THEORETICAL EXAMINATION OF THE LONGITUDINAL BEHAVIOUR OF EMBEDDED RAILS

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Abstract

The longitudinal behaviour of embedded rails significantly differs from that of ballasted or slab tracks with direct rail fastenings. As the ambient temperature rises and falls, rails expand and contract, thereby imposing stress on the channel and embedding compound. In the article the author presents the creation of a method that makes the calculation and the inspection of embedded rails simpler and closer to the everyday engineering mentality. The design process includes an analysis of the displacement and normal forces along a rail axis.

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- Embedded rail,
- longitudinal behaviour,
- breathing length of ERS.

1 INTRODUCTION

The dilatational behaviour of embedded rails significantly differs from that of ballasted or slab tracks with direct rail fastenings (Ludvig, 2002). Fig. 1 shows an embedded rail. While the initial flexible resistance is typical of the order of a ~ 10 mm displacement, in the second and third cases, after a relatively small displacement of (≤ 2 mm), the linear flexible period is ideally followed by a plastic period (Nemesdy, 1966). The flexible longitudinal resistance of embedded rails has been investigated by Kulcsár and Major (2014) and the properties of railway ballast by Horvát and Fischer (2011a,b) and Fischer (2011).

Fig. 1 Embedded rail structure.

This latter behaviour can be modelled with software that enables non-linear calculations (providing spring stiffness and limit force, see Tab. 1); and in practical calculations the flexible period can be ignored, and the plastic resistance of the ballast can be applied using well-known formulas (Lichtberger, 2005; Führer, 1978).

In the second graph there are force-displacement diagrams that are valid for ballasted tracks, slab tracks with direct rail fastenings according to EN 1991-2 and Calçada et al. (2009), and for embedded rail structures with a $30 \text{ kN} \cdot \text{mm}^{-1} \cdot \text{m}^{-1} / 2$ rail spring constant.

Fig. 2 Value of the shear resistance in the case of ballasted tracks, slab tracks with direct rail fastenings (Calçada et al., 2009), and embedded rail structures with $30,000 \text{ kN} \cdot \text{m}^{-2} / 2$ rails spring constant.

Note: Figure 2 above applies to 2 rails!

Tab. 1 Features of bilinear spring characteristics (Major, 2012; Major, 2013).

Sign	Name	Spring constant [kN.m ⁻²]	Limit force [kN.m ⁻¹]
A	Rail fastening on loaded track	120,000	60
B	Ballast resistance on loaded track	30,000	60
C	Rail fastening on unloaded track	80,000	40
D	Ballast resistance on unloaded track	10,000	20

The two different kinds of dilatational behaviour (flexible and plastic) show a significant difference when the graphs of the dilatational forces and rail end movements are compared. The author calculated the behaviour of a 60E1 rail in the case of a 45 °C temperature change and applied a 10 [kN/m⁻¹/rail] value as plastic shear resistance. The author described the longitudinal stiffness of a rail embedding with a value of 5,000 [kN.m⁻²/rail]. When the author established the length of the breathing period, a 0.01 mm displacement was set as a limit condition and not complete stillness (Kormos, 2002). The elasticity modulus of the rail is 206,000,000 [kN.m⁻²]; the value of the linear heat expansion factor is $1.2 \cdot 10^{-5}$ [°C⁻¹].

Figs. 3-6 below describe the changes in the dilatational forces and the displacement evolving along the length of the rail.

2 ANALYTICAL DESCRIPTION OF LONGITUDINAL BEHAVIOUR

A method for the analytical description of longitudinal behaviour can be found in the book “Modern Railway Track” (2d d.) by Coenraad Esveld (2001).

Linear shear resistance is described by the following formula:

$$\tau = ku, \quad (1)$$

where:

- τ : flexible shear/sticking resistance [kN.m⁻¹],
- k : longitudinal spring constant [kN.m⁻²],
- u : displacement [m].

The differential equation describing the analysed problem is given by the following formula:

$$\frac{d^2u}{dx^2} - \frac{k}{EA}u = 0, \quad (2)$$

where:

- x : distance from the moving rail end [m];
- E : modulus of elasticity of the rail material [kN.m⁻²];
- A : cross sectional area of the rail [m²].

After solving the differential equation, we get the following equation for the displacement along the length of the rail (leaving out the deduction):

$$u(x) = \pm \frac{\alpha \Delta T}{\mu} e^{-\mu x}, \quad (3)$$

$$\mu = \sqrt{\frac{k}{EA}}, \quad (4)$$

where:

- α : linear expansion coefficient of the rail material [°C⁻¹],
- ΔT : temperature change [°C].

Fig. 3 The changes in the dilatational force along the length of the rail in the case of flexible shear resistance ($k=5,000$ kN.m⁻², “breathing length”=123.500 m).

Fig. 4 The changes in the dilatational force along the length of the rail in the case of plastic shear resistance ($p=10$ kN.m⁻¹, “breathing length”=86.978 m).

Fig. 5 The displacement of the rail along the length of the rail in the case of flexible shear resistance ($k=5,000$ kN.m⁻², “breathing length”=123.500 m).

Fig. 6 The displacement of the rail along the length of the rail in the case of plastic shear resistance ($p=10$ kN.m⁻¹, “breathing length”=86.978 m).

The formula valid for a change in the normal forces:

$$N(x) = \pm EA\alpha\Delta T(1 - e^{-\mu x}) \quad (5)$$

In engineering practice the analysis of two factors is essential. The first one is the size of the complete moving length of the rail (if there is a rail fracture, the breathing length is examined); the other one is the extent of the displacement of the moving rail end ($u_{\max}$).

To determine the breathing length in a closed form is impossible based on the above equation; the value of the maximum displacement can be determined from the equation below:

$$u_{\max} = u(0) = \pm \frac{\alpha\Delta T}{\mu} = \pm \frac{\alpha\Delta T}{\sqrt{\frac{k}{EA}}} = \pm \alpha\Delta T \sqrt{\frac{EA}{k}} \quad (6)$$

The author has created an Excel program based on the above formula, which can calculate and present the changes in the normal force and displacements along the rail. The rail's complete moving length can be calculated so long as the rail displacement reaches the limit value of 0.01 mm.

For the purposes of validation, the author compared the results of the Excel program to the results of a Finite Element Model (FEM) calculation. The material characteristics, spring constant, and cross sectional data were as seen in Tab. 2. The results are summarized in Tab. 3.

Tab. 2 Input parameters of the model examined.

E=	206,000,000	[kN.m ⁻²]
A=	0.007247	[m ²]
α =	$1.2 \cdot 10^{-5}$	[°C ⁻¹]
ΔT =	45	[°C]
k=	10,000	[kN.m ⁻²]

Tab. 3 Results.

	FEM calculation	Excel calculation
Max. normal force $N_{\max}$ [kN]	806.16	806.16
$u_{\max}$ [mm]	6.33	6.59
Breathing length z [m]	~60.00	79.50

There is only one significant difference in the breathing length among the results above. The reason is that the FEM calculations do not produce the value of the breathing length directly; it can only be estimated with the help of a graph, where there is no substantial change in the normal force along the length of the rail. But the Excel program delivers an exact value based on the limit value of the displacement (0.01 mm). The normal forces at a breathing length of 60 m are very similar: in the FEM calculations it is 800.22 kN; in the Excel calculations it is 800.21 kN. So the Excel program devised by the author can be used for practical calculations.

3 NEW FORMULAS FOR THE PRACTICE

In the previous paragraph the analytical aspect of the dilatational behaviour of an embedded rail was described in the interrelation (6).

Using the “c” system factor for each rail profile, $u_{\max}$ can be re-written into this very simple formula:

$$u_{\max} = u(0) = \pm c\Delta T \sqrt{\frac{1}{k}} \quad (7)$$

where:

$$c = \sqrt{\alpha^2 \cdot EA} \quad (8)$$

The “c” system factor depends on the material features (α , E) of the rail and the cross sectional area in each rail profile (A). In this form the relationship is simpler, and the need for calculations is much lower.

Tab. 4 shows the values of the „c” system factors for different rail profiles.

Tab. 4 Values of the “c” system factors.

Rail profile	c [kN ^{0.5} .°C ⁻¹]
For heavy haul tracks	
49E1	13.662
54E1	14.386
60E1	15.084
For urban tracks	
51R1	13.933
53R1	14.148
59R1	14.928
60R1	15.132
Ts52	14.182
35GPB	14.662
SA42-13	12.598

Note: During the calculations the modulus of elasticity of the rail material was $E=206,000,000$ kN.m⁻². The linear expansion coefficient was $\alpha = 1.2 \cdot 10^{-5}$ °C⁻¹.

To determine the value of the complete moving length of the rail in the relation of the longitudinal spring constant is the next step. Considering a 45°C change in the rail temperature, the breathing length can be given with the following power function:

$$z(k) = ak^b \quad [m] \quad (9)$$

where:

$z(k)$: breathing length [m].

This general correlation can be derived from an examination of the results of the Excel calculations. The breathing length values were determined with different spring constants (k) and different rail profiles. Parameters “a” and “b” are summarized in Tab. 5. The quality of the approximation is $R^2=0.9999$.

This formula to be applied provides the calculations for the breathing length in a closed form, for which no possibility previously existed. Figs. 7-8 below describe the changes in the breathing length for heavy haul and urban tracks.

Fig. 7 Breathing length versus the spring constant for heavy haul tracks ($\Delta T=45^\circ\text{C}$).

Tab. 5 Values of the “a” and “b” factors of the power functions.

Rail profile	a	b
For heavy haul tracks		
49E1	14,738	-0.5759
54E1	15,716	-0.5763
60E1	16,598	-0.5763
For urban tracks		
51R1	15,201	-0.5767
53R1	15,323	-0.5757
59R1	16,260	-0.5753
60R1	17,039	-0.5787
Ts52	15,725	-0.5781
35GPB	16,053	-0.5762
SA42-13	13,629	-0.5774

4 EXAMINATION OF THE POSSIBILITY OF POURING THE EMBEDDING MATERIAL FOR A RAIL CHANNEL ABOVE THE UPPER LIMIT OF A NEUTRAL TEMPERATURE ZONE

The necessity of pouring the embedding material for a rail channel above the upper limit of a neutral temperature zone can happen in the summer season under the pressure of a construction deadline. Of course, the temperature regulations given in the technical instructions for the pouring material have to be maintained.

The previous equation in paragraph 3 is modified by a relation depending on a change in temperature over 45°C . The “ $t(\Delta T)$ ” factor makes a connection between the breathing length at a change in temperature over 45°C and the values at higher temperature changes.

$$z(k, \Delta T) = z(k) \cdot t(\Delta T) = a k^b \cdot t(\Delta T) \quad [\text{m}] \quad (10)$$

In the formula above the only unknown factor is “ $t(\Delta T)$ ”, which was defined for more rail profiles. Based on the Excel calculations, the following formula was determined:

$$t(\Delta T) = p \cdot \Delta T + q \quad [-] \quad (11)$$

The relations were determined for a $45^\circ\text{C} < \Delta T \leq 65^\circ\text{C}$ rail temperature change (the temperature during construction is $+35^\circ\text{C}$ and the weld breakage at -30°C ; the temperature change is $\Delta T = 65^\circ\text{C}$). Tab. 6 shows the “p” and “q” factor values for each rail profile.

Fig. 8 Breathing length versus the spring constant for urban tracks ($\Delta T=45^\circ\text{C}$).

Tab. 6 The values of “p” and “q”.

Rail profile	p	q
For heavy haul tracks		
49E1	0.0030	0.8652
54E1	0.0029	0.8706
60E1	0.0030	0.8672
For urban tracks		
51R1	0.0030	0.8670
53R1	0.0029	0.8724
59R1	0.0029	0.8715
60R1	0.0031	0.8608
Ts52	0.0029	0.8703
35GPB	0.0029	0.8704
SA42-13	0.0030	0.8680

The author has also dealt with the practical problem of when the pouring of the rail channel happens over the upper limit of the neutral temperature zone. In this case there are two questions to answer:

1) What size is the opening gap in the case of a possible rail or weld breakage in the winter?

2) Is there any damage to the adhesion of the embedding compound (e.g., a tear from the channel wall) or in the embedding material itself (an internal shear fracture)?

Let us assume that the permitted value of the opening gap in the case of a rail/weld rupture cannot be over 20 mm; this corresponds to a 10-10 mm rail end displacement. This can be considered as the permissible limit of any displacement.

According to the earlier studies, the damage to the embedding compound does not occur at a “ u_{sup} ” displacement, so it can be considered as the permissible limit of any displacement for the embedding material. Using the former correlations:

$$u_{\text{sup}} = \pm c \Delta T \sqrt{\frac{1}{k_{\text{min}}}} \quad (12)$$

$$k_{\text{min}} = \left(\pm \frac{c \Delta T}{u_{\text{sup}}} \right)^2 = \Delta T^2 \left(\pm \frac{c}{u_{\text{sup}}} \right)^2 \quad (13)$$

$$f = \left(\pm \frac{c}{u_{\text{sup}}} \right)^2 \quad (14)$$

$$k_{\text{min}} = \Delta T^2 f \quad (15)$$

The “f” damage factor appearing in the formula above makes a connection between the rail profile and the maximum permissible rail end displacement. (“c” is the system factor). The maximum permissible displacement is $u_{sup} = 7.5$ mm; there is no tearing of the embedding material from the rail or the channel wall, and the shear strength of the embedding compound is not exhausted. So a longitudinal lower limit for the spring constant value can be determined. If the spring constant is not lower than the k_{min} value, the pouring material will not be damaged after a rail fracture in the winter time if the pouring was done at a higher temperature above the upper limit of the neutral temperature zone. Tab. 7 shows the “f” factor values for each rail profile.

If the value of the spring factor is higher than the necessary k_{min} , the structure is resistant to the given change in temperature in addition to the automatic fulfilment of the traffic security requirements.

Note: Rail stresses must be checked in cases of the fulfilment of k_{min} .

5 CONCLUSIONS

In this article the author has presented the creation of a method that makes the calculation and inspection of embedded rails simpler and closer to the everyday engineering mentality. The other great advantage of the method created is that it does not require a FEM program; it is time-saving compared to FEM, because it requires no building of a model. Using the factors introduced during the deduction, the author has provided answers to practical problems such as the judgment for working out a neutral temperature zone.

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Tab. 7 The values of the “f” factors.

Rail profile	f [kN.°C ⁻² .m ⁻²]
For heavy haul tracks	
49E1	3.3182
54E1	3.6792
60E1	4.0449
For urban tracks	
51R1	3.4512
53R1	3.5585
59R1	3.9617
60R1	4.0707
Ts52	3.5756
35GPB	3.8218
SA42-13	2.8215