EQUIVALENT IMPERFECTIONS IN ARCHED STRUCTURES

Marian DALLEMULE^{1*}

Abstract

There are currently three design methods to verify the in-plane buckling of an arched structure: substitute member method, the method of equivalent imperfection with recommendations for arched bridges, and the equivalent unique global and local initial imperfection method (EUGLI), which uses the critical elastic buckling mode as an imperfection. The latter method is included in the EN 1993-1-1 cl. 5.3.2 (11) since 2002; however, to this day it is neither utilized in the design practice nor is it incorporated in ordinary structural analysis software. The main purpose of this article is to show the application of the proposed methods in a step-by-step manner to the numerical example considered and to compare these design methods for various arched structures. Verification of the in-plane buckling of an arch is explained in detail.

Address

¹ Ingsteel spol. s r. o., Tomášikova 17, P. O. Box 82, 820 09, Bratislava
Department of Steel and Timber Structures, Faculty of Civil Engineering, Slovak University of Technology in Bratislava, Radlinského 11, 813 68 Bratislava, Slovakia

* **Corresponding author:** dallemule@ingsteel.sk,
marian.dallemule@stuba.sk

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1 INTRODUCTION

Arched structures are often used as supporting structures for bridges or other long-span structures. A well-designed arch should be purely under compression, but in the real world, it is also bent due to the nature of a live load or various imperfections. From a design point of view, arches are complicated and interesting and are usually a part of important civil structures. Verification of an in-plane buckling of an arched structure (Fig. 1) is provided, using three different design methods according to the Eurocodes:

- 1) The substitute member method (*SM*) according to (STN EN 1993-1-1)
- 2) Equivalent imperfections for arched bridges (*TAB. D.8*) according to STN EN 1993-2 Annex D, D.3, Table D.8
- 3) The Equivalent Unique Global and Local Initial imperfection method (*EUGLI*) according to (STN EN 1993-1-1) cl. 5.3.2 (11)

A comparative parametric study was carried out for hingeless (fixed), 3-hinge and 2-hinge arches.

2 THE NUMERICAL EXAMPLE CONSIDERED

Static and buckling analyses were carried out using the 1st order theory for the *SM* method and the 2nd order theory for *TAB. D.8* and the *EUGLI* method. The 2nd order analysis used in this article is not in fact a *large deflection analysis* – the so-called Newton-Raphson method (in terms of numerical analysis) – but a simplified approach, which only takes into account only the contribution of axial forces on the displacements. The difference in using this simplified approach and a large deflection analysis is negligible for ordinary frame structures. The graphic results for both theories are similar, almost identical in shape, and can be seen in Fig. 2. For the numerical results, see Table 1.

Arch:	$L = 40\text{ m}$; $f = 8\text{ m}$; $L_{\text{real}} = 43.93\text{ m}$; shape = parabola
Load:	$g_{\text{Ed},1} = 48\text{ kN/m}$; $g_{\text{Ed},2} = 36\text{ kN/m}$
Material:	$f_y = 275\text{ MPa}$; $g_{\text{M1}} = 1.1$; $E = 210\text{ GPa}$
Cross-section:	$A = 0.03\text{ m}^2$; $I_y = 1.2186 \cdot 10^{-3}\text{ m}^4$; $W_{\text{el},y} = 5.0775 \cdot 10^{-3}\text{ m}^3$; $a_y = 0.49$ CS = class 3; CS is not susceptible to lateral-torsional buckling

Fig. 1 Static scheme of the example given.

The arch was divided into 80 elements with 81 nodes for the FEM analysis. The elements were numbered 1~80 from left to right. The nodes were numbered as follows: the leftmost node is No. 1; the topmost node (at the crown) is No. 2; and the rightmost node is No. 3. The nodes on the left side are numbered from left to right as 4~42 and on the right side as 43~81. The uniform loading was replaced with nodal loads in a vertical direction (plane):

$$Q_{Ed,1} = 12 \text{ kN}; Q_{Ed,4-42} = 24 \text{ kN}; Q_{Ed,2} = 21 \text{ kN}; \\ Q_{Ed,43-81} = 18 \text{ kN}; Q_{Ed,3} = 9 \text{ kN}.$$

Note: Based on the buckling analysis the 1st buckling mode $h_{cr}(x)$ is obtained. The buckling mode is usually normalized through one of its displacement components, either u (the horizontal component), v (the vertical component) or η (the nodal rotation), whichever is greatest. For the EUGLI method purposes, the buckling mode is normalized based on the maximum absolute vector size $|\eta_{cr,max}|$. This is achieved in the example at node No. 22: $|\eta_{cr,22}| = 1.000$, where $\eta_{cr,22,u} = 0.554$ and $\eta_{cr,22,v} = -0.833$.

3 VERIFICATION

3.1 Substitute Member Method

The *substitute member method* is the most used method for verifying compressed members. It was derived for members with a uniform axial force and cross-section throughout their lengths. This condition is not satisfied in the example, but the values of the axial

Tab. 1 Short list of the results.

Quantity	1 st order analysis		2 nd order analysis	
	value	node / elem*	value	node / elem*
$R_{H,L}$ kN	1048.69	1	1050.20	1
$R_{H,R}$ kN	1048.69	3	1050.20	3
$R_{V,L}$ kN	900.00	1	898.51	1
$R_{V,R}$ kN	780.00	3	781.49	3
$N_{Ed,max}$ kN	-1373.35	1*	-1373.62	1*
$V_{Ed,max}$ kN	-58.51	41*	-71.13	41*
$M_{Ed,max}$ kN.m	307.88	23	397.25	23
def_{max} mm	69.367	23	88.064	23
η_{cr} -	4.3936	-	4.3886	-

* According to the FEM analysis, the axial and shear forces are uniform along the element, the number for which represents the element's ordinal number. The other results are unique for each node, such as the reactions, bending moments, displacements, and buckling modes (displacements), in which case the number represent the node's ordinal number. These numbers are subsequently written as subscripts. To simplify the verification further, the values of the axial force in the adjacent elements could be averaged – as if they were represented at a node. No simplification was performed in these calculations; thus for the verification, it has to be distinguished for each element as to which node is verified. This is irrelevant for the SM method.

force change very slightly over the member's length; therefore, this method can be used for a comparison with a relatively good degree of accuracy.

The effect of any imperfection is hidden in the reduction factor χ through the imperfection factor, which in this case is $\alpha_y = 0.49$ (buckling curve 'c' for the given cross-section). All the internal forces are calculated using the 1st order analysis. The $N_{Ed,max}$ and $M_{Ed,max}$ are taken into account with the proper interaction factor k_{yy} , because they are not in the same position. Using α_{cr} and $N_{Ed,max}$, the quantities $\lambda_{rel} \chi$ and $N_{b,Rd}$ (the design buckling resistance of a member) could be determined.

Fig. 2 Results of the static and buckling analyses ($N_{max} = N_{Ed,max}$; $V_{max} = V_{Ed,max}$; $M_{max} = M_{Ed,max}$).

$$\lambda_{rel} = \sqrt{\frac{A \cdot f_y}{\alpha_{cr} \cdot N_{Ed,max}}} = \sqrt{\frac{0.03 \cdot 275000}{4.3936 \cdot 1373.35}} = 1.169$$

$$\phi = 0.5 \cdot \left(1 + \alpha_y \cdot (\lambda_{rel} - 0.2) + \lambda_{rel}^2 \right) = 0.5 \cdot \left(1 + 0.49 \cdot (1.169 - 0.2) + 1.169^2 \right) = 1.421$$

$$\chi = 1 / \left(\phi + \sqrt{\phi^2 - \lambda_{rel}^2} \right) = 1 / \left(1.421 + \sqrt{1.421^2 - 1.169^2} \right) = 0.449$$

$$N_{b,Rd} = \chi \cdot A \cdot \frac{f_y}{\gamma_{M1}} = 0.449 \cdot 0.03 \cdot \frac{275000}{1.1} = 3365.12 \text{ kN}$$

$$k_{yy} = C_{my} \cdot \left(1 + 0.6 \cdot \lambda_{rel} \cdot \frac{N_{Ed,max}}{N_{b,Rd}} \right) = 0.9 \cdot \left(1 + 0.6 \cdot 1.169 \cdot \frac{1373.35}{3365.12} \right) = 1.1577$$

$$U = \frac{N_{Ed,max}}{N_{b,Rd}} + k_{yy} \cdot \frac{M_{Ed,max}}{M_{Rd}} = \frac{1373.35}{3365.12} + 1.1577 \cdot \frac{307.88}{1269.38} = 0.6889 < 1$$

3.2 Equivalent Imperfections For Arched Bridges

An equivalent imperfection for an arched structure could be obtained according to STN EN 1993-2, annex D, chapter D.3 *Arched bridges*, Table 8. *Shape and amplitudes of imperfections for in-plane buckling of arches*. When dealing with imperfections, internal forces have to be calculated using the 2nd order analysis. The equivalent imperfection has the form of two asymmetric half-waves of the sine function – see Table 2. The amplitude for this type of imperfection depends on the buckling curve assigned for the given cross-section. In our case (buckling curve “c”), $e_0 = \pm L/400 = \pm 40/400 = \pm 0.1 \text{ m}$.

Tab. 2 Table D.8, taken from STN EN 1993-2 Annex D.

	shape of imperfection (sinus or parabola)	e ₀ according to classification of cross section to buckling curve			
		a	b	c	d
		$\frac{l}{600}$	$\frac{l}{500}$	$\frac{l}{400}$	$\frac{l}{300}$

The imperfection could be (for the sake of the simplification) replaced by a uniform asymmetric load along the arch – see (1).

$$q_{eq} = \pm \frac{8 \cdot R_H \cdot e_0}{(0.5 \cdot L)^2} = \pm \frac{8 \cdot 1050 \cdot 20 \cdot 0.1}{(0.5 \cdot 40)^2} = 2.1 \text{ kN/m} \quad (1)$$

Taking into account the imperfection, the equivalent design uniform load is:

$$g_{Ed,1,eq} = g_{Ed,1} + q_{eq} = 48 + 2.1 = 50.1 \text{ kN/m}$$

$$g_{Ed,2,eq} = g_{Ed,2} - q_{eq} = 36 - 2.1 = 33.9 \text{ kN/m}$$

Note: The difference in internal forces between using the equivalent load and the original load acting on an imperfect structure is negligible.

The maximum stress has been achieved at element 19, node No. 22, where:

$$U = \frac{N_{Ed,19}}{N_{Rd}} + \frac{M_{Ed,22}}{M_{Rd}} = \frac{1144.97}{7500} + \frac{533.37}{1269.375} = 0.5728 < 1$$

3.3 The Eugli Imperfection Method

The *Equivalent Unique Global and Local Initial Imperfection* (sometimes referred as the “EUGLI imperfection”) is a relatively new design method that has not been well utilized in design practice. It can be found in (STN EN 1993-1-1) clause 5.3.2 (11). This method will be described below.

The main idea is that the shape of the imperfection has the form of the critical elastic buckling mode. The SM method is also based on the same idea, where the imperfection for a simply supported member with a uniform cross-section and axial force all over the length of the member has the shape of a half-wave of the sine function. Chladný (2013) generalized this procedure for frame structures. This method is also designated for the verification of members with a non-uniform cross-section and/or non-uniform axial compressive force Kováč (2012). In recent times, it was successfully used in the design of two similar arched bridges, i.e., the *Apollo bridge* (Slovakia 2005) and the *Pentele bridge* (Hungary 2007).

The advantage is that the equivalent imperfection includes both global and local aspects for the verification of stability. This shape is considered to be the most effective shape for the loss of stability in general. Other methods according to (STN EN 1993-1-1) 5.3.2 separate local and global imperfections in terms of a *local bow imperfection* and the *initial sway of columns* (for frame structures). Almost all of the structural analysis software today provides stability calculations, which results in α_{cr} and the shape of the critical elastic buckling mode $\eta_{cr}(x)$. Both of these results are necessary not only for this method to be used but also for the 2nd derivation of $\eta_{cr}(x)$, which, however, most of the software does not provide, but that is just a matter of a numerical evaluation. The problem with this verification approach is that there are no experimentally established and statistically evaluated amplitudes for these types of imperfections. Considering the complexity of frame structures, it is probably not possible. Because no amplitudes have been established, it is possible to use the ones determined by the **Basic requirement**, i.e., *The buckling resistance of a frame structure with axially loaded members shall be equal to the flexural buckling resistance of the substitution member* (Chladný, 2013). This premise is the main contribution of Chladný’s paper.

The imperfection shape $\eta_{init,m}(x)$ shall be affine to the 1st buckling mode $\eta_{cr}(x)$. Its maximum amplitude $\eta_{init,m,max}$ (m is the critical cross-section) is based on $e_{0,d}$ (the design value of the initial bow imperfection) and $M_{\eta_{cr},m}$ (the bending moments due to the shape of $\eta_{init,m}(x)$); see an illustrative example in Fig. 3.

Fig. 3 An illustrative example. Note the ‘hidden’ location of $e_{0,d,m}$.

The results of the *Basic requirement* are that if the utilization factor of the member is less than 1.0 according to the *SM* method, then, according to the *EUGLI* method, the utilization factor for the same member is even lower. In the event the utilization factor of the member is equal to 1.0 according to the *SM* method, then it is also equal to 1.0 according to the *EUGLI* method. This is caused by the behaviour of the 2nd order analysis. The equivalent initial imperfection $\eta_{init,m}(x)$ is determined by multiplying the shape $\eta_{cr}(x)$ with the maximum amplitude $\eta_{init,m,max}$. It covers both the global and local imperfections for the entire structure. From the imperfect structure analysis, the addition of bending moments $M_{\eta_{cr}}(x)$ (as an influence of the axial force acting on an imperfect structure) can be clearly separated from the bending moments due to the external load. Note that $e_{0,d,m}$ is at the point where the curvature of $\eta_{init,m}(x)$ is at its maximum, and it can be graphically interpreted, as can be seen in Fig. 3. The cross-section verification, instead of the member verification, can be carried out. All the internal forces should be calculated using the 2nd order analysis. This method is iterative in general, because the critical cross-section x_m has to be found. It is the point where the interaction of $N_{Ed,m}$ and $M_{Ed,m} + M_{\eta_{init,m,m}}$ has the overall maximum effect. There are four combinations in the relation between the uniform or non-uniform axial force and the uniform or non-uniform cross-section. Only in the case where both the cross-section and axial force are uniform is the calculation not iterative; the position of the critical cross-section x_m could be found directly.

The reader can find more information and examples developed in Baláz (2008), Baláz (2009), Baláz and Koleková (2010) and Baláz et al. (2010). The basic formulae with the description for the *EUGLI* method (taken from the references) are listed below:

Characteristic value of the equivalent initial bow imperfection:

$$e_{0,k,m} = \alpha \cdot (\bar{\lambda}_m - 0.2) \cdot \frac{M_{Rk,m}}{N_{Rk,m}} \quad (2)$$

Design value of the equivalent initial bow imperfection:

$$e_{0,d,m} = e_{0,k,m} \cdot \delta_e \quad (3)$$

where:

$$\delta_e = \frac{1 - \chi \cdot \bar{\lambda}_m^2 / \gamma_{M1}}{1 - \chi \cdot \bar{\lambda}_m^2} \quad (4)$$

The bending moment in the critical cross-section m due to the $\eta_{cr}(x)$ is:

$$|M_{\eta_{cr}}|_m = E \cdot I_m \cdot |\eta_{cr}''(x)|_m \quad (5)$$

The design value of the maximum amplitude for the *EUGLI* imperfection $\eta_{init,m}(x)$:

$$\eta_{init,m,max} = \frac{\alpha_{cr} \cdot e_{0,d,m} \cdot N_{Ed,m}}{|M_{\eta_{cr}}|_m} \cdot |\eta_{cr}|_{max} \quad (6)$$

Tab. 3 Results.

elem	node	N_{Ed}	M_{Ed}	$M_{\eta_{cr}}$	U_{N+M}	λ_{rel}	χ	δ_e	e_{0d}	$\eta_{init,m,max}$
17	19	1164.21	384.08	4.805	0.4578	1.271	0.401	1.167	0.104	32.528
	20		390.44	4.836	0.4628	1.271	0.401	1.167	0.104	32.320
18	20	1154.07	390.44	4.836	0.4615	1.276	0.399	1.169	0.104	32.241
	21		394.75	4.837	0.4649	1.276	0.399	1.169	0.104	32.234
19	21	1144.33	394.75	4.837	0.4636	1.282	0.397	1.170	0.105	32.157
	22		397.01	4.810	0.4653	1.282	0.397	1.170	0.105	32.343
20	22	1135.02	397.01	4.810	0.4641	1.287	0.394	1.171	0.106	32.268
	23		397.25	4.754	0.4643	1.287	0.394	1.171	0.106	32.643

The shape of the $\eta_{init,m}(x)$ imperfection:

$$\eta_{init,m}(x) = \eta_{init,m,max} \cdot \frac{\eta_{cr}(x)}{|\eta_{cr}|_{max}} \quad (7)$$

Bending moments on the imperfect structure due to the action of axial force N_{Ed} :

$$M_{\eta_{init,m}}(x) = -E \cdot I(x) \cdot \frac{\eta_{init,m,max}}{\alpha_{cr} - 1} \cdot \frac{\eta_{cr}''(x)}{|\eta_{cr}|_{max}} \quad (8)$$

If the critical point m could be located directly, (8) could be simplified into (9):

$$M_{\eta_{init,m,m}} = k \cdot e_{0,d,m} \cdot N_{Ed,m} \quad (9)$$

where:

$$k = \frac{1}{1 - 1/\alpha_{cr}} \quad (10)$$

$|\eta_{cr}|_{max}$ is the maximum amplitude of the critical buckling mode, which usually equals 1.0 if the buckling mode is normalized; $\bar{\lambda}_m$ is the relative slenderness for the critical cross-section m ; α is the imperfection factor for the relevant buckling curve; and α_{cr} is the minimum force amplifier for the axial force configuration N_{Ed} in the members to reach the critical elastic force.

The verification according to the *EUGLI* method could be divided into five steps:

1) After carrying out the static and buckling analyses, the curvature of the buckling mode $\eta_{cr}''(x)$ should be calculated. These calculations can be done 'by hand' or by using software, e.g. Rubin et al. (2004) (the author does not know of any other software that can provide these calculations). 'By hand' means to use one of the methods of numerical differentiation, e.g. (20). To obtain the curvature of $\eta_{cr}(x)$ either the 2nd derivation of the displacements or the 1st derivation of the rotations has to be performed. Using the displacements in the case of arches is questionable due to the nature of the node displacement. A node displacement of a frame structure (in general) always has two components (vertical and horizontal) versus a single member – where this question would never arise. The rotation of the nodes, which is already the 1st derivation of $\eta_{cr}(x)$, is known from the buckling analysis too. To obtain the required $\eta_{cr}''(x)$, a differentiation of these rotations is needed. This method is considered to be less accurate in general as opposed to the 2nd differentiation of the displacements, but it seems to be less questionable too. It is advisable to use the second approach, i.e., differentiating the rotations.

$$y_i' \approx \frac{y_{i+1} - y_{i-1}}{2 \cdot h} \quad (11)$$

y_i' is in fact the $\eta_{cr}''(x)$; y is the node rotation; h is the distance between two adjacent nodes.

Note: To preserve accuracy, all the distances (length of the elements) should be equal.

2) The cross-section at each element has a different axial force for which it is necessary to calculate quantities λ_{rel} and $e_{0,d}$. Because the $M_{\eta_{cr}}$ is different for each node, $\eta_{init,m,max}$ has to be calculated for both ending nodes of an element – see the results in Table 3.

For the sake of clarity, a representative calculation for element 19, node 22, is provided below:

$$M_{\eta_{cr},22} = E \cdot I \cdot \left| \eta_{cr}''(x_{22}) \right| = 210 \cdot 10^6 \cdot 1.2186 \cdot 10^{-3} \cdot 0.018794 \cdot 10^{-3} = 4.810 \text{ kN.m}$$

$$k = \frac{1}{1 - 1/\alpha_{cr}} = \frac{1}{1 - 1/4.3886} = 1.2951$$

$$U_{N19+M22} = \frac{N_{Ed}}{N_{Rd}} + \frac{M_{Ed}}{M_{Rd}} = \frac{N_{Ed,19}}{A \cdot f_y / \gamma_{M1}} + \frac{M_{Ed,22}}{W_{el,y} \cdot f_y / \gamma_{M1}}$$

$$U_{N19+M22} = \frac{1144.33}{0.03 \cdot 275 \cdot 10^3 / 1.1} + \frac{397.01}{5.0775 \cdot 10^{-3} \cdot 275 \cdot 10^3 / 1.1} = 0.1526 + 0.3127 = 0.4653$$

$$\lambda_{rel,19} = \sqrt{N_{Rk} / N_{cr,19}} = \sqrt{\frac{A \cdot f_y}{\alpha_{cr} \cdot N_{Ed,19}}} = \sqrt{\frac{0.03 \cdot 275 \cdot 10^3}{4.3886 \cdot 1144.33}} = 1.282$$

$$\phi_{19} = 0.5 \cdot (1 + \alpha \cdot (\lambda_{rel,19} - 0.2) + \lambda_{rel,19}^2) = 0.5 \cdot (1 + 0.49 \cdot (1.282 - 0.2) + 1.282^2) = 1.586$$

$$\chi_{19} = 1 / \left(\phi_{19} + \sqrt{\phi_{19}^2 - \lambda_{rel,19}^2} \right) = 1 / \left(1.586 + \sqrt{1.586^2 - 1.282^2} \right) = 0.397$$

$$e_{0,d,19} = e_{0,k} \cdot \delta_{e,19} = \alpha \cdot (\lambda_{rel,19} - 0.2) \cdot \frac{M_{Rk}}{N_{Rk}} \cdot \frac{1 - \chi_{19} \cdot \lambda_{rel,19}^2 / \gamma_{M1}}{1 - \chi_{19} \cdot \lambda_{rel,19}^2}$$

$$e_{0,d,19} = 0.49 \cdot (1.282 - 0.2) \cdot \frac{5.0775 \cdot 10^{-3} \cdot 275 \cdot 10^3}{0.03 \cdot 275 \cdot 10^3} \cdot \frac{1 - 0.397 \cdot 1.282^2 / 1.1}{1 - 0.397 \cdot 1.282^2} = 0.105$$

$$\eta_{init,m,19,22} = \frac{k \cdot e_{0,d,19} \cdot N_{Ed,19}}{M_{\eta_{cr},22}} = \frac{1.2951 \cdot 0.105 \cdot 1144.33}{4.810} = 32.343$$

3) If the results are prepared, the critical cross-section m for the first iteration could be estimated. Because the shape of the axial forces is changing very slightly over the length and because both maximum bending moments, i.e., $M_{Ed,max}$ and $M_{\eta_{cr,max}}$, are located

approximately at the first quarter of the span, the location of point m is also expected at this location, i.e., somewhere between nodes 19–23. But it is generally best to follow the stresses or the utilization factors U (as in this case) and to choose the point with the maximum stress due to the $N_{Ed} + M_{Ed}$. Therefore, the element 19, node 22, has been chosen for the 1st iteration step, $U_{N19+M22} = 0.4653$, which is the maximum value of U_{N+M} .

4) The 1st iteration: Based on the value $\eta_{init,m,19,22}$, $M_{\eta_{init,m}}$ is calculated for the surrounding nodes. A point with the highest utilization factor (or stress) has to be found; see Table 4.

There is also another approach that could be used to find this ultimate point. Using formula (7) the exact form of the imperfection for the given $\eta_{init,m}$ is obtained. Executing the 2nd order static analysis on a specific imperfect structure will result in a verification based only on the stresses. The disadvantage is that the bending moments M_{Ed} and additional $M_{\eta_{init}}$ cannot be separated in the verification. This approach is not used in this article.

Element 18, node 21, has the highest U , for which the calculations are as follows:

$$M_{\eta_{init},18,21} = \eta_{init,m,19,22} \cdot M_{\eta_{cr},21} = 32.343 \cdot 4.837 = 156.45 \text{ kN.m}$$

$$U = \frac{N_{Ed,18}}{N_{Rd}} + \frac{M_{Ed,21} + M_{\eta_{init},18,21}}{M_{Rd}} = \frac{1154.07}{7500} + \frac{394.75 + 156.45}{1269.375} = 0.5881$$

5) Next iteration: Because the cross-section with the highest U (element 18, node 21) is not the same as the one initially expected (element 19, node 22), step 4 has to be repeated until they are the same. Now, based on the new value $\eta_{init,m,18,21}$, the $M_{\eta_{init,m}}$ is calculated for the surrounding nodes. For the 2nd iteration results, see Table 5.

Element 18, node 21, has the highest U , for which the calculations are as follows:

$$M_{\eta_{init},18,21} = \eta_{init,m,18,21} \cdot M_{\eta_{cr},18} = 32.234 \cdot 4.837 = 155.92 \text{ kN.m}$$

$$U = \frac{N_{Ed,18}}{N_{Rd}} + \frac{M_{Ed,21} + M_{\eta_{init},18,21}}{M_{Rd}} = \frac{1154.07}{7500} + \frac{394.75 + 155.92}{1269.375} = 0.5877 < 1$$

The node with the highest utilization factor is the same as expected at the end of the previous iteration step. This verification converged quickly, i.e., after just two iterations, because in this case, the axial force is almost uniform; it changes very slightly along the structure. The critical point m is usually ‘moved’ from the point of the maximum U_M to the point of the maximum U_N . The $M_{\eta_{init}}$ and the final verification can be interpreted graphically, as seen in Fig. 4.

Tab. 4 Results for the 1st iteration.

1 st iteration: $\eta_{init,m,max} = 32.343$				
elem	node	$\eta_{init,m,max}$	$M_{\eta_{init}}$	U
17	19	32.528	155.42	0.5802
	20	32.320	156.42	0.5860
18	20	32.241	156.42	0.5847
	21	32.234	156.45	0.5881
19	21	32.157	156.45	0.5868
	22	32.343	155.55	0.5879
20	22	32.268	155.55	0.5866
	23	32.643	153.76	0.5854

Tab. 5 Results for the 2nd iteration.

2 nd iteration: $\eta_{init,m,max} = 32.234$				
elem	node	$\eta_{init,m,max}$	$M_{\eta_{init}}$	U
17	19	32.528	154.89	0.5798
	20	32.320	155.89	0.5856
18	20	32.241	155.89	0.5843
	21	32.234	155.92	0.5877
19	21	32.157	155.92	0.5864
	22	32.343	155.03	0.5875
20	22	32.268	155.03	0.5862
	23	32.643	153.25	0.5850

Bending moments due to the EUGLI imperfection

Utilization factors (EUGLI method verification)

Fig. 4 The final verification.

4 DISCUSSION AND CONCLUSION

The comparison of these three design methods according to the Eurocodes has been carried out for the spectrum of $f/L = 0.1 \sim 0.5$ for various arches: hingeless (Fig. 5), 2-hinge (Fig. 6) and 3-hinge (Fig. 7).

Comparing Figs. 5~7, the best ‘match’ in the verification using the three different design methods is for the hingeless arches. The differences are almost negligible. For the 2-hinge and 3-hinge arches, the *substitute member method* is about 10-20% on the ‘safe’ side compared to the equivalent imperfection methods. The impossibility of determining the ultimate cross-section position (point m) in a structure is a drawback of the *SM* method. For the 3-hinge arches, there is a really good ‘match’ in the verification between the *Tab. D.8* method and the *EUGLI* method, which is caused by the fact that there are no bending moments due to the external load, because of the nature of the load relative to the static scheme and the shape of the arches. It has to be noted that none of these methods could be used for the verification of flat arches, i.e., a snap-through of arches. For considering such a phenomenon, there is a condition in [2], annex D, statement (D.5) and Tab. D.5. No snap-through analysis is needed for the given example, not even for arches with $f/L = 0.1$.

Among these three design methods, the *EUGLI* method seems to be the most complicated, but it describes the buckling behavior in the best way. It also has the most efficient shape for the loss of stability, compared to the other equivalent imperfection methods. The iterative verification process would be exactly the same for members with a non-uniform cross-section as described in section 2.3. The bending and axial stiffness in this case should be different but uniform for each element. The procedure is best suited for use with structural software, where the buckling analysis is provided. Using this method, the addition of the bending moments due to the imperfection can be clearly separated in the final verification. In some very simple cases, where the position of the critical cross-section m could be directly determined, the effect of the 2nd order analysis on the imperfect structure could be calculated by just using the simplified formula (9). As there are no experimentally established amplitudes for the *EUGLI* imperfections, the proposed procedure for determining the $e_{0,d,m}$, which is based on the *substitute member method*, has to be used. This step might be confusing for EC users, who may be unfamiliar with the background of the *EUGLI* method.

Hingeless arch

2-hinge arch

3-hinge arch

Fig. 5-7 Results of the parametric study.

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