ESTIMATION OF AN EFFECTIVE YOUNG'S MODULUS OF ELASTICITY IN THE LOCALITY OF THE GABČÍKOVO HYDROLOGY POWER PLANT BY GEOMETRIC LEVELING

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Abstract

The Gabčíkovo hydroelectric power plant is located in a complicated geological environment (gravel sub-soil and a high groundwater level). Excavation work started after the withdrawal of water in the autumn of 1984 and lasted until the autumn of 1986. A basic geodetic control network with a special monument was founded before the excavation work began. This network served for the setting-out of the hydroelectric power plant as well as for the control of the excavation work. The repeated geodetic control measurements have been evaluated and presented at many seminars and conferences. Monitoring the horizontal and vertical stability of the geodetic control network during the general site excavation showed significant horizontal and vertical deformations. The paper is focused on an estimation of an effective Young's modulus of elasticity in the area studied.

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Key words

- Elasticity deformation,
- Boussinesq's homogeneous elastic half-space,
- Young's modulus,
- Green's function.

1 INTRODUCTION

Information about the Earth's surface deformations due to the transfer of materials during construction in the vicinity of large structures is very important for the establishment of a geodetic network. This problem manifests itself in the form of certain patterns that are important to know for the surveyors who prepare the establishment of the network for the setting-out and monitoring of a structure. The repeated geodetic measurements (permanent or non-permanent - repeated after some time) have a high degree of accuracy and can serve for the monitoring of the Earth's surface deformations. Mass movements during the construction produce horizontal and vertical changes in the Earth's surface. Conventional surveying techniques, such as precise leveling and angle and distance measurements served in the past for monitoring vertical and horizontal deformations. An analysis of the geodetic measurements showed that the Earth's surface deformations are caused by seasonal, regional and local phenomena such as the surface water load, snow load, groundwater load or the load of the atmosphere pressure. A current analysis of the permanent Global Navigation Satellite Systems (GNSS) measurements shows a season-

al variation in the measured signals in the form of a winter snow load or unloading due to the melting of snow in the spring and summer.

The process of the deformations can be modelled using a purely elastic model based on Green's function (Pinel et al., 2007). This is then compared to the measured values, which can be used to study the structural properties of the sub-soil. The modelling of the ground deformation process and its measurement enables the detection of patterns that are very important for practical activities. The next section will only focus on the modelling of the Earth's surface deformations in a vertical direction. Leveling measurements with a high degree of precision in the locality of the Gabčíkovo hydroelectric power plant served for this purpose.

2 MEASURING VERTICAL CHANGES IN THE BASIC SETTING-OUT NETWORK AT GABČÍKOVO

The basic control network of the Gabčíkovo hydroelectric power plant consists of 12 specially stabilized points. Steel tubes with a di-

Tab. 1 Mean kilometer error of the levelling.

Date	Mean kilometer error of the levelling (mm)
Autumn 1984	0.39
Autumn 1986	0.30

iameter of 30 cm and iron rods were filled with concrete up to a depth of five meters. These pillars were up to five meters high above the ground (Janek, 1984). The height network (leveling network) for the precise leveling consisted of a main leveling line, with the first point at the Gabčíkovo church (point 31a of the Slovak Levelling Network), which is identical to the GK point and the end point at the Gabčíkovo port (point 90.3 of the Slovak Levelling Network). This main leveling line with ten points between the endpoints was six kilometers long. The main levelling line was measured first. The side levelling line consisted of 12 fundamental points and three detailed survey points of the setting-out network. A standard procedure was chosen for this measurement, i.e., back and forth measurements. The side leveling line was measured after the main leveling line. The measurements were performed with the Zeiss Ni 002 rectified leveling instrument and two invar leveling rods (Mitáš and Klajban, 1987). The first leveling measurements were performed in the autumn of 1984. The excavation work for the hydroelectric power plant began at the same time. The last leveling measurement used for the analysis was carried out in the autumn of 1986, when the excavation work was almost completed.

Tab. 1 presents the mean basic kilometer errors for the first and second measurements. These values document the high precision of the leveling measurements. Tab. 2 presents the height differences between the first and second measurements in the autumn of 1984 and the autumn of 1986.

3 MODELLING OF THE EARTH'S SURFACE DEFORMATIONS

The effect of the cylinder load on the Earth's surface was analyzed in detail in (Kollár *et al.*, 2009). The numerical calculations require a realistically estimated Young's modulus of the sub-soil. An error in the Young's modulus estimated would cause unrealistic results. The effect of the unloading of the Earth's surface is a process that occurs in the vicinity of buildings due to the mass displacements during excavation works. These changes result in deformations in both the horizontal as well as in the vertical directions. The deformations impact local geodetic networks implemented for the purpose of monitoring structures.

Tab. 2 Height differences between the first and second measurements.

Measured point	Difference (Autumn 1984 – Autumn 1986) (mm)
GK	0.0
NB1	0.0
NB2	0.0
NB4	7.5
PB7	21.0
PB9	20.0
PB11	8.0
DB11	11.0
PB10	7.0
PB8	27.0
PB6	31.0
PB5	12.0
PB3	14.0
DB4	15.0
DB1	25.0
PB4	12.0
DB5	18.0
PB1	5.0
HVB64	-2.0
NB6	-4.5
PR	-4.0
90.3	-1.0

The theoretical basis for modelling the Earth's surface deformations in the case of homogeneous elastic half-spaces was founded by Boussinesq (1885). It deals with the solution of linear differential equations, which have been developed by many researchers in analytical or numerical forms using the finite element method. This is performed for different forms of deformation elements. Green's functions are a mathematical tool for solving linear differential equations. These functions are obtained in an analytical form for a specific problem. Equation 1 represents the elastic response to a rectangular load. It is expressed as a product of Green's function and the load (Kollár, 2010, *unpublished*), where E is the Young's modulus; μ is the Poisson number; x, y are the Cartesian coordinates of the calculation points; and a, b are the Cartesian coordinates of the object causing the deformation. The symbol p in Equation 1 represents the pressure on the area bounded by the coordinates a, b (Fig. 1).

Fig. 1 The horizontal Cartesian coordinates of the calculation points.

Fig. 2 Excavation pits for the hydroelectric power plant and navigation locks in the local coordinate system (LCS).

$$v_z(x, y) = \frac{(1-\mu^2)p}{\pi E} \left[\begin{aligned} & (y-b_1) \ln \frac{x-a_1 + \sqrt{(x-a_1)^2 + (y-b_1)^2}}{x-a_2 + \sqrt{(x-a_2)^2 + (y-b_1)^2}} + \\ & (y-b_2) \ln \frac{x-a_2 + \sqrt{(x-a_2)^2 + (y-b_2)^2}}{x-a_1 + \sqrt{(x-a_1)^2 + (y-b_2)^2}} + \\ & (x-a_1) \ln \frac{y-b_1 + \sqrt{(x-a_1)^2 + (y-b_1)^2}}{y-b_2 + \sqrt{(x-a_1)^2 + (y-b_2)^2}} + \\ & (x-a_2) \ln \frac{y-b_2 + \sqrt{(x-a_2)^2 + (y-b_2)^2}}{y-b_1 + \sqrt{(x-a_2)^2 + (y-b_1)^2}} \end{aligned} \right] \quad (1)$$

The numerical solution of the Boussinesq problem by the finite element method is given, for instance, in Zienkiewicz (1972).

4 EFFECTIVE ESTIMATION OF THE YOUNG'S MODULUS OF ELASTICITY

The Young's modulus of elasticity for sub-soil can be determined by laboratory measurements or by measuring the propagation of seismic waves in the soil examined. A local static Young's modulus (Pinel *et al.*, 2007) can also be estimated from geodetic measurements using the least squares method, i.e., searching for the minimum differences between the vertical measured and modelled changes according to Equation 1. The effective Young's modulus of elasticity is the one in which the standard deviation between the above-mentioned differences reaches the minimum (Grapenthin *et al.*, 2006). This approach assumes a constant Young's modulus in the whole area studied.

The effective estimation of the Young's modulus was performed by calculating the vertical deformations at different points with the height changes measured. This was done for different elastic modulus. An effective estimation of Young's modulus corresponds to the minimum value of the standard deviation obtained by Equation (2), where v_i is the difference between the measured and modelled changes in height. The symbol $\bar{v}$ represents the average of these differences. The calculation was performed for 19 points (the symbol n in Equation (2)). The points where a subsidence was observed (points at the port) were not included in these calculations. The reason for that

is the possible impact of changes in the water level of the Danube River.

$$\sigma_E = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (v_i - \bar{v})^2} \quad (2)$$

The projected dimensions of the structure's pits were used for the calculations, i.e., 380 m x 210 m for the upper part of the hydroelectric power plant and 220 m x 50 m for the bottom with a depth of 32 m. The following dimensions of the navigation locks were used: 180 m x 420 m for the upper part, 80 m x 320 m for the bottom, and a depth of 22 m (see Fig. 1). The constant density of 1800 kg/m³ determined from laboratory measurements of the sub-soil (Slávik, 2011) was used for the calculations of the pressure (p) in Equation 1.

Tab.3 and Fig. 3 presents the changing standard deviations of the difference between the modelled and measured deformations with respect to the Young's modulus. The minimum of the function is located at the point that corresponds to the effective Young's modulus of elasticity and has a value of 2914 MPa.

Tab. 3 Standard deviation computation for selected Young's modulus of elasticity.

Yuang's Modulus (MPa)	2 000	2 500	3 000	3 500	4 000	4 500
Standard Deviation (mm)	6.069	5.031	4.869	5.036	5.317	5.611

Fig. 3 Effective estimation of the Young's modulus of elasticity.

Fig. 4 Overall situations - the points of the levelling line and deformations caused by the structure of the pits.

Fig. 5 Details of the situations - the points of the levelling line and deformations caused by the structure of the pits.

Fig. 6 Leveling line with measured height differences, modelled deformation ($E = 2914$ MPa) and their differences.

5 EXPERIMENTAL MODELLING OF THE EARTH'S SURFACE DEFORMATIONS AFTER THE UNLOADING

The experiment of the Earth's surface deformations due to the unloading caused by the excavation work at the building pits was

based on Equation 1. An effective Young's modulus of $E=2914$ MPa, Poisson number of $\mu = 0.3$, and density of 1800 kg/m^3 were used for this purpose. The results of the modelling of the vertical deformation are presented in Figs. 4 to 7.

Fig. 7 Detailed situation of the levelling line with the height differences measured, modelled by $E = 2914 \text{ MPa}$, and their differences.

Tab. 4 The measured and modelled deformations.

Point	Measured (mm)	Modelled (mm)	Difference (mm)
GK	0.0	1.5	-1.5
NB1	0.0	2.1	-2.1
NB2	0.0	2.9	-2.9
NB3	3.0	4.0	-1.0
NB4	7.5	7.1	0.4
PB7	21.0	19.2	1.8
PB9	20.0	22.8	-2.8
PB11	8.0	14.6	-6.6
DB11	11.0	20.9	-9.9
PB10	7.0	14.3	-7.3
PB8	27.0	21.9	5.1
PB6	31.0	22.5	8.5
PB5	12.0	13.1	-1.1
PB3	14.0	14.7	-0.7
DB4	15.0	25.3	-10.3
DB1	25.0	25.4	-0.4
PB4	12.0	18.0	-6.0
DB5	18.0	19.6	-1.6
PB1	5.0	13.4	-8.4
HVB64	-2.0	2.7	-4.7
NB6	-4.5	2.3	-6.8
PR	-4.0	2.0	-6.0
90.3	-1.0	1.8	-2.8

6 STATISTICAL EVALUATION OF THE EXPERIMENT

The efficiency of the proposed modelling of the vertical deformations can be statistically assessed by the use of the differences between the modelled and measured deformations. Table 4 presents the measured and modelled values. The largest discrepancy (-10 mm) is at point DB4. The standard deviation of the differences reached a value of 4.6 mm.

7 CONCLUSION

The experiment presented allows for the formulation of the following conclusions:

1. Geodetic measurements can be used for the estimation of an effective Young's modulus or a mean Young's modulus in homogeneous sub-soil and flat areas. This modulus can serve for sufficient modelling of the vertical deformations of the Earth's surface.
2. The value of the effective Young's modulus of elasticity estimated based on the geodetic measurements was 2914 MPa in the locality of the Gabčíkovo hydroelectric power plant.
3. The estimation of the Young's modulus with the presented method could be uncertain in an inhomogeneous environment or a broken relief. This however, requires experimental verification.
4. The accuracy achieved (4.6 mm) in the modelling of the vertical deformation could be increased if the control measurements were realized after the removal of all the materials. This was not observed in the case of the Gabčíkovo hydroelectric power plant as evidenced by the publication (*Janek, 1998*).
5. The main levelling line in the locality of the Gabčíkovo hydroelectric power plant was located in the deformation zone caused by the excavation work. The modelling showed that the height of the starting point of the levelling line at the Gabčíkovo church was changed due to the excavation work (1.5 mm). At least one point should be chosen outside the deformation zone.

The Gabčíkovo experiment would be difficult to repeat. This enormous structure provided a non-standard space for information about the Earth's surface deformations. This information is valuable for physicists, geologists, geophysicists, structural mechanics, geotechnical engineers and surveyors. The solution of such problems requires an integrated approach. The hidden patterns discovered can be applied not only for the construction of sophisticated structures, but also for monitoring the stability of existing structures.

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