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M. BALÁŽ, M. DANÁČOVÁ, J. SZOLGAY

ON THE USE OF THE MUSKINGUM METHOD FOR THE SIMULATION OF FLOOD WAVE MOVEMENTS

Miroslav BALÁŽ
email: miroslav.balaz@stuba.sk
Research field: flood routing, Muskingum method

Michaela DANÁČOVÁ
email: michaela.danacova@stuba.sk
Research field: flood routing, flood forecasting

Ján SZOLGAY
email: jan.szolgay@stuba.sk
Research field: rainfall – runoff modelling, flood routing, flood forecasting

Address:
Department of Land and Water Resources Management, Faculty of Civil Engineering Slovak University of Technology, Radlinského 11, 813 68 Bratislava, Slovak Republic

ABSTRACT

The Muskingum method is a hydrological flow routing model with lumped parameters, which describes the transformation of discharge waves in a river bed using two equations. The first one is the continuity equation (conservation of mass) and the second equation is the relationship between the storage, inflow, and outflow of the reach (the discharge storage equation). These equations are applied within a river reach between two cross sections of a river. The parameters of the model can be estimated by several methods. Here the classical graphic method is compared with two new methods where a genetic algorithm and harmony search was used for optimization. The discrete state space formulation of the Muskingum method was applied on the lower Morava reach between Moravský Svätý Ján and Záhorská Ves. The results showed a good degree of accuracy of all three methods, which were assessed by the Nash-Sutcliffe efficiency coefficient.

KEY WORDS

- *Muskingum method,*
- *transformation of a flood wave,*
- *calibration of parameters,*
- *genetic algorithm.*

INTRODUCTION

Many methods have been sought for predicting the characteristic features of the movement of a flood wave along a river in order to determine the actions necessary for protecting life and property from the effects of flooding and to improve the management of water related systems along natural or manmade watercourses. The literature abounds with a wide spectrum of flow routing models (e.g. Fread, 1985; Liggett, Cunge, 1975; Linsley, Kohler, Paulhus, 1986) which are sufficiently accurate when used within the bounds of their limitations.

Flow routing may be classified as either lumped or distributed. In lumped flow routing or hydrologic routing, the flow is computed as a function of time at one location along a watercourse, but in distributed flow routing or hydraulic routing, the flow is computed as a function of time simultaneously at several cross sections along a watercourse (Maidment, Fread, 1993).

Distributed routing models are mostly based on the numerical solution of the Saint-Venant equations. According to Maidment and Fread (1993), dynamic routing models are required for (1) slowly rising flood waves in mild sloping channels, i.e., slopes roughly less than 0.10 percent; (2) situations where backwater effects are important owing to tides, significant tributary inflows, natural constrictions, dams, and/or bridges; and (3) situations where waves propagate upstream from large tides, storm surges or very large tributary inflows (Maidment, Fread, 1993). As the trend in increased computational speed for computers and storage capabilities with decreasing costs continues, the economic feasibility of using

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dynamic routing models for a wider range of applications will steadily increase, since dynamic models have the capability to correctly simulate the widest spectrum of wave types and waterway characteristics.

However, due to the moderate data requirements and operational costs, in several practical applications (e.g. flood forecasting), hydrological routing models will still be in use. To translate and attenuate an upstream runoff hydrograph into a downstream hydrograph in lumped routing, the continuity of a mass can be expressed as

$$I_{(t)}\text{-}Q_{(t)} = \frac{dS}{dt} \approx \frac{\Delta S}{\Delta t} \tag{1}$$

in which *I* and *O* are the inflow and outflow, respectively, during the incremental time *dt* (or Δ*t*), and *S* is the storage.

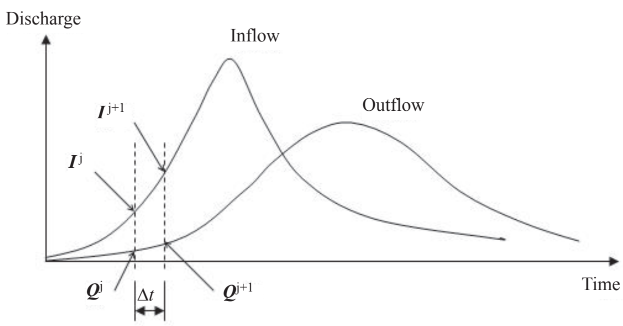

Fig. 1 *Schematic diagram of upstream (I) and downstream (O) flood hydrograph*

For the hydrograph shown in Figure 1, the approximation of continuity equation can be expressed in terms of the average inflow (upstream) and average outflow (downstream) between times t_1 and t_2, (during the interval $\Delta t = t_1 - t_2$). Expressing the equation in terms of the average inflow and average outflow, at times t_1 and t_2 the change of storage in the reach can be expressed as

$$\frac{1}{2}(I_1 + I_2) - \frac{1}{2}(O_1 + O_2) = \frac{S_2 - S_1}{\Delta t} \tag{2}$$

In hydrologic routing Equation 1 is accompanied by the lumped storage-outflow relationship for *Q(t)*. Both graphic and mathematical techniques can be used to solve this system of two in general nonlinear equations. The attractiveness of lumped flow routing is its relative simplicity when compared with distributed flow routing. However, lumped flow routing methods for rivers ignores backwater effects and are not accurate for rapidly rising hydrographs routed through mild-to-flat sloping rivers. They are also inaccurate for rapidly rising hydrographs in long reservoirs.

MUSKINGUM METHOD

The *Muskingum method,* which was developed by McCarthy (1938), is a popular lumped flow routing technique. The Muskingum Method is an alternative for routing hydrographs through stream reaches, which is well established in the hydrological literature (e.g. McCarthy, 1938; Ponce and Yevjevich, 1978; Ponce and Theurer, 1982; Perumal, 1992; Ponce and Chaganti, 1994; Tang, et al., 1999; Birkhead, James, 2002; Al-Humoud and Esen, 2006) and its modest data requirements make it attractive for practical use. The Muskingum method sometimes produces unrealistic initial negative dips in the computed hydrograph. However, it provides reasonably accurate results for moderate-to-slow rising floods propagating through mild-to-steep sloping watercourses (Maidment, Fread, 1993).

Muskingum routing is based on an assumed linear relationship between a channel's storage and inflow and outflow discharge; and consequently, it accounts for prism and wedge storage. The storage under a line parallel to the streambed is called prism storage; the water located between this line and the actual profile is wedge storage. The routing parameters in the models are usually derived by calibration using measured discharge hydrographs (Birkhead, James, 2002).

In the Muskingum method the storage *S* in the routing reach is represented by the following discharge-storage equation:

$$S = K\,[X\,I + (1 - X)\,Q] \tag{3}$$

in which the rism storage in the reach is *KQ,* where *K* is a proportionality coefficient, and the volume of the wedge storage is equal to *KX (I – Q),* where *X* is a weighting factor having a range of $0 \le X \le 0{,}5$ (most streams (Maidment, Fread, 1993) have *X* values between 0,1 and 0,3). A general rule of thumb is that *K* can be estimated by the travel time through a reach, and a value of 0,2 (McCuen, 2004) can be used for *X*. A value of 0,5 for *X* is usually considered to be the upper limit of rationality. Another recommendation to be followed in practice is that the ratio Δ*t/K* should be approximately 1 (McCuen, 2004).

The time rate of the change of storage *dS/dt* in Equation 3 is represented substituting Equation 3 into Equation 2 as follows:

$$\frac{dS}{dt} = \frac{S^{j+1} - S^{j}}{\Delta t^{j}} = \frac{K\left\{\left[X\,I^{j+1} + (1-X)\,Q^{j+1}\right] - \left[X\,I^{j} + (1-X)\,Q^{j}\right]\right\}}{\Delta t^{j}} \tag{4}$$

where the superscripts *j* and *j+1* denote the times separated by the interval Δt^{j}.

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For the outflow of the reach we subsequently get this relationship:

$$Q^{j+1} = C_1 I^{j+1} + C_2 I^j + C_3 Q^j \tag{5}$$

which is the classical Muskingum flow routing equation, where

$$C_1 = \frac{\Delta t - 2KX}{2K(1-X)+\Delta t} \tag{6}$$

$$C_2 = \frac{\Delta t + 2KX}{2K(1-X)+\Delta t} \tag{7}$$

$$C_3 = \frac{2K(1-X) - \Delta t}{2K(1-X)+\Delta t} \tag{8}$$

and where $C_1+C_2+C_3=1$, and $K/3 \le \Delta t \le K$ is usually the range for Δt. The values for K and X can be determined from the observed inflow and outflow hydrographs (McCarthy, 1938; Linsley, et al. 1986; Morris, Wiggert, 1972; Chow, 1964). Using Equation 4 and the left side of Equation 3, K can be expressed as (Maidment, Fread, 1993):

$$K = \frac{0.5\Delta t\left[\, I^{j+1} + I^j - \left(Q^{j+1}+Q^j\right)\right]}{X\left(I^{j+1}+I^j\right)+(1-X)\left(Q^{j+1}-Q^j\right)} \tag{9}$$

If, at each time interval, the values of the numerator are plotted against those of the denominator, a loop by the data is formed. Iteratively, varying X will tend to close the loop and that value of X which causes the plot to be closest to a single line is the engineering estimate value for the reach. Then K may be computed from the average value determined from Equation 9 for the correct value of X. Lateral inflows can also be included in the calibration of the Muskingum method (O'Donnell, Pearson, Woods, 1988).

The Muskingum method can also formulated within the state space framework, which enables us to more flexibly change conditions of the system's dynamics (see, for example, Szolgay, 1982). For the discrete state space equations the Muskingum equation is defined as:

$$S = KO + KX(I-O) \tag{10}$$

where S is the storage, K is the time proportionality coefficient, X is a weighting factor (having the range $0 \le X \le 0{,}5$), O is the outflow discharge from the river reach and I is the inflow discharge to the river reach. In combination with a continuity equation:

$$\frac{dS}{dt} = \mathbf{I}(t) - \mathbf{O}(t) \tag{11}$$

we obtain the following ordinary differential equation

$$\frac{dS}{dt} = \frac{1}{K(X-1)}\mathbf{S}(t) + \frac{1}{1-X}\mathbf{I}(t) \tag{12}$$

which describes the instantaneous change of storage in the river reach. In addition, the output from the reach can be calculated as

$$\mathbf{O}(t) = \frac{1}{K(1-X)}\mathbf{S}(t) + \frac{X}{1-X}\mathbf{I}(t) \tag{13}$$

The last two equations describe the continuous state space representation of the Muskingum method where $\boldsymbol{S}$ is the system state variable. The general solution is:

$$\mathbf{S}(\mathrm{t}) = e^{(1/K(X-1))t}\mathbf{S}_0 + \int_0^t e^{(1/K(X-1))(t-\tau)}\frac{1}{1-X}\mathbf{I}(\tau)\,d\tau \tag{14}$$

Under the assumption that inputs to the model are considered constant during sampling interval of the length T between a and $a+1$, the state transition will have the form

$$\mathbf{\Phi}(a+1,a) = e^{T/(K(X-1))}. \tag{15}$$

and the transition of the input will be

$$\mathbf{\Psi}(a+1,a) = K - Ke^{T/(K(X-1))} \tag{16}$$

The discreet state equations then have the form:

$$\mathbf{S}(a+1) = \mathbf{\Phi}(a+1,a)\cdot\mathbf{S}(a) + \mathbf{\Psi}(a+1,a)\cdot\mathbf{I}(a) \tag{17}$$

$$\mathbf{O}(a+1) = \frac{1}{K(1-X)}\mathbf{S}(a+1) + \frac{X}{1-X}\mathbf{I}(a) \tag{18}$$

THE MODEL CALIBRATION APPROACH PROPOSED FOR RIVER REACH OF THE MORAVA RIVER

One of the most important parameters in flood routing is the wave speed (or alternatively the travel time) at which the flood wave travels along the river reach downstream. Strictly speaking, this wave speed (celerity) is the speed at which the flood wave moves downstream. This speed can be readily approximated from characteristic points on the recorded hydrographs at either end of a reach (Weinmann a Laurenson, 1979), or it may be estimated from the rating curve at a particular cross-section (Wang, et al.,

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2006). Here it is proposed to use the such an estimate of the travel time in the parameterisation of the Muskingum routing method.

We have analyzed the relationship between the discharge and travel time of characteristic points from a set of flood waves for a reach of the Morava River. The reach between the gauging stations at Moravský Svätý Ján and Záhorská Ves, with a length of 34.76 km and a slope of 0,2 ‰, was chosen. The selected river reach has a typical lowland character. In our analysis of the flood wave travel time, we used hourly discharge data from the years 1992 to 2002 (data were provided by the Slovak Hydrometeorological Institute). In the river reach there are two significant inflows measured, the Zaya (data were provided by the TU Vienna) and the Rudava Rivers. The inter-basin area of the basin is 1,392 km^2. The unmeasured lateral inflows of the whole reach had to be estimated (based on the known data) using hydrological analogy as explained in Danáčová (2008). There were no other corrections of the data sets made.

The classical model calibration approach has been compared in this work with the new method prosed here, in which variable travel time parameter is estimated by stochastic optimization techniques. Two methods, the genetic algorithms (GA) and harmony search (HS), were used. Genetic algorithms are common techniques in optimization and in hydrologic modeling (e.g. Sekaj, 2005; Čistý et al., 1999; Wang et al., 2007; Szolgay et al., 2007; Čistý and Bajtek, 2009; Mohan, 1997). HS is a music-inspired evolutionary algorithm, mimicking the improvisation process of music players (Geem et al., 2001; Kim et al., 2001; Kosinski et al., 2008).

The effectiveness of a GA or HS depends on the setup of their parameters (e.g. in the case of the GA the size of the population, the number of generations, the kinds of mutations, selection, crossover), the choice of the objective function and boundaries of the search space. In particular it is necessary setting up the upper boundaries (UB) and lower boundaries within which a Genetic algorithm or Harmony search will search for the results. In this case it was the range of the search space for the wave travel time (*K)*. As an objective function the Nash-Sutcliffe coefficient (Nash; 1970) was optimized to the minimum value. It was also used for the model validation since in practice it is very often used to assess the predictive power of hydrological models (Nash, 1970; McCuen, Knight and Cutter, 2006; Szolgay et al., 2009; Szolgay, 2004; Hlavčová et al., 2004; and Parajka, 2001).

The calibration of the travel time parameter vs. discharge relationship was accomplished at one of largest flood wave subjectively selected from large waves at the river reach researched. For verification we used randomly selected flood waves with a range of discharges and shapes within the selected time period.

Following calibration approaches were compared: first we assumed optimal *X* and *K* as constant values estimated for every flood wave separately (Column 4, Tab. 1) by the classical calibration approach as improved by Valent (2008). This parameter set served as a baseline representing the optimal model performance (as achieved by the standard calibration procedure). In the second approach we have taken *X* and *K* (both constant) as the average of all optimal pairs from the baseline set estimated by the classical approach (Column 5 in Tab. 1). This approach was representing the usual situation in practical simulation and forecasting applications, where the optimal parameters are not known during the event and the average parameters for the given reach have to be used. In the third approach (Columns 6 and 7 of Tab.1) we assumed that *K* is varying with discharge and the parameter *X* was taken as the average of all flood waves (as in approach two).

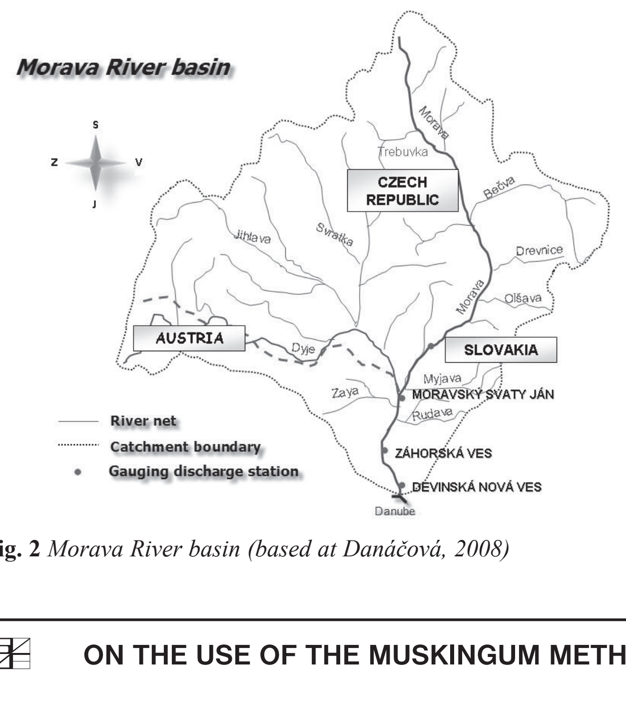

Fig. 2 *Morava River basin (based at Danáčová, 2008)*

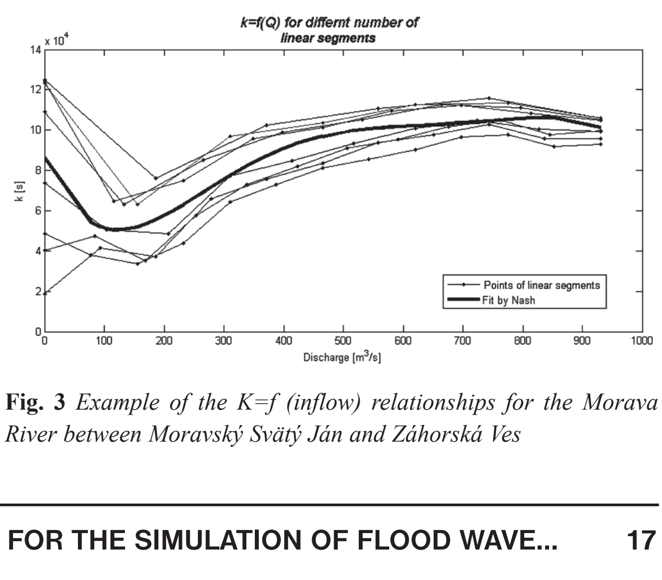

Fig. 3 *Example of the K=f (inflow) relationships for the Morava River between Moravský Svätý Ján and Záhorská Ves*

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Tab. 1 *Nash-Sutcliffe values for each of the tested methods*

No.	Flood wave	Peak flow [m^3/s]	Values of the Nash-Sutcliffe coefficient			
			Optimal K and X for each flood Classical calibration method	Average K and X Classical calibration method	Average X and variable K using GA	Average X and variable K using HS
1	2	3	4	5	6	7
1	24.09.1996 - 02.10.1996	170,2	0,9896	0,9297	0,9313	0,9184
2	11.09.1998 - 27.09.1998	305,3	0,9627	0,9553	0,9611	0,9620
3	01.04.1994 - 07.05.1994	428,5	0,9659	0,9409	0,9664	0,9704
4	16.03.1992 - 23.04.1992	595,6	0,9744	0,9678	0,9844	0,9850
5	15.03.1996 - 04.06.1996	633,8	0,9579	0,9762	0,9833	0,9836
6	29.06.1997 - 29.08.1997	923,1	0,9906	0,9794	0,9846	0,9849
Average			0,9735	0,9582	0,9685	0,9674

The relationships *K=f* (inflow) were first taken as composed from a preset number of linear segments (lines). For this the discharge range was divided into intervals of the same length. From 4 to 12 linear segments were used and for each set of segments the optimal chain was estimated (see Fig. 3) by constrained optimization on one large calibration flood wave both with the help of a Genetic algorithm and Harmony search. For the final mathematical description the *K=f* (inflow) relationship was fitted by 5th degree polynomial into these chains of linear segments by the least squares.

RESULTS

The models compared were verified on 6 flood waves covering a wide range of flood peaks using the Nash-Sutcliffe coefficient as performance criterion (Tab. 1). As it can be seen in Table 1,

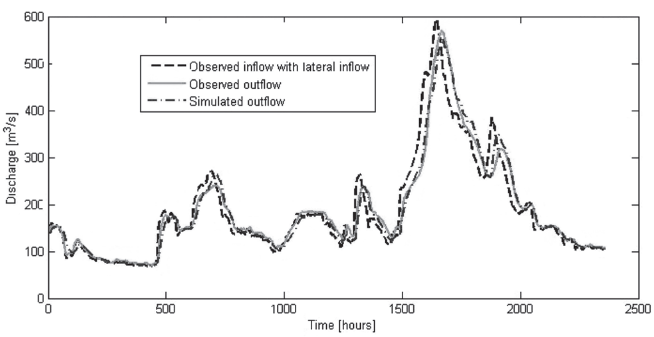

Fig. 4 *Example of the simulated wave for the variable K=f (inflow) relationship for the Morava River between Moravský Svätý Ján and Záhorská Ves*

the application of average parameters (as expected) degrades the performance of the model when compared to the optimal performance. On the other hand, the use of GA or HS for the forecasting of wave travel time (*K*) results in an increased level of model accuracy. The new models outperform the average model and are near optimal. It is also important to note even if the accuracy of the fit for simulated discharges was not the best, but the visual comparison clearly shows, that the match of the timing of the intermediate peaks is better than in the case of the average model (constant travel time) (see Fig. 4). Other advantage of the approach used is that the *K=f* (inflow) relationship was calibrated at one wave. Moreover the main aim of this research was to find a feasible methodology for evaluating flood wave travel time and incorporating it into the Muskingum model. Based on the results one can conclude, that the proposed approach could be further developed. The advantages of HS in comparison with GA are: slightly better results and increased saving of time (HS needed approximately 10% less required time than GA in this work). HS also almost always showed a smaller variability of the estimated function when compared with GA.

CONCLUSION

There are a variety of techniques for estimating the required *X* and *K* parameters associated with the Muskingum method. The methods that were used in this work for the Muskingum equation were the classical approach and a new method based on the estimation of the relationship between the travel time parameter *K* and the discharge. Also relatively new optimization techniques such as Genetic algorithms and Harmony search were used and compared.

The Muskingum method was applied to the River Morava within the Moravský Svätý Ján – Záhorská Ves reach. The function of travel time parameter and discharge was estimated for one of the largest flood. The results were validated using the Nash-Sutcliffe coefficient on 6 floods. The results obtained by the new method presented in this work were satisfactory and, because of the advantage of the small of wave travel time or for the forecasting of discharge. Based on the results one can amount of data required for calibration, this method could be used in suitable river reaches for the estimation conclude, that the proposed approach could be further developed.

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