A] \\ f_A^{-1}(x) & \text{if } x \in UP_A \\ 1 & \text{if } x \in [1_A^-, 1_A^+] \\ g_A^{-1}(x) & \text{if } x \in DOWN_A \end{cases}, \tag{1}$$

In this way, we have obtained the membership function $\mu_A(x)$, $x \in \mathbb{R}$. When the functions f and/or g are not invertible or condition 2) is not satisfied then the membership curve (or relation) can be defined in the plane $x{-}y$, composed of the graphs of f and g and the line $y = 1$ over the core $\{x \in [f(1), g(1)]\}$. Notice that in general $f(1)$ needs not be less than $g(1)$. In this way we can obtain improper intervals for $[l_A, 1_A^-]$ or $[1_A^+, p_A]$ which have been already discussed in the framework of extended interval arithmetic by Kaucher (Kaucher, 1980).

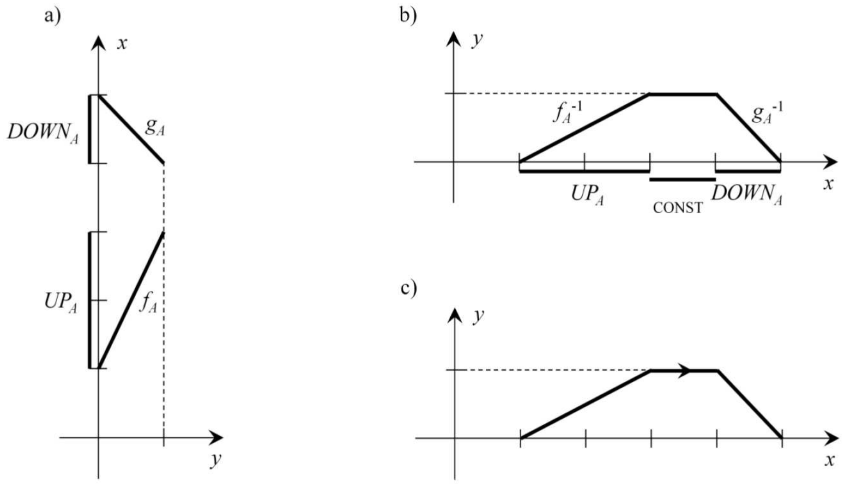

Figure 1. a) Ordered Fuzzy Numbers, b) Ordered Fuzzy Numbers with membership function, c) Arrow denotes the order of inverted functions and the orientation

In Figure 1 to ordered pair of two continuous functions, here just two affine functions, corresponds a membership function of a convex fuzzy number with an extra arrow denoting the orientation of the closed curve formed below. A pair of continuous functions (g, f) determine a different Ordered Fuzzy Number than the pair (f, g); graphically the corresponding curves determine two different orientations of Ordered Fuzzy Numbers.

Let $A = (f_A, g_A)$, $B = (f_B, g_B)$ and $C = (f_C, g_C)$ be Ordered Fuzzy Numbers. The sum $C = A + B$, product $C = A \cdot B$ and division $C = A/B$ are defined in $\boldsymbol{R}$ as follows:

$$f_C(y) = f_A(y) \times f_B(y) \quad \text{and} \quad g_C(y) = g_A(y) \times g_B(y) \tag{2}$$

where "$\times$" works for "$+$", "$\cdot$", and "$/$", respectively, and where A/B is defined when $f_B \neq 0$ and $g_B \neq 0$ for each $y \in [0, 1]$.

Let $r \in \mathbb{R}$ and denote by r' the constant function $r'(s) = r$ for any $s \in [0,1]$. Then $r^*(s) = (r', r')$ is the Ordered Fuzzy Number represented in $\boldsymbol{R}$ by (the crisp real) number r. Subtraction in $\boldsymbol{R}$ is defined as addition of the corresponding negative number, i.e. $-A = (-f_A, -g_A)$. It is obvious that $A+(-A) = 0^*$. Multiplication by a scalar is defined by $rA = (rf_A, rg_A)$.

A relation of partial ordering in the space of all OFN can be introduced by defining the subset of 'positive' Ordered Fuzzy Numbers: a number $A = (f, g)$ is not less than zero, and by writing

$$A \geq 0 \text{ iff } f \geq 0, \quad g \geq 0. \tag{3}$$

Definition 2

A map ϕ from the space $\boldsymbol{R}$ of all OFN's to reals is called a defuzzification functional if it satisfies:

1. $\phi(c^*) = c$
2. $\phi(A + c^*) = \phi(A) + c$
3. $\phi(cA) = c\phi(A)$ for any $c \in R$ and $A \in \boldsymbol{R}$,
4. $\phi(A) \geq 0$ if $A \geq 0$

where $c^*(s) = (c', c')$, $s \in [0,1]$, represents crisp number (a real) $c \in R$.

The linear functionals, as MOM (*middle of maximum*), FOM (*first of maximum*), LOM (*last of maximum*) are given by specification of h_1 and h_2

$$\phi(f_A, g_A) = \int_0^1 f_A(s)dh_1(s) + \int_0^1 g_A(s)dh_2(s), \tag{4}$$

where $h_1 \geq 0$, $h_2 \geq 0$ are of bounded variation and

$$\int_0^1 dh_1(s) + \int_0^1 dh_2(s) = 1. \tag{5}$$

3. Inventory optimization

Every firm has the challenge of matching its supply volume to customer demand. How well the firm manages this challenge has a major impact on its profitability. Also, the amount of inventory held has a major impact on available cash. With working capital at a premium, it is important for companies to keep inventory levels as low as possible and to use or sell inventory as quickly as possible. For most of analysts their opinion concerning a company's performance to make earnings forecasts and buy and sell recommendations, inventory is always one of the top factors they consider.

The challenge of managing inventory is still increasing. Inventory optimization models can be deterministic – with every set of variable states uniquely determined by the parameters in the model, stochastic – with variable states described by probability distributions or fuzzy. In this paper we proposed the last one.

3.1. Deterministic model

The enterprise inventory management is an integral part of operating activities, as it affects the liquidity of its financial performance and competitive advantage of the company. The purpose of inventory management is to have it in the amount necessary to operate, incurring the lowest possible operating costs.

The present formulation is in the frame of the economic order quantity model (EOQ). We consider an abstract inventory item. To estimate the cost of inventory we formulate main assumptions for EOQ model:

1. the abstract inventory item is split into units,
2. we are referring to some time unit, say one year,
3. the demand is constant in time,
4. the sales are uniform in time and known,
5. the next supply arrives when the stock is on one day level.

Let us start with deterministic formulation in which the following objects appears:

D – annual inventory demand, measured in number of units,

$D/360$ – daily demand for supply (assume that a year has 360 days),

Q – order quantity, measured in number of units,

ΔQ – daily consumption of inventory,

D/Q – frequency of the deliveries,

$360/((D/Q)) = t_0$ – time between successive deliveries,

c_p – unit price of purchase,

c_t – transportation cost of a single delivery,

c_s – unit inventory cost,

$r(Q)$ – discount function on purchase,

$s(Q)$ – discount function on stored inventory,

$K(Q)$ – total cost,

K_p – purchase cost,

K_f – frozen capital cost,

K_t – transportation (delivery) cost,

K_s – storage cost,

R – banking percentage rate to calculate cost of frozen capital.

We can write the general expression for the total cost $K(Q)$, as the sum of the purchase cost K_p, the frozen capital cost K_f, the transportation (delivery) cost K_t and the storage cost K_s, i.e.

$$K(Q) = K_p + K_f + K_t + K_s. \tag{6}$$

Suppose that we get the discount $r(Q)$ on purchase (Fig. 2) and the discount $s(Q)$ on stored inventory (Fig. 3) depending on the amount of Q as step functions:

$$r(Q) = \begin{cases} r_0 = 0 & \text{if } 0 < Q < Q_1^r \\ r_1 & \text{if } Q_1^r \leq Q < Q_2^r \\ r_2 & \text{if } Q_2^r \leq Q \leq D \end{cases} \tag{7}$$

and

$$s(Q) = \begin{cases} s_0 = 0 & \text{if } 0 < Q < Q_1^s \\ s_1 & \text{if } Q_1^s \leq Q < Q_2^s \\ s_2 & \text{if } Q_2^s \leq Q \leq D \end{cases} \tag{8}$$

where Q_1^r, Q_2^r, Q_1^s and Q_2^s are fixed amounts of item's quantity (here 3 steps were assumed, however, more steps can be also considered).

The purchase cost K_p depends on the amount of the single deliver Q, the frequency of the deliveries D/Q, the discount $r(Q)$ and the unit price c_p, and is given by

$$K_p = c_p \cdot (1 - r(Q)) \cdot Q \cdot \frac{D}{Q} = c_p \cdot (1 - r(Q)) \cdot D. \tag{9}$$

Due to discount function $r(Q)$ we can see that it is a piecewise constant function.

The cost of frozen capital depends on number of deliveries D/Q, money spent on a single delivery, banking percentage rate R, and the amount of the single delivery Q. Deduced form of the purchase cost K_p leads to the following cost K_f of frozen capital:

$$K_f = c_p \cdot (1 - r(Q)) \cdot Q \cdot \frac{D}{Q} \cdot \frac{R}{\frac{D}{Q}} = c_p \cdot (1 - r(Q)) \cdot Q \cdot R. \tag{10}$$

We can see that the expression K_f represents a step function, which is linear at each step.

The cost of the transportation (delivery) K_t depends on annual frequency of deliveries D/Q and the transportation cost of a single delivery c_t, i.e.

$$K_t = c_t \cdot \frac{D}{Q}. \tag{11}$$

According to the assumptions from 3 to 5, the storage cost K_s depends on annual frequency of deliveries D/Q, the discount $s(Q)$, the unit inventory cost c_s and the level of inventory between successive deliveries. As shown in Figure 4 the level of inventory is given by

$$\int_0^{t_0} \left(-\frac{Q}{t_0} \cdot t + Q + \Delta Q \right) dt = \left(\frac{Q}{2} + \Delta Q \right) \cdot t_0. \tag{12}$$

and storage cost by

$$\begin{aligned} K_s &= \frac{D}{Q} \cdot c_s \cdot (1 - s(Q)) \cdot \left(\frac{Q}{2} + \Delta Q \right) \cdot t_0 = \\ &= \frac{D}{Q} \cdot c_s \cdot (1 - s(Q)) \cdot \left(\frac{Q}{2} + \Delta Q \right) \cdot \frac{360}{\frac{D}{Q}} = \\ &= 180 \cdot c_s \cdot (1 - s(Q)) \cdot (Q + 2 \cdot \Delta Q). \end{aligned} \tag{13}$$

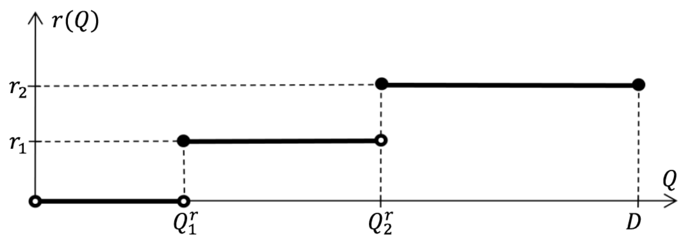

Figure 2. Step function of the discount $r(Q)$ on purchase

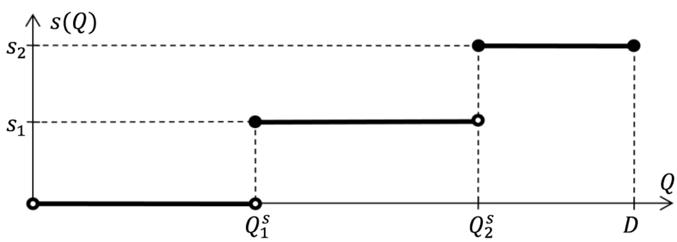

Figure 3. Step function of the discount $s(Q)$ on stored inventory

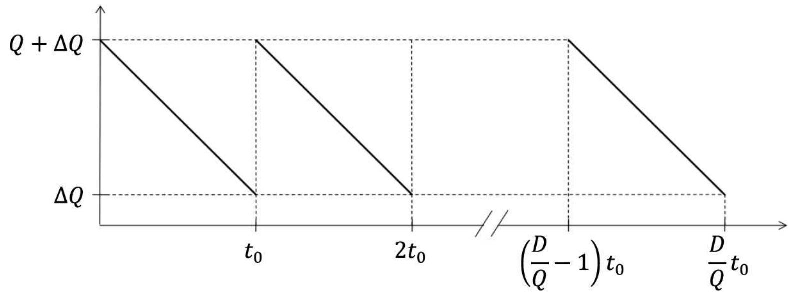

Figure 4. The level of inventory between successive deliveries

Hence the function of total cost $K(Q)$ can by expressed by

$$\begin{aligned} K(Q) &= c_p \cdot (1 - r(Q)) \cdot D + c_p \cdot (1 - r(Q)) \cdot Q \cdot R + \\ &+ c_t \cdot \frac{D}{Q} + 180 \cdot c_s \cdot (1 - s(Q)) \cdot (Q + 2 \cdot \Delta Q) = \\ &= c_p \cdot (1 - r(Q)) \cdot (D + Q \cdot R) + c_t \cdot \frac{D}{Q} + \\ &+ 180 \cdot c_s \cdot (1 - s(Q)) \cdot (Q + 2 \cdot \Delta Q). \end{aligned} \tag{14}$$

3.2. Optimalization problem

The inventory optimization problem requires to find the minimum of the cost function $K(Q)$. The argument of the minimum gives the optimal value of the order quantity. Notice that in $K(Q)$ the first and the last component does depend on Q in a piecewise way. Suppose that

$$0 < Q_1^r < Q_1^s < Q_2^r < Q_2^s < D. \tag{15}$$

The search for the optimal value should be performed in a piecewise way, i.e. in each subinterval $L_0 = (0, Q_1^r)$, $L_1 = [Q_1^r, Q_1^s)$, $L_2 = [Q_1^s, Q_2^r)$, $L_3 = [Q_2^r, Q_2^s)$, $L_4 = [Q_2^s, D]$ (Fig. 5). Then the global optimum is that which is minimal over those values calculated from each subintervals. Because

$$\frac{\partial K(Q)}{\partial Q} = c_p \cdot (1 - r(Q)) \cdot R - c_t \cdot \frac{D}{Q^2} + 180 \cdot c_s \cdot (1 - s(Q)) \tag{16}$$

and

$$\frac{\partial K(Q)}{\partial Q} = 0 \Leftrightarrow Q^* = \sqrt{\frac{c_t \cdot D}{c_p \cdot (1 - r(Q)) \cdot R + 180 \cdot c_s \cdot (1 - s(Q))}} \tag{17}$$

on each of those subintervals L_k, $k = 0, 1, 2, 3, 4$ the local optimum is attained at

$$Q^*_{ij} = \sqrt{\frac{c_t \cdot D}{c_p \cdot (1 - r_i) \cdot R + 180 \cdot c_s \cdot (1 - s_j)}}, \qquad i, j = 0, 1, 2. \tag{18}$$

If $Q^*_{ij} \in L_k$, $i, j = 0, 1, 2$, $k = 0, 1, 2, 3, 4$ then the optimal value is calculated according to

$$\begin{aligned} Q_{opt} = \arg\min\{&K(Q^r_1), K(Q^s_1)K(Q^r_2), K(Q^s_2), K(D), \\ &K(Q^*_{00}), K(Q^*_{10}), K(Q^*_{11}), K(Q^*_{21}), K(Q^*_{22})\}. \end{aligned} \tag{19}$$

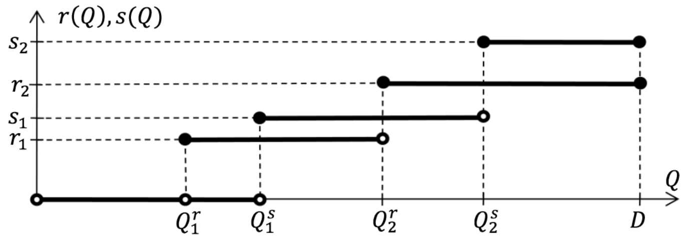

Figure 5. Step functions of the discount $r(Q)$ and $s(Q)$

3.3. Fuzzy optimization problem

The present formulation is in the frame of the economic order quantity model (EOQ) and similar to that proposed in the set of CFN by (Vujošević, Petrović, and Petrović, 1996) and repeated by (Kuchta, 2001). Fuzzy approach to economical and management accounting problems was already applied in the CFN setup by Buckley in (Buckley, 1992) and in the OFN one in (Chwastyk and Kosiński, 2013; Kosiński, Kosiński, and Kościeński, 2013).

Our aim is to give general solution of the optimization problem with the total cost function in (14) when D, c_p, c_t and c_s are fuzzy and represented by Ordered Fuzzy Numbers (OFN).

It will be easy to see that the arithmetic of OFN manifests its superiority over the arithmetic of Convex Fuzzy Numbers (CFN), and the complex calculation performed by other authors of (Vujošević et al., 1996) and (Kuchta, 2001) can be omitted. What we need only is the choice of the defuzzification functional, which suits the decision maker the most.

Let $\phi(\cdot)$ be the defuzzification functional chosen by the decision maker. Then the problem of its minimal value on the fuzzy cost $K(Q)$ gives us the economic order quantity. Writting explicitly

$$\text{find} \arg\{\min \phi(K(Q)) : Q \in \boldsymbol{R}\}. \tag{20}$$

The question arises: how to find the minimum of the functional? The answer is rather obvious and comes from the physics, and it is formulated as the stationary action principle: the minimum of the functional appears at the argument Q at which its first variation (the *Gâteaux* derivative) vanishes. Calculating the first variation of $\phi(K(Q))$ with respect to Q under given D, c_p, c_t and c_s, we get

$$\delta\phi(K(Q)) = \partial_K \phi(K) \partial_Q K(Q) \partial Q. \tag{21}$$

Here $\partial_K \phi(K)$ and $\partial_Q K(Q)$ denote functional derivative. Due to the arbitrariness of ∂Q, the vanishing $\delta\phi(K(Q)) = 0$ implies

$$\partial_K \phi(K) \partial_Q K(Q) = 0, \tag{22}$$

and the argument Q^*, at which the product of the derivatives vanishes, constructs the solution of our optimization problem.

To illustrate let us consider a class of linear functionals given by (4). Let us denote branches of the fuzzy number $K(Q)$ by (f_K, g_K), and for the remaining quantities we assume the denotation with the appropriate subscripts, i.e.

$$\begin{gathered} D = (f_D, g_D), \quad Q = (f_Q, g_Q), \\ c_p = (f_p, g_p), \; c_t = (f_t, g_t), \; c_s = (f_s, g_s). \end{gathered} \tag{23}$$

Then the linear functional on the fuzzy cost $K(Q)$ will have the form

$$\phi(K(Q)) = \phi(f_K, g_K) = \int_0^1 f_K(s) dh_1(s) + \int_0^1 g_K(s) dh_2(s) \tag{24}$$

where compare (14),

$$\begin{gathered} f_{K_{ij}}(s) = f_p(s)(1 - r_i)(f_D(s) + R f_Q(s)) + \frac{f_t(s) + f_D(s)}{f_Q(s)} + \\ +180 f_S(s)(1 - s_j)(f_Q(s) + \Delta Q), \quad i, j = 0, 1, 2 \end{gathered} \tag{25}$$

and

$$\begin{gathered} g_{K_{ij}}(s) = g_p(s)(1 - r_i)\left(g_D(s) + R g_Q(s)\right) + \frac{g_t(s) + g_D(s)}{g_Q(s)} + \\ +180 g_S(s)(1 - s_j)(g_Q(s) + \Delta Q), \quad i, j = 0, 1, 2. \end{gathered} \tag{26}$$

Now we perform the differentiation in (21) under (24), to get

$$\begin{gathered} \delta\phi(K(Q)) = \int_0^1 \left(f_M(s) - \frac{f_t(s) \cdot f_D(s)}{(f_Q(s))^2} \right) \delta f_Q(s) dh_1(s) + \\ + \int_0^1 \left(g_M(s) - \frac{g_t(s) \cdot g_D(s)}{(g_Q(s))^2} \right) \delta g_Q(s) dh_2(s), \end{gathered} \tag{27}$$

where $(f_{M_{ij}}(s), g_{M_{ij}}(s))$, $i,j = 0,1,2$ represent five fuzzy numbers $\tilde{M}_{ij}$ as OFN

$$f_{M_{ij}}(s) = f_p(s)(1-r_i)R + 180 f_S(s)(1-s_j) \tag{28}$$

and

$$g_{M_{ij}}(s) = g_p(s)(1-r_i)R + 180 g_S(s)(1-s_j) \tag{29}$$

where $i,j = 0,1,2$.
We can consider two cases:
Case A: Functions h_1 are h_2 are absolutely continuous, and
Case B: Functions h_1 and h_2 are singular, i.e. almost everywhere $h_1'(s)$ and $h_2'(s)$ vanish.

In (Chwastyk and Kosiński, 2013) we have discussed less complex case and proved that in both cases the forms of h_1 and h_2 in (24) are not important for optimal value of Q. In the present case, however, we formulate theorem.

Theorem. If the total inventory cost $K(Q)$ arising from the fuzzy annual demand D, the unit cost of purchase c_p, the transportation cost of a single delivery c_t, the unit inventory cost c_s, the discount function $r(Q)$ and $s(Q)$, and banking percentage rate R, is given by (14) and the decision maker chooses the defuzzification functional ϕ in (24), then in **Case A** economic order quantity is given by two phase optimization procedure:

- First phase: on each subinterval L_k, $k = 0,1,2,3,4$ optimal values are found

$$q_{ij}^* = \phi(K(Q_{ij}^*)), \text{ where } Q_{ij}^* = (f_{Q_{ij}^*}, g_{Q_{ij}^*}), \tag{30}$$

where $f_{Q_{ij}^*}$ and $g_{Q_{ij}^*}$ are given by equations

$$f_{Q_{ij}^*}(s) = \sqrt{\frac{f_t(s)\cdot f_D(s)}{f_p(s)\cdot(1-r_i)\cdot R + 180\cdot f_S(s)\cdot(1-s_j)}} \tag{31}$$

and

$$g_{Q_{ij}^*}(s) = \sqrt{\frac{g_t(s)\cdot g_D(s)}{g_p(s)\cdot(1-r_i)\cdot R + 180\cdot g_S(s)\cdot(1-s_j)}} \tag{32}$$

where $s \in [0,1]$ and $i,j = 0,1,2$.

- Second phase: from these five expressions the optimal value is calculated according to

$$\arg\{\min \phi(f_{K_{ij}}, g_{K_{ij}}) : i,j = 0,1,2\} \tag{33}$$

under the denotation (23), and when in the expressions for $f_{K_{ij}}$ and $g_{K_{ij}}$ in (25) and (26) the formulas (31) and (32) substitute the pair (f_Q, g_Q).

Remark. If the rebate function is just a constant, i.e. $Q_0 = Q_1 = Q_2$, then the economic order quantity is given by

$$q^* = \phi(K(Q^*)), \text{ where } Q^* = (f_{Q^*}, g_{Q^*}), \tag{34}$$

with

$$f_{Q^*}(s) = \sqrt{\frac{f_t(s) \cdot f_D(s)}{f_p(s) \cdot (1 - r_2) \cdot R + 180 \cdot f_S(s) \cdot (1 - s_2)}} \tag{35}$$

and

$$g_{Q^*}(s) = \sqrt{\frac{g_t(s) \cdot g_D(s)}{g_p(s) \cdot (1 - r_2) \cdot R + 180 \cdot g_S(s) \cdot (1 - s_2)}} \tag{36}$$

where $s \in [0, 1]$.

In particular case, when $\phi = \phi_{MOM}$, then the minimal cost is given by

$$q^* = \frac{K(f_{Q^*}(1)) + K(g_{Q^*}(1))}{2} \tag{37}$$

with the function K given by (14).

4. Example

In (Kuchta, 2001) the author considered the problem of minimizing the value of the fuzzy cost $K(Q)$ of a firm in which

$$K(Q) = D \cdot c + c_t \cdot \frac{D}{Q} + c_s \cdot \frac{Q}{2}. \tag{38}$$

It correspond to our problem by neglecting the cost of frozen capital, discount of purchase cost, discount stored inventory cost as well as the influence of safety stock corresponding to daily consumption of inventory. Then the storage cost is equal $K_s = c_s \cdot Q/2$ instead of (13). It correspond to the case formulated as the optimization problem from Section 3.1.

Kuchta in her paper (Kuchta, 2001) considered first the crisp (deterministic) case with the following data: $D = 1000$, $c = 10$, $K_t = 8$ and $K_s = 7$. In her calculation the final economic order quantity Q_k was 46 and the total cost $K(Q_k)$ corresponding to this order value was 10329. Unfortunately, in our calculation we get $Q_k = 47.8$ and the cost value $K(Q_k) = 10334.7$. Those values are different from that of Kuchta in (Kuchta, 2001).

Then she considered the fuzzy case with the same crisp values of D and c, but with the fuzzy transportation cost $\tilde{K}_t$ represented by the triangular membership function $(7, 8, 9)$ and the fuzzy storage cost $\tilde{K}_s$ represented by the triangular membership function $(1.5, 7, 15)$. Determination of the economic order quantity in that fuzzy case is not unique, and is based on some estimation to be done by the decision maker if he/she is supplied with a set of fuzzy cost value determined with the help of (38) in which the fuzzy values $\tilde{K}_t$ and $\tilde{K}_s$ appear, together with $2M+1$ crisp values of Q from the vicinity of Q_k, where M is a natural number determined by the decision maker (in Kuchta's paper it was 50). Then the decision maker has to choose from those $2M+1$ fuzzy cost values the most suitable for him/her.

The same example will here be considered for OFN. Adapting our general solution formula (31) and (32) with vanishing discount functions we obtain

$$f_{Q^*}(s) = \sqrt{\frac{2 \cdot f_t(s) \cdot f_D(s)}{f_S(s)}} \text{ and } g_{Q^*}(s) = \sqrt{\frac{2 \cdot g_t(s) \cdot g_D(s)}{g_s(s)}}.$$

In contrast to Kuchta's approach, if we apply our method and the linear defuzzification functional (24), then from the Theorem for the Case A (absolutely continuous h_1 and h_2 in (24)), we get the explicit expression for the fuzzy EOQ

$$f_{K^*}(s) = f_D(s) \cdot c + f_t(s) \cdot \frac{f_D(s)}{f_{Q^*}(s)} + f_s(s) \cdot \frac{f_{Q^*}(s)}{2}$$

and

$$g_{K^*}(s) = g_D(s) \cdot c + g_t(s) \cdot \frac{g_D(s)}{g_{Q^*}(s)} + g_s(s) \cdot \frac{g_{Q^*}(s)}{2}.$$

To this end let us choose the representation of two convex triangular fuzzy numbers $\tilde{K}_t$ and $\tilde{K}_s$ as Ordered Fuzzy Numbers. We know that to each CFN correspond two OFNs, they differ the orientation. Hence for $\tilde{K}_t$ we have $(9-s, 7+s)$ and $(7+s, 9-s)$, with $s \in [0,1]$ (Fig. 6). On the other hand for $\tilde{K}_s$ we have $(15-8s, 1.5+5.5s)$ and $(1.5+5.5s, 15-8s)$ (Fig. 7). For $\tilde{K}_t$, if we take the first OFN, which has the so-called negative orientation, then it means that our estimation of future transportation cost is rather optimistic, the cost is at most around 8, on the other hand if we take the second OFN, namely $(9-s, 7+s)$, then we are rather pessimistic: the transportation cost is at least around 8.

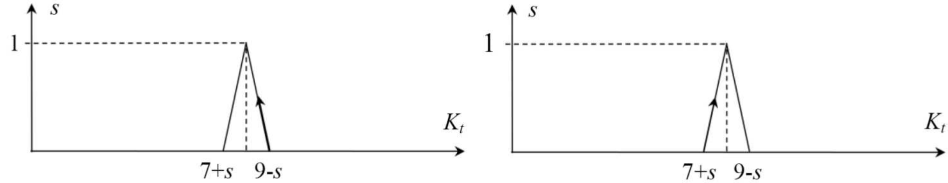

Figure 6. OFN of fuzzy transportation cost

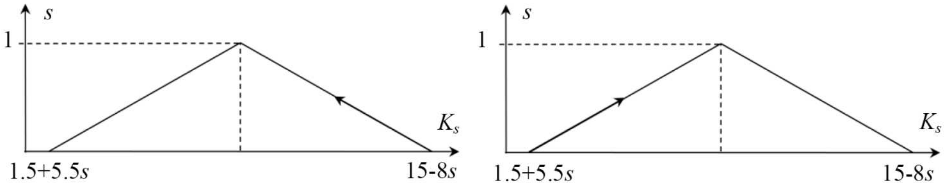

Figure 7. OFN of fuzzy storage cost

For further calculation we assume the optimistic viewpoint and take for $f_t(s) = 9-s$ and $g_t(s) = 7+s$, while for $f_s(s) = 15-8s$ and $g_s(s) = 1.5+5.5s$. Notice that we could assume 3 different cases and, consequently, 3 different solutions for fuzzy EOQ could follow.

Applying the formula appearing in Remark,[1] with $f_D(s) = 1000$, $f_M(s) = (f_s(s))/2$ and $g_D(s) = 1000$, $g_M(s) = g_s(s)/2$ we obtain the fuzzy EOQ as the Ordered Fuzzy Number

$$f_{Q^*}(s) = \sqrt{\frac{2000 \cdot (9-s)}{15-8s}}, \; g_{Q^*}(s) = \sqrt{\frac{2000 \cdot (7+s)}{1.5+5.5s}}, \; s \in [0,1].$$

From the last expression we could calculate easily the fuzzy minimal inventory cost $K(Q^*)$. Notice that neither Q^* nor $K(Q^*)$ can be represented in the form of CFN with triangular membership function. We could draw figures for them by substituting values of s from $[0,1]$ interval. By applying a particular defuzzification functional we could calculate the crisp values corresponding to Q^* and $K(Q^*)$. Before the end this section we point out the characteristic values of Q^*, namely

$$f_{Q^*}(0) = \sqrt{\frac{2000 \cdot 9}{15}} = 34.6,$$

$$f_{Q^*}(1) = g_{Q^*}(1) = \sqrt{\frac{2000 \cdot 8}{7}} = 47.8,$$

$$g_{Q^*}(0) = \sqrt{\frac{2000 \cdot 7}{1.5}} = 96.6.$$

Notice that by applying the defuzzification functional $\phi = \phi_{MOM}$ to Q^* we obtain the crisp EOQ $\phi_{MOM}(Q^*) = f_{Q^*}(1) = 47.8$, and equal to Q_k from the deterministic case.

Corresponding to those values the characteristic values of the cost are:

$$f_{K^*}(0) = 10000 + f_t(0) \cdot \frac{1000}{f_{Q^*}(0)} + f_s(0) \cdot \frac{f_{Q^*}(0)}{2} =$$

$$= 10000 + 9 \cdot \frac{1000}{34.6} + 15 \cdot \frac{34.6}{2} = 10519.6,$$

$$f_K^*(1) = g_K^*(1) = 10000 + f_t(1) \cdot \frac{1000}{f_{Q^*}(1)} + f_s(1) \cdot \frac{f_{Q^*}(1)}{2} = 10334.6,$$

$$g_{K^*}(0) = 10000 + g_t(0) \cdot \frac{1000}{g_{Q^*}(0)} + g_s(0) \cdot \frac{g_{Q^*}(0)}{2} =$$

$$= 10000 + 7 \cdot \frac{1000}{96.6} + 15 \cdot \frac{96.6}{2} = 10144.9,$$

If we look at the data of fuzzy cost values in (Kuchta, 2001) then we can see in Table 7.1 on page 112 that domains of triangular membership functions of those values vary from 10138 to 10513. Moreover, fuzzy values of cost are related to the range of order quantities from 91 to 36. In our calculation this range is 34.6 to 96.6.

5. Conclusions

Here we have solved a problem originating from management of inventory, using the model of Ordered Fuzzy Numbers, and we have demonstrated its applicability in modelling the influence of imprecise quantities and preferences of decision maker.

Thanks to well-defined arithmetic of OFN one can construct an efficient decision support tool when data are imprecise. Then in the next paper we will introduce some dynamics in management of inventory and show that the OFN can be successfully applied in the presentation of stock prices giving transparent image of the stock exchange.

NOTES

[1] Notice that in our example D is crisp and is represented by the pair of constant functions (1000',1000').

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