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APPLICATION OF SENSITIVITY AND UNCERTAINTY ANALYSES IN THE CALCULATION OF THE HEAT TRANSFER COEFFICIENT OF A WINDOW

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ABSTRACT

The type of analysis we choose to use depends on exactly what we intend to explore. The Monte Carlo (MCA) method, which can be applied to a sensitivity analysis (SA) and an uncertainty analysis (UA), is based on the random selection of a random variable generated by all the input parameters X . This paper discusses a realistic model of a window and is focused on the uncertainties of the input and a sensitivity analysis of the output parameters - a window is a heat transfer coefficient. A case study is described to evaluate the necessity of the use of uncertainty and sensitivity analyses.

KEY WORDS

- Sensitivity analysis,
- Uncertainty analysis,
- Monte Carlo (Latin hypercube sampling),
- Morris method.

1. INTRODUCTION

Sensitivity analyses (SA) and uncertainty analyses (UA) should be applied in a phase of a project or in the design phase of a system. We try to describe a real situation by a mathematical model in order to analyse and optimize it. We usually consider a mathematical analysis model to be very accurate, but the fact is that an actual system cannot be fully identified due to measurement uncertainties, and the theoretical intentions of the designer cannot be precisely satisfied due to manufacturing tolerances. Therefore mathematical models are often intentionally idealized and simplified in view of the need for simpler solutions.

Many tasks cannot be solved without simplifying a model using various simulation software solutions, which allow other computational algorithms to determine the uncertainty (Kotek 2007).

The input data needed in the construction of thermal performance calculation methods can be divided into three groups: climatic data, envelope data and building use data. The probability density

functions of these data were assigned after a detailed bibliographic research. The Monte Carlo Latin Hypercube Sampling (MC-LHS) technique was used to assess the confidence interval of the building energy needs and the uncertainty of an energy performance class. A sensitivity analysis based on the Morris method was performed for different building heat balance terms, in order to identify the most important parameter set that takes into account the uncertainty in the model output (Corrado 2009). The above mentioned approach was applied to a case of the calculation of a window heat transfer coefficient, considering the uncertainty of the window parameters.

2. SENSITIVITY AND UNCERTAINTY ANALYSIS TYPES

There is a wide range of sensitivity and uncertainty analysis methods. The use of a particular type depends on our intention - what we precisely want to explore. The MCA method can be applied to the SA and UA. The MCA is based on the random selection of a random

variable generated by all of the input parameters X . We can get the value of a random phenomenon by using all the random variables X calculated as a computational mathematical model. Just one set of random variables is selected from all of the input parameters in each step of the simulation. One of the other methods based on a variation of the variables is called the Morris method, and it is used only for an SA evaluation. It works on the following principle: there is only one random selection variable in the calculation chosen, and the other input parameters remain constant. Then the calculation is carried out, and the change in Y is evaluated, which continues until the last parameter (Campolongo, 1997; Saltelli, 1997).

3. SENSITIVITY AND UNCERTAINTY ANALYSIS TYPES

Estimating the statistical distribution of each input datum is a difficult task; however, some studies have tried to describe and quantify the uncertainty through suitable probability functions (Macdonald, 2002; Clarke, 1990).

The density function $f(x)$ and its integral cumulative distribution function $F(x)$ are utilized for probability distribution analyses. The two functions are correlated; therefore, the probability of the variable X being less than the value y is given by:

$$P(X < y) = F(y) = \int_{-\infty}^y f(x) dx \quad (1)$$

4. ANALYSIS METHODOLOGY

In the engineering research field, several techniques have been used for uncertainty and sensitivity analyses. A good overview can be found in (Saltelli, et al. 1990; Campolongo, et al. 1997). The MC-LHS and Morris methods are considered to be the most suitable methods for our application.

4.1 DESCRIPTION OF THE STOCHASTIC MODEL

A stochastic model describes a system that takes into account both random and scheduled events. We assume the basic computing core of the physical processes modeled in the energy calculations in the form:

$$Y = F(X_i) \quad (2)$$

Where:

F – the deterministic computational model

X_i – the input parameter of the random variables

In a case where the computational model describes the physical phenomenon of the outcome, variable Y is provided by the experimental measurements.

4.2 APPROACHES TO THE UNCERTAINTY AND SENSITIVITY ANALYSES

Latin hypercube sampling (LHS) is a form of stratified sampling that can be applied to multiple variables. The method is commonly used to reduce the number of runs necessary for a Monte Carlo simulation in order to achieve a reasonably accurate random distribution. LHS can be incorporated into an existing Monte Carlo model fairly easily and works with variables following any analytical probability distribution.

Monte-Carlo simulations provide statistical answers to problems by performing many calculations with randomized variables and analyzing the trends in the output data. There are many resources available that describe Monte-Carlo simulations (<http://mathieu.fenniak.net/latin-hypercube-sampling/>).

MC-Latin Hypercube Sampling: a Monte Carlo analysis is based on repeated simulations; the outputs of interest are evaluated for each sample element x_{ij} of the sample matrix M_{nk} , where n is the input factor number and k is the sample size. The output vector $Y = [y_i]$ is then generated:

$$y_i = f(x_{i1}, x_{i2}, \dots, x_{in}) \quad i = 1, 2, \dots, k \quad (3)$$

The Latin Hypercube Sampling achieves a better coverage of the sample space of the input factors. The sample space S of an input factor X_i is partitioned into h disjoint intervals $S_1 \dots S_h$ of equal probability $1/h$. One random input value is then selected from each interval S_p and the process is repeated k times, where k is the sample length (see Fig. 1).

Fig. 1 Illustration of the LHS method results.

The mean value:

$$\mu = \frac{1}{n} \sum_{i=1}^n y_i \quad (4)$$

and the standard deviation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (y_i - \mu)^2}{(n-1)}} \quad (5)$$

of the vector outputs are calculated and used to interpret the results.

Coefficient of variation:

In probability theory and statistics, the coefficient of the variation (CV) is a normalized measure of the dispersion of a probability distribution. The coefficient of the variation (CV) is defined as the ratio of the standard deviation σ to the mean μ .

$$c_v = \frac{\sigma}{\mu} \quad (6)$$

Morris method: The aim of the Morris method is to determine which factors can be considered to have effects which are negligible, linear and additive, or non-linear and the effect of an interaction. The Morris method is based on the so-called “elementary effect” of an input parameter X_i .

Set up of the p-level grid: The p-level grid of an input X_i corresponds to the set of input data values at p-quantiles. $x_{i,j}$ can take one value from $\{a_1, a_2, \dots, a_p\}$ as $P(X_i < a_1) = 1/2p$, $P(X_i < a_2) = 1/2p + 1/p$, ..., $P(X_i < a_j) = 1/2p + j/p$, ..., $P(X_i < a_p) = 1 - 1/2p$. For instance, if X_i is normally distributed with $\mu = 7$ and $\sigma = 2$ and for a 4-level grid, the set of inputs is $\{4.71, 6.36, 7.63, 9.29\}$. Repeating this process for the n input data, we obtain the n-dimensional space p-level grid.

Sample generation: The general procedure for assessing one single sample is as follows: Initially, an n -input vector is assigned as a random base value (from the discretized grid defined previously). A ‘path’ of steps through the discretized grid is followed, changing one at a time, while the values of the other factors are held at their last values.

Finite distribution F_i : The elementary effect Δx is calculated as:

$$\Delta_i(x) = \frac{|y(x_{i,j}, \dots, x_{i-1,j}, x_{i,j+1}, x_{i+1,j}, \dots, x_{n,j}) - y(x_{i,j}, \dots, x_{i-1,j}, x_{i,j}, x_{i+1,j}, \dots, x_{n,j})|}{\delta} \quad (7)$$

where δ is a multiple of $1/p$. Thus n finite distributions F_i of the elementary effects for the different input factors are obtained.

The distribution F_i is characterized through its mean and standard deviations (μ_{MP} , σ_M). A high mean indicates a factor with an important overall influence on the output; a high standard deviation indicates either a factor interacting with other factors or a factor whose effect is nonlinear (Saltelli, et al., 2004).

The analysis procedure developed is shown in Figure 2. It includes the definition of the input data and uncertainties, the random sampling (through the Monte Carlo simulation software Simlab 2.2), the thermal modeling (ISO 13790:2008) and the post processing analysis (Simlab 2.2).

Fig. 2 Uncertainty and sensibility simulation procedure (Corrado 2009).

4.3 SETUP

This case study of a window was performed with 60 and 1000 simulations. There were a total of six input parameters used: frame length, length of glazing in the x , length of glass in the y , heat transfer coefficient of the glazing, heat transfer coefficient of the frame, and linear thermal transmittance.

It includes the definition of the input data and uncertainties, random sampling (by means of the Monte Carlo simulation software Monte Carlo SimLab 2.2), namely of the value *seed* (the starting value of the sequence) and # of executions (N - Number of running the model for simulation tools is the value of the number of simulations.), heat simulation (ISO 13790:2008) and analysis processing (Kotek 2007). In the SimLab it is provided that the value of the *seed* should be a seven-seeder number. The number of the model's execution, meaning the # of the execution is, according to the instructions, recommended as the 1.5 fold, optimally the decuple of the number of the input parameters. It is preferable because the bigger the number N (S) is, the more exact the result (e.g. from N random choices from a normal dividing of the input parameter, we get N number in the interval $\pm 3\sigma$). The aim of this study was to find the

Fig. 3 Technical parameters a) scheme (plastic) window 800 x 1500 (mm) b) frame and wing profile.

most sensitive parameter for the calculation of the energy transition coefficient of the window (Kotek 2007).

The MC-LHS was used to generate the sample matrix method. The LHS sampling is a particular case of stratified sampling, which is meant to achieve a better coverage of the sample space of the selected input parameters. There are several different techniques in SimLab available for the sensitivity analysis. The one chosen for demonstrating the results is the standard regression coefficient (SRC). The SRC provides a measure of the linear relation between any given input X and the output, cleaned of any effect due to the correlation between X and any other input (Saltelli, et al., 2005).

5 CASE STUDY

A case study was analysed in order to establish the influence of the input parameter uncertainties on a building's energy performance. The aim of this example is to find an input parameter of equation (8), which most influences the resulting heat transfer coefficient U_w of a window.

$$U_w = \frac{U_f \cdot A_f + U_g \cdot A_g + \Psi_g \cdot l_g}{A} \quad [\text{W}/(\text{m}^2 \cdot \text{K})] \quad (8)$$

Where (STN 73 0540, 2002):

- U_f – the heat transfer coefficient of the frame $[\text{W}/(\text{m}^2 \cdot \text{K})]$;
- A_f – the area of frame $[\text{m}^2]$;
- U_g – the heat transfer coefficient of the glazing $[\text{W}/(\text{m}^2 \cdot \text{K})]$;
- A_g – the area of glazing $[\text{m}^2]$;
- Ψ_g – the linear thermal transmittance $[\text{W}/(\text{m} \cdot \text{K})]$;
- l_g – the total length of frame $[\text{m}]$.

Parameter	Value
U_f Heat transfer coefficient of the frame	1.1 $\text{W}/(\text{m}^2 \cdot \text{K})$
U_g Heat transfer coefficient of the glazing	1.3 $\text{W}/(\text{m}^2 \cdot \text{K})$
A_f Area of the frame	0.465024 m^2
A_g Area of glazing	0.734976 m^2
Ψ_g Linear thermal transmittance	0.08 $\text{W}/(\text{m} \cdot \text{K})$
l_g Length of glazing perimeter in the frame	4.6 m

After substituting for the relation (8,) the U_w values for these 6 input parameters are the following: $U_w = 1.484 \text{ W}/(\text{m}^2 \cdot \text{K})$.

Selecting the input parameters (for the calculation and determination of their standard deviation) is very difficult. The frame length (l_f) was transferred to more than one value frame and statistically evaluated by the average of the values (mean)

and the standard deviation. The variations of the length input parameters of the glazing in the x (l_{gx}), length of the glazing in the y (l_{gy}) were selected as 10%. The other parameters, the heat transfer coefficient of the glazing (U_g) (Fang, Y., Eames, P., Norton, B., 2002), the heat transfer coefficient of the frame (U_f) (Experimental Heat Transfer, 1996) and the linear thermal transmittance ψ (Svendsen, Laustsen, 2004) are based on the literature and measurements.

In the practical example (case study) in Table 1, we can imagine the need to obtain information about the parameters and which is the most sensitive parameter to be optimized.

5.1 RESULTS OF THE UNCERTAINTY ANALYSIS (UA)

The MCA was executed with 60 simulations and then 1000 simulations. The UA shows the distribution of the output, which

is caused by the uncertainties in the inputs. An uncertainty of the output causes a wide spread, which is shown in the histogram in figure 4c (Function of the result distribution for 1000 simulations). Figure 4 demonstrates how far the distribution matches the assumptions by illustrating a normality plot. Due to the fact that the results follow the line, it can be concluded that the output for the heat transfer coefficient of a window has a normal distribution, because if the number of simulations has increased as in Fig. 4c, we can see how the histogram is directed towards the normal distribution.

5.2 RESULTS OF THE SENSITIVITY ANALYSIS (SA)

We can determine the order of factors that affect the outcome of U_w using the sensitivity analysis. As the evaluation criterion, the Standardised Regression Coefficient (SRC) was chosen.

Fig. 4 Density of U_w result probability, a) Probability of distribution function for 60 simulations b) Cumulative distribution function, c) Function of the result distribution for 1000 simulations.

Tab. 1 Deviation of the input parameters for the calculation of the U_w material window: mean (μ) and standard deviation (σ).

Sign of window	Input parameter	Mean μ	Standard deviation σ /(PDF)	Variance in %
l_f	Frame length	0.112	0.016/ Δ	14.3
l_{gx}	Length of glazing in the x	0.576	0.058/ Δ	10
l_{gy}	Length of glazing in the y	1.276	0.12/ Δ	10
U_g	Heat transfer coefficient of the glazing	1.1	0.05/ Δ	4.5
U_f	Heat transfer coefficient of the frame	1.3	0.10/ Δ	7.6
ψ	Linear thermal transmittance	0.08	0.01/ Δ	12.5

Parameter	SRC
1. Frame length	-3.721e-002
2. Length of glass in the direction of the x- axis	-4.682e-002
3. Length of glass in the direction of the y- axis	-1.215e-002
4. Heat transfer coefficient of the glazing	0.2309
5. Heat transfer coefficient of the frame	0.9350
6. Linear thermal transmittance	0.2384

The SRC value informs us that if the parameter x increases, the output value of the parameter y decreases.

The sensitivity analysis shows us that the frame heat transfer coefficient U_f is the strongest parameter that affects our U_w value.

A negative sign of the other parameters means that when we increase the parameter, the U_w value decreases.

5.3 Morris method

The results of an identical analysis are in Figure 5.3, which were given by the values (μ) and (σ): the frame length, the length of the glazing in the direction of the x-axis, the length of the glazing in the direction of the y- axis, the heat transfer coefficient of the glazing, the heat transfer coefficient of the frame, and the linear thermal transmittance. However, out of all of these factors, the frame heat transfer coefficient U_f has the biggest influence, which is shown in the black circle.

Fig. 5.1 Graphical interpretation of SRC coefficient.

Fig. 5.2 Graphic interpretation of the SRC sensitivity coefficients.

Fig. 5.3 Morris sensitivity of the 6 input factors.

5.4 RESULTS OF UNCERTAINTY HEAT TRANSFER COEFFICIENT OF A WINDOW

A sensitivity coefficient has to be defined in order to compare the relative effects of the individual data. There are various definitions which allow for a direct comparison of the effects of different changes. The most straightforward method considers the coefficient of the variation for a distribution (v), defined as the ratio of the standard deviation to the mean value (Corado 2009).

Tab. 2 Calculation coefficient of variation.

	I_f	I_{gx}	I_{gy}	U_g	U_f	ψ
μ	0.112	0.576	1.276	1.100	1.300	0.080
σ	0.016	0.058	0.120	0.050	0.100	0.010
c_v	0.14	0.10	0.09	0.05	0.07	0.13

6 CONCLUSION

An uncertainty analysis and sensitivity analysis were performed for the windows (plastic) case study. Using MC-LHS and the Morris method, the following variables in the model output have been studied:

- U_f Heat transfer coefficient of the frame
- U_g Heat transfer coefficient of the glazing
- A_g Area of glazing
- Ψ_g Linear thermal transmittance
- l_g Length of glazing perimeter in the frame

The Monte Carlo Latin Hypercube Sampling (MC-LHS) technique was used to assess the heat transfer coefficient of the window. A sensitivity analysis based on the Morris method was also performed in order to identify the most important parameter

(the highest degree of sensitivity) set that takes into account the uncertainty in the model output.

By the calculation of the window heat transfer coefficient using the MC-LHS, it has been found that the sensitivity analysis leads to the following results: The sensitivity analysis shows us that the frame heat transfer coefficient U_f is the strongest parameter that affects our U_w value. A negative sign of the other parameters means that when we increase this parameter, the U_w value decreases. An identical analysis is in *Figure 5.3*, which was defined by the values (μ) and (σ) for the frame length, the length of the glazing in the x -axis, the length of the glazing in the direction of the y -axis, the heat transfer coefficient of the glazing, the heat transfer coefficient of the frame, and the linear thermal transmittance. The frame heat transfer coefficient U_f has the biggest influence on all of these factors.

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