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ESTIMATION OF YOUNG'S MODULUS OF ELESTICITY BY THE FORM FINDING OF GRID SHELL STRUCTURES BY THE DYNAMIC RELAXATION METHOD

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Abstract

The paper is basically focused on the process of form finding by the dynamic relaxation method (DRM) with the aid of computational tools that enable us to make many calculations with different inputs. There are many important input values with a significant impact on the course of the calculations and the resulting displacement of a structure. One of these values is Young's modulus of elasticity. This value has a considerable impact on the final displacement of a grid shell structure and the resulting internal forces.

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Key words

- Grid shell structure,
- dynamic relaxation method,
- young's modulus of elasticity.

1 INTRODUCTION

Free shaped transparent roofs and facades are an integral part of today's modern architecture. Developments in the sector of widespread roofs has led to the utilization of lightweight grid structures made of steel, wood and glass. Finding an appropriate shape for a grid shell allows you to make the correct selection of the material and also to optimize the material requirements. This is an important factor in these types of structures. Therefore, many methods have been developed to calculate curved grid shell geometry according to the equilibrium conditions. The process of finding the stable state of a structure is usually called the stabilizing process. Through this process, highly efficient structures that are stressed only by compression without the effect of bending moments can be determined. Structures obtained by the form-finding process can also effectively resist external loads.

2 BASIC EQUATION OF THE DYNAMIC REALXATION METHOD

The dynamic relaxation method is a numerical optimization method for solving highly non-linear problems. The basis of the method is the tracking of the movement of each structural node from the initial unloaded position during each time step until the structure reaches a static equilibrium as a result of artificial damping. It means that the sum of all the forces acting on the node is equal to zero. According to D'Alembert's Principle, the equilibrium condition for the motion of the system, which must be fulfilled at each time step by considering the inertial and damping forces, can be written as follows (1):

$$M^n \cdot \ddot{v}^n + C^n \cdot \dot{v}^n + K^n \cdot v^n = P^n \tag{1}$$

where M^n, C^n and K^n are the fictitious mass, viscous damping, stiffness and the P^n vector of externally applied loads, $\ddot{v}^n$and $\dot{v}^n$ are derivations of the displacement vector v^n. The superscript *n* marks an iterative step (a fictitious time step).

In this method, there are two types of damping, i.e., kinetic and viscous. Kinetic damping is used due to the rapid and stable convergence in the calculations. Compared with viscous damping, which assumes that the link between nodes contains a viscous force component, artificial kinetic damping has no real influence. In this paper we will only utilize kinetic damping. Therefore, the viscous damping component in equation (1) can be ignored:

$$M^n \cdot \ddot{v}^n + K^n \cdot v^n = P^n \tag{2}$$

At the beginning the whole structure is divided into a set of discrete points. The movement of each node is caused by a residual force (3), which in the first step is equal to the applied external force f_{ext} and is monitored at every single time interval. The state of equilibrium is reached when the residual forces, i.e., the summation of the internal forces ($K^n \cdot v^n$) and external forces (P^n) acting on the node, are close to zero (Fig.1). The fictitious masses, which are concentrated on each node of the grid, become stabilized in its state of equilibrium on the basis of the initial acceleration and a few iterative steps:

$$R^n = M^n \cdot \ddot{v}^n = P^n - K^n \cdot v^n \tag{3}$$

When more than one element contributes to the force at node *i*, eq. (3) can then be formulated for the x direction as (4):

$$R_{ix}^n = P_{ix}^n - K_{ix}^n \cdot v_{ix}^n = P_{ix}^n - \sum_{m=1}^{q} \frac{E_m \cdot A_m}{l_m^2}(x_j - x_i) \cdot \Delta l_m \tag{4}$$

Where E_m is the Young's modulus for element *m*, A_m is the cross section area, l_m the length of element *m* between nodes *i* and *j*, $x_{i,j}$ the x coordinate of nodes *i, j*, Δl_m the elongation of element *m,* and *q* the number of elements.

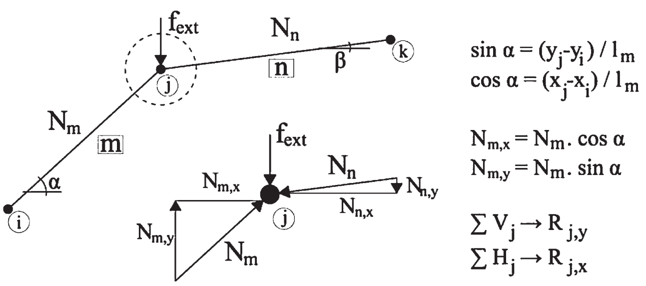

Fig. 1 *Equilibrium state in DRM.*

According to the method of constant acceleration, the nodal velocity varies linearly for the time increment Δ*t*. The acceleration vector of the *n*th iteration is estimated by the linear interpolation of the velocity vector at time Δ*t*.

$$\ddot{v}_i^n = \frac{\dot{v}_i^{n+1/2} - \dot{v}_i^{n-1/2}}{\Delta t} \tag{5}$$

Then the nodal velocity of the *n+1/2*th iteration can be written as:

$$\dot{v}_i^{n+1/2} = \dot{v}_i^{n-1/2} + \frac{\Delta t}{M_i^n} \cdot R_i^n \tag{6}$$

In eq. (6), M_i^n and R_i^n are members of the fictitious mass matrix and the residual force in the *n*th iteration. The positions of all the nodes of the iteration *(n+1)* are calculated based on the new velocities:

$$v_i^{n+1} = v_i^n + \Delta t \cdot \dot{v}_i^{n+1/2} \tag{7}$$

During the entire calculations the kinetic energy (8) of the whole structure and the single energy peaks are controlled. When a peak is detected, all the nodal velocities are set to zero, and the current coordinates are the input values for the next iteration. The iterative calculations are carried out until an energy peak, which is smaller than the chosen limit value, is reached. According to (Alamatian, 2012), the convergence criterion for an out of balance force and kinetic energy are e_R=1.0E-6 and e_K=1.0E-12.

$$E_K^n = \frac{1}{2} \cdot \sum_{i=1}^{q} M_i^n \cdot (\dot{v}_i^{n+1/2})^2 \tag{8}$$

In this method, the movement of the construction is fictitious. Static problems are solved using a fictitious dynamic analysis. To solve them, they are transformed into a pseudo-dynamic system by the introduction of fictitious values of inertia and damping. DRM is a static analysis method using a dynamic equilibrium equation. The mass is used to check the convergence of the calculations. There are many approaches to solving this problem, for example (Adriaenssens, 2000), (Day, 2000), (Han, 2003), (Rezaiee-Pajand, 2011) and (Topping, 1994). Based on the structural dynamics, we are able to obtain the frequency and period of a single node. The time increment is obtained by the multiplication of the period by the non-dimensional variable k_m. According to the calculations, k_m is equal to 0.1. The mass vector (9) is obtained by derogation from these equations.

$$M^n = \frac{K^n}{k_m^2 \cdot 4 \cdot \pi^2} \tag{9}$$

At the beginning the initial kinetic energy, displacement and velocities are set to zero. It should be noted that the velocity varies linearly between time steps *n+1/2* and *n-1/2* and then $\dot{v}^{n+1/2} = -\dot{v}^{n-1/2}$. For the first iterative step, when *n = 0*, the residual forces are equal to the external applied loads. From these assumptions the resulting equation for the first velocities is:

$$\dot{v}_i^{n+1/2} = \frac{\Delta t}{2 \cdot M_i^n} \cdot R_i^n \tag{10}$$

Equation (10) is also used for the first velocities of the new iterations, when the kinetic peak is reached, and all the nodal velocities are set to zero.

Substituting the initial displacement ($v^0 = 0$) into eq. (7) for the nodal displacement in the first iterative step implies:

$$v_i^{n+1} = \Delta t \cdot \dot{v}_i^{n+1/2} \tag{11}$$

According to (Alamatian, 2012) and (Bel Hadj Ali, 2011), if the calculated displacements of the *n+1* iteration are utilized as the initial coordinates of the new analysis, the convergence may not be achieved. Therefore, the displacement of the starting point of the new analysis should be set to the time step *n-1/2*. The displacement can be formulated using the forward finite differences in half of the time step *n*, i.e., between *n-1/2* and *n*:

$$v_i^{n-1/2} = v_i^n - \frac{\Delta t}{2} \cdot \dot{v}_i^{n-1/2} \tag{12}$$

By derivation v_i^n from eq. (7) and $\dot{v}_i^{n-1/2}$ from eq. (6) and by substituting into eq. (12), for the initial displacement of the restarted analysis, eq. (13) is implied:

$$v_i^{n-1/2} = v_i^n - \frac{3}{2} \cdot \Delta t \cdot \dot{v}_i^{n+1/2} + \frac{1}{2 \cdot M_i^n} \cdot \Delta t^2 \cdot R_i^n \tag{13}$$

Fig. 2 summarizes the entire calculations for DRM. The figure shows the whole iterative process. The expression "Initial conditions" means that E_k, $\dot{v}^0$ and v^0 are equal to zero, and when *n = 0,* then $R^0=P^0$. At the beginning it is necessary to estimate the limit values e_R = 1.0E-6 and e_K = 1.0E-12 on which the decision process runs through. The calculations begin with the equations for the nodal masses, initial velocities and displacements. The first step of the iterative process is to calculate the out-of-balance force vector and subsequently check the convergence criterion. The iteration proceeds with updating the fictitious velocity and checking the second criterion. Through the

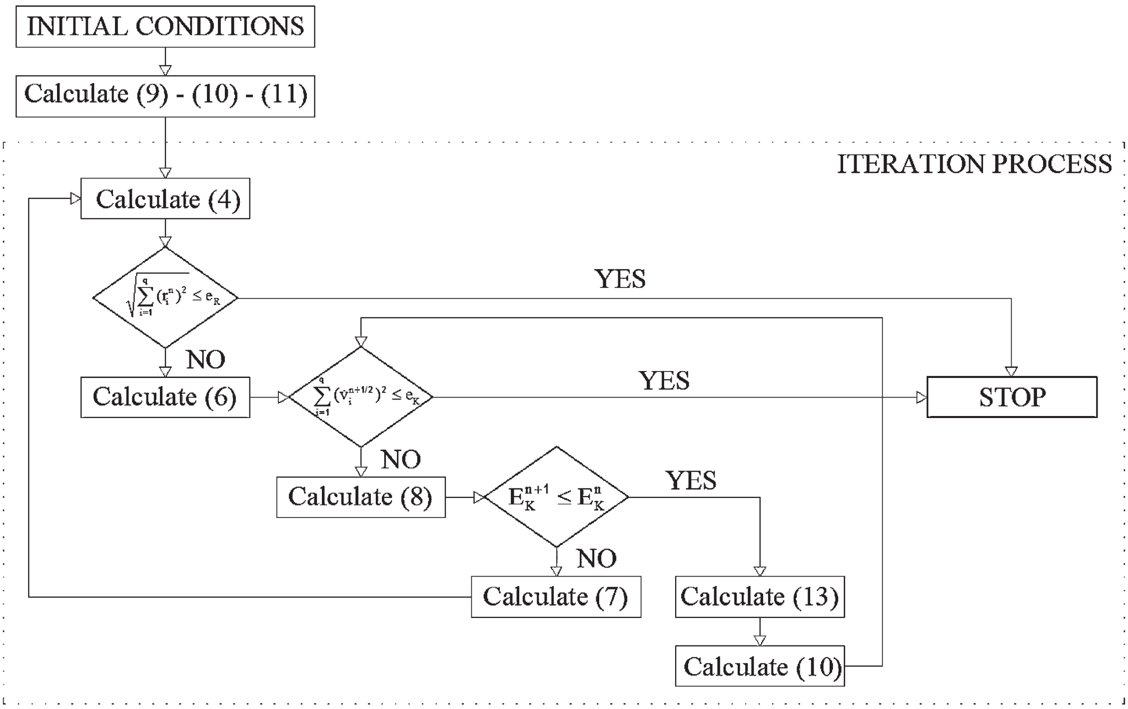

Fig. 2 *The DRM flowchart.*

estimation of the kinetic energy, the achievement of the energy peak is controlled. If it is not reached, the displacements are updated, and the iteration returns to the first step. This describes the main iteration process. When an energy peak occurs, the calculations continue with an estimation of the initial displacement of the restarted analysis, an estimation of the internal force vector from the extension of the element, and the estimation of the initial velocities of the restarted analysis. The iteration process ends when all the convergence conditions are fulfilled.

3 EFFECT OF THE INPUT VALUES ON THE FINAL SHAPE

The resulting point positions depend on multiple input parameters. The best way to show their effects is to change one of the parameters and keep all the others fixed. Out of the many components that can vary, several of the most important ones have been chosen. Namely, the results of the DRM optimization will be affected by the following parameters: the fictitious mass, Young's modulus of elasticity, ultimate stress, type of damping, grid pattern, cross-section, the load, and the support combinations. When choosing the material for grid shell structures, there are two important parameters that affect the resulting shape and internal forces. The first value is Young's modulus of elasticity. Its impact is described and analyzed in this paper in the section below. Another aspect is the value of the ultimate stress. Material with a higher ultimate stress can resist major bending moments, and the radius of the curvature will be minor.

3.1 ESTIMATION OF YOUNG'S MODULUS OF ELASTICITY

As already mentioned, in the DRM calculations, there are many values with a significant impact on a structure's displacement. One of these is E, the modulus of elasticity. The camber of a grid shell depends on this value. With a high value of E internal forces rise considerably too. This leads to the position of the equilibrium of a structure in more iterative steps due to the steady state based on a zero residual force. It is difficult to estimate the right value. The value of the modulus should be low enough in order to minimize the bending moments resulting from shaping the shell. Conversely, its value should be high enough in order to provide sufficient global buckling stiffness.

To achieve a better camber of a structure we have to determine the value of the modulus corresponding to our selected strain. That means we can use the secant modulus, which represents the slope of the line joining the origin and a selected point on the stress-strain curve; this could represent a vertical line at a 10% strain.

3.2 FORM-FINDING OF A GRID SHELL STRUCTURE

The final shape was obtained by software based on the DRM terms, with which it is possible to analyze single shapes with different ground plans, boundary conditions, and loading. It also allows tracing the process of form finding and seeing the effect of the input values on the calculations and on the final form of the grid shell.

For the numerical verification of the kinetic DRM and the effect of the modulus of elasticity, some analyses have been made. The shaping process started with an initially flat rectangular ground plan with dimensions of 10 m x 15 m. The boundary conditions were set at all four edges and considered to be pin joined. An external load with a value of 5kN was applied onto each free node in the negative z-direction. The mesh type was a hexagonal grid with steel profiles of an 80 mm diameter and a 4 mm wall thickness. All the necessary input values have been inserted in Tab. 1.

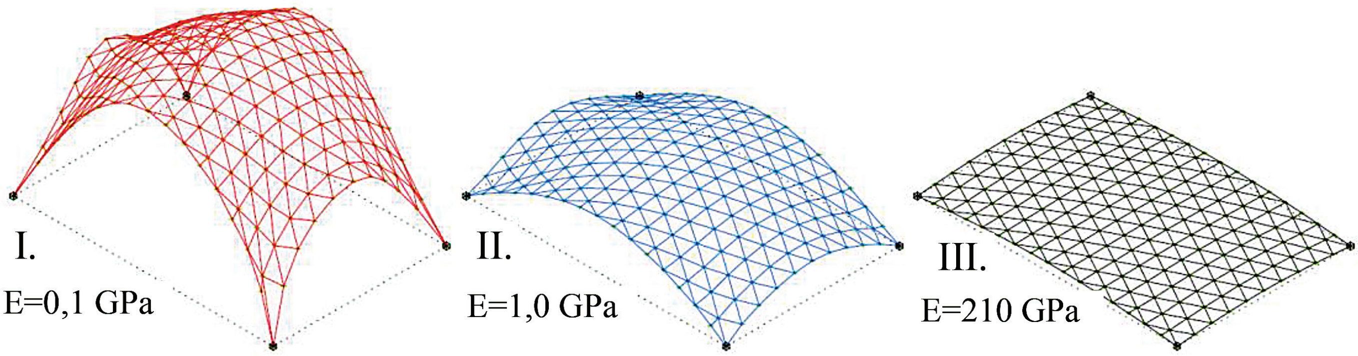

Fig. 3 *Resulting shapes of grid shells according to the value of Young's modulus.*

Tab. 1 *Material parameters and constants for analysis.*

Parameter		
Young's modulus	E	0.1 / 0.5 / 1.0 / 50 / 100 / 150 / 210 GPa
Poisson ratio	ν	0.3
Cross-sectional area	A	9.55E-4 m^2
External load	P	-5 kN
Mass factor	k_m	0.1

Fig. 3 shows the final geometries obtained from the form-finding process of DRM. The shapes of the grid shells are changing according to Young's modulus. An increase in the value of the modulus invokes an increase in the stiffness, thereby resulting in the reduction of the structure's curvature. The final shape is obtained after more iterative steps. According to equation (4), the stiffness forces are much higher than the external load, and the equilibrium of the node can occur after more iterative steps. Fig. 4 shows the process of the displacement of the node during the calculations with various elastic modules. The diagram shows the displacement of one node in the middle of the ground plan, with a maximal z-dir. displacement at the end of the calculations. The changes in the deformations and number of iterative steps are clear from the diagram. The diagram in Fig. 5 indicate the relation between Young's modulus and the displacement. It is obvious that the increasing value of the modulus causes a smaller displacement of the node and can change the overall shape of the structure.

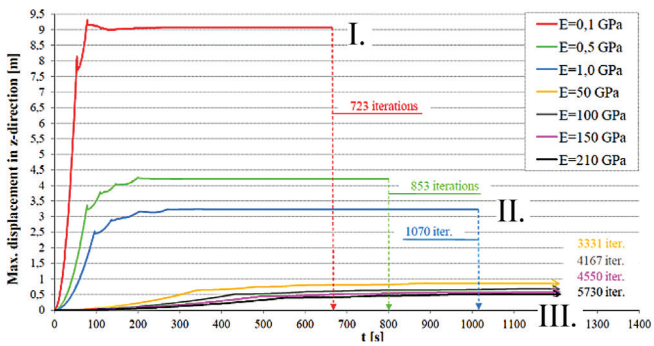

Fig. 4 *Change in the maximal z-dir. displacement according to a variation of Young's modulus.*

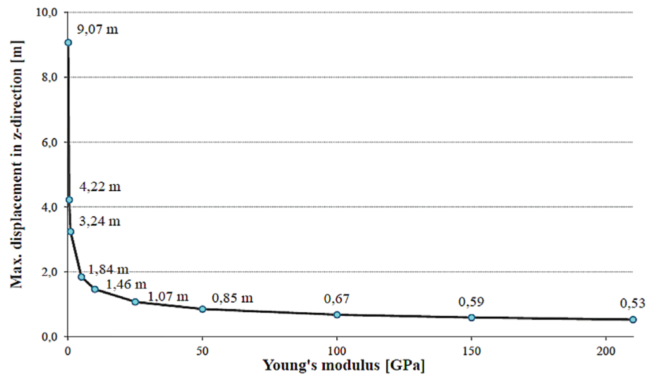

Fig. 5 *Young's modulus – Displacement diagram*

3.3 STRUCTURAL ANALYSIS BY FEM MODELING

The geometry from the DRM was loaded into the Scia Engineer FEM software. The grid profiles were modeled using beam elements with the same material and cross-sectional properties as in the DRM software. The rigid connections were modeled between the profiles. In the corners pinned boundary conditions were used. The calculations considered only one load case, i.e., an external load with a value of -5 kN. This load was applied onto each node of the structure in a negative z-dir. (the same as in the DRM calculation). The FEM analysis had one change compared to the DRM, i.e, the elastic modulus was considered with a value of E=210 GPa in all the cases. Under these conditions the calculations were carried out. When comparing the two calculations, the axial forces from FEM were approximately equal to the axial forces from DRM (Tab. 2). Differences occurred because in DRM the internal forces are calculated during the shaping process on a pre-deformed structure, and in FEM the internal forces are calculated on a deformed structure under the influence of an external load. Fig. 6 shows the normal stresses on grid shells. From this figure and Table II, it is clear that the curvature of the structure has a significant impact on the internal forces. When considering a material strength capacity equal to 235 MPa and a cross-section as in the DRM calculations, it can be seen that structure III is not suitable in terms of the ultimate stresses. The results were obtained from a static analysis; no dynamic aspects or stability was taken into account. Therefore, the optimal material and cross-section for a specific grid shell depends on the global stability and the material's ultimate strength.

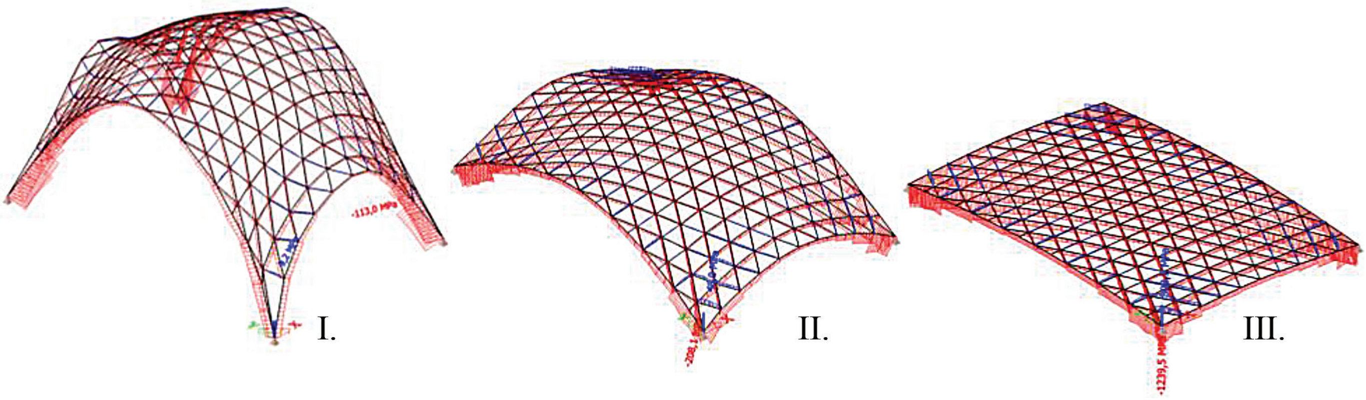

Fig. 6 *Normal stresses of single grid shells (I, II, III) under an external load.*

Tab. 2 *Values of the axial forces and normal stresses of the most stressed beam.*

Structures	Axial Force FEM [kN]	Axial Force DRM [kN]	Normal Stress [MPa]
I.	-93,1	-88,1	-113,0
II.	-191,7	-196,1	-208,1
III.	-1090,4	-1133,6	-1239,5

4 CONCLUSION

The final geometry of a grid shell structure results from the form-finding process, where the input values can significantly affect the node's displacements. There are many methods for finding an appropriate shape of a structure. This paper is focused on the dynamic relaxation method (DRM) and the influence of Young's modulus of elasticity on the radius of the curvature. Single shapes were first obtained and then calculated with different values of the modulus by software working according to the DRM equations. The result is that, by increasing the elastic module, the curvature is reduced, and the final shapes significantly changed. These geometries were loaded into the computing program and calculated by FEM. All the material and geometry inputs and loading were the same as in the DRM; only the value of the elastic modulus was considered with a value for the steel of 210 GPa. From the results introduced in this article, the influence of the curvature on the internal forces of a grid shell structure and the necessity of changing the elastic modulus during the form finding process of these structures by DRM is clear.

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