Studia Humana
Volume 3:1 (2014), pp. 13—26
DOI: 10.2478/sh-2014-0002

Philosophical Problems of Foundations of Logic

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Abstract:
In the paper the following questions are discussed: (i) What is logical consequence? (ii) What are logical constants (operations)? (iii) What is a logical system? (iv) What is logical pluralism? (v) What is logic? In the conclusion, the main tendencies of development of modern logic are pointed out.

Keywords: modern logic, criticisms of logic, normative logic, abstract logic, universal logic, combining logic

This work is supported by Russian Foundation for Humanities, grant No. 11-03-00761.

1. I want to bring into consideration the main logic development trends at the end of 20th century and the beginning of the 21st century. In the same way that the problem of the foundations of mathematics has risen hundred years ago, now there is the problem of the foundations of logic.

The following sections refer to:

(i) What is logical consequence?
(ii) What are logical constants (operations)?
(iii) What is a logical system?
(iv) What is logical pluralism?
(v) What is logic?

Each of the above-mentioned problems has been discussed intensively for recent decades, and there is plenty of literature on each issue. I fragmentary discussed some of these issues in [67], [68], and [69].

2. The standard definition of a subject of logic is the following: the science which studies *the principles of correct reasoning*. However, such a definition does not solve the problem of exact area of the given subject, i.e. what is the area of application of logic? For traditional logic, it is *syllogistic* reasonings, and there are 24 equally correct syllogisms. The nature of reasonings can vary greatly. For example, mathematical logic studies *mathematical* reasonings: "If […] his researches are devoted first of all to study of mathematical reasonings, the subject of his investigations can be called mathematical logic" (see [90]). In turn, *fuzzy* logic studies fuzzy reasonings, i.e. it deals with reasoning that is approximate rather than fixed and exact ([42]), informal logic studies *informal* reasonings (see [48]), and philosophical logic, as a result, studies *philosophical* reasonings. Then psychologistic reasonings are studied by *psychologistic* logic… In order to avoid similar senselessness, it is necessary to select the nucleus or base concepts with which the given science deals.

3. Such a nucleus undoubtedly is the concept of '*logical consequence*'. Logical consequence is the relation between premises and conclusion of a valid reasoning. Alfred Tarski in 1936, as one of creators of modern logic, sketched its essence in the work with the characteristic title "On the Concept of Logical Consequence" (see [119])[1]. However, we can add there the methodological aspects: in what terms it is defined or what paradigm of the offered answer is. Approaches to the answer concerning the sphere of application of logic, its basic concepts, which are used by the conception of logical consequence, may be completely different: model-theoretic, semantic set-theoretic, proof-theoretic, constructive, combinatory, etc. As we shall see, Tarski's answer is within the framework of the semantic approach:

> "The sentence *X follows logically* from the sentences of the class *K* if and only if every model of the class *K* is also a model of the sentence *X*" [119, p. 417].

Nowadays Tarski's concept of logical consequence is regarded as debatable. Tarski's work has more philosophical, nontechnical character and allows to interpret it in various conflicting ways, for example, there is an opinion that Tarski's definition is incorrect from the point of view of modern mathematical logic (see [24]) or that it should be generally rejected (see [30]). Van McGee in [86] has continued this attack. An interesting analysis of Tarski's work is proposed in [102], where he examines three basic concepts of logical inference, each of them envelops an important part of argument and each of them is accepted by logical community. The interesting conclusion of the author is that Tarski does not tell, what the logical consequence is, but considers what the logical consequence is similar to. Ray in [98] and Sher in [110] has defended Tarski's analysis against Etchemendy's criticisms in his big article (see the reply in [4] and [51]. Of particular interest is the article of Gómes-Torente (see [45] where the author discusses, analyses and defends from a historical perspective some of the aspects of Tarski's definition of logical consequence. As noted by Shapiro in [106, pp. 132, 148]: "There have been, and still are, a variety of characterizations of intuitive idea that a sentence (or proposition) Φ is a logical consequence of a set Γ of sentences (or propositions)", and he leads not an exhaustive list of definitions (with his ten definitions), beginning with Aristotle.

The debate continued in the 21st century (see [14], [111], [28], [88], [5], [108], [63], [77], [78], [31],[47], [89], [25], [101], and [104]). In the last work the author rejects the standard definition of logical consequence and suggests a sufficiently general form of the consequence relation between *abstract signs*.

The basic objections against Tarski's definition of the concept of logical consequence are as follows. Anywhere in [119] it is not stipulated that the data domain should vary, as it is in modern logic (see [24, p. 43]). Logical properties, in particular the general validity of the argument of logical consequence, should be independent of a separately selected universal set of reasonings, in which language appears interpreted. Otherwise, many statements about a cardinality of data domain at a special interpretation of language can be expressed only by means of logical constants and, as result, they should appear logically true. However, Tarski himself considers the idea of the term 'logic' as excluding from the logical truths any statements about a cardinality, let even of logical area. Another objection is directed against Tarski's acceptance of the ω-rule (the rule of infinite induction) at formalizing first-order arithmetic. However, actually it was only a version of this rule in the simple theory of types. In connection with these objections it is necessary to make some general notes. Tarski knew very well Gödel's works about the completeness, where the theorem is proved on the basis trueness of statements at all possible interpretations, as well as about the incompleteness (ω-incompleteness) of first-order arithmetic. In the first case one showed a concurrence of logical consequence in the first-order classical logic with syntactic consequence, in the second case one did not. From Tarski's works it clearly follows that he considers the logical consequence and deductability as different concepts and the first as much wider one than the second. The basic intention of Tarski was to define the logical inference, applicable to very wide class of languages, so wide that, as we shall see further, there are problems of the whole other level relating to the question ‘What is logic’.

For now note that the concept of logical consequence has taken the central place in logic and therefore the following problem seems to be very important: *What does this mean for the conclusion A to be inferred from premises Γ*? The following criterion is considered conventional: A follows from premises Γ if and only if any case, when each premise in Γ is true, is the case, when A is true. Significantly, the famous Russian logician Andrey Markov (the founder of constructive mathematics in USSR) connects this principle to the definition of what logic is: “Logic can be defined as a science about good methods of reasoning. By “good” methods of reasoning it is possible to mean ones, where from true premises we infer a true conclusion” (see [82, p. 5]). As a result, the essence of logical consequence is preservation *of truth* in all cases. There are many ways, when, using Tarski’s concept of logical consequence, it is possible to represent all laws of classical logic as valid. Thus, we obtain a standard definition of this logic together with all its logical operations. For instance, the conjunction of two formulas $A \wedge B$ is true at a situation (in a possible world) w if and only if A is true in w and B is true in w.

But we have much more problems there. Why do we call the obtained logic classical and what does this mean? We still consider this problem. What does ‘the standard setting of truth conditions for logical connectives’ mean? Finally, what should we consider as logical constants (operations)? The concept of truth is directly connected to the understanding of logical consequence, given by Tarski, and altogether results in objects which we call ‘logical laws’: *they are deductions preserving the truth*. But how can we define the logical law, not having defined what we should consider as logical constants, while we have a natural variability and instability of non-logical objects of reality. If we consider all objects as logical terms: variables, numbers, etc., then a model-theoretic interpretation of each term should be fixed and, therefore, only one model should exist. It would make the concept of logical truth empty.

4. Tarski in the end of his paper notes that the definition of the notion of logical consequence strictly depends on the distinction between logical and extra-logical constants. Because, if all the primitive terms are counted as logical constants, then logical consequence collapses into material consequence: A is consequence of Γ if and only if either A is true or at least one member of Γ is false. On the other hand, me must include the implication sign or the universal quantifier among the logical constants, otherwise “our definition of the concept of consequence would lead to results which obviously contradict ordinary usage” (see [119, p. 418]. Tarski writes that he does not know any objective basis for strict differentiation of these two groups of terms, and he concludes that this distinction between logical and extra-logical constants is the next big unsolved problem.

It is obvious that this problem did not give rest him and in thirty years he comes back to it in the lecture “What are logical notions?” read in 1966 in London Bedford College, in the same year in the Tbilisi Computer Center, and later in SUNY, Buffalo in 1973 (published posthumously in [120]). Tarski extends an area of discourse of applying Klein’s *Erlanger Program*, where one proposed a classification of various geometries in accordance with the space transformation, when geometrical concepts are invariant. For example, concepts of Euclid’s metric geometry are invariant relatively to isometric transformations. In the same way, algebra can be considered as study of concepts, invariant relatively to automorphisms of such structures as rings, fields, etc. The basic idea consists in that logical notions, i.e. sets, classes of sets, classes of classes of sets, etc., quantifiers, truth functions (implication, conjunction, disjunction, negation, etc.), should be “invariant under all possible one-one transformations of the world onto itself” (see [120, 149]). In other words, Tarski identifies logical notions with those notions that are invariant under all permutations of the universe of discourse (data domain). A similar idea had been previously maintained in [83]. Lindenbaum and Tarski in [73] showed that all logical notions from *Principia Mathematica* are invariant in this sense.

In one form or another an idea of an invariant permutability was discussed in various works in mathematical and philosophical logic (see [91], [92], [93], [84], [85], [114], [8], [109], [112], [87], [105], [32], [34], [125], [46], [7], [59], [21], [15], [76], and [16]). In the last work the authors come from the close connection between logical constants and logical consequence, and they investigate a function extracting the constants of a given consequence relation.

In [109, p. 53] is given a characterization of logical constants relatively isomorphic invariance which is a generalization of Tarski's approach. In the important work (see [87]), where criterion for logicality is invariance under bijections across universes, it is shown that if Tarski's thesis is accepted, then logical operations are defined in the full infinitary language $L_{\infty,\infty}$. Recall that the language $L_{\infty,\infty}$ is a language of conventional first-order logic (FOL) with equality (Frege's language), but admits conjunctions and disjunctions of an arbitrary length and as well as an arbitrary length of sequence of universal and existential quantifiers. This language is very rich – it contains the whole second-order logic (SOL), which is the extension of FOL by allowing quantifiers not just over individuals in the domain of discourse, but also over subsets of that domain and over relations and functions on the domain. Not only arithmetic, but also a set theory are included in SOL (natural numbers, sets, functions, etc. are there logical notions), as a result, all set-theoretic problematics, including the continuum hypothesis and many other important mathematical statements, are contained in SOL (see [81]). Thus, *mathematics is a part of logic*. Depending on expressive means of new logic, we come to logic of natural numbers, logic of real numbers, logic of topological spaces, etc. In the end, McGee accepted the Tarski-Sher thesis as a necessary condition for an operation across domains to count as logical, but not a sufficient one.

In connection with these problems Feferman's article [32] seems to be very interesting. In this article Feferman criticizes McGee's proposal and one of objections is that there is an assimilation of mathematics by logic. But the main objection is the following: "No natural explanation is given by it of what constitutes the same logical operation over arbitrary basic domains" (p. 37). The solution is to introduce invariance under mappings ("homomorphism invariance") instead of invariance under bijections. Such operations, according to Feferman, are logical and, it is the most remarkable, they exactly coincide with operations of the first-order logic without equality. However, here again there is a problem whether the equality may be considered as a logical operation? See the discussion of this problem in [96, pp. 61 ff], where Quine leans toward the positive answer. As a value of his approach, Feferman considers that the operations of FOL are defined in terms of homomorphic invariant operations of one-place type. Thus, he refers to [69], where the central role of one-place predicates in human thinking is shown by the example of the natural language.

Continuation of Feferman's ideas is the article [21], where the author characterizes the invariant operations as definable in a fragment of FOL. According to his notion of invariance, negation, arbitrary conjunctions and universal quantification are not invariant. As Casanovas notices, "… it is not easy to accept that universal quantification and conjunction are less logical than existential quantification and disjunction" (p. 37). On the other hand, it follows from his results that some particular forms of equality are invariant. Casanovas' work makes you think seriously about the criteria of invariance.

Now it is clear that the characterization of logical operations entails the characterization of the logic as a whole.

5. Note that the characterization of FOL can be given in terms of fundamental model-theoretic properties of the theory T in the first-order language. These properties are:

The compactness theorem (for countable languages). *If each finite set of propositions in T has a model*, *then T has a model*.

The compactness takes place, as only the finite set of premisses is used in deductions. This property was revealed by Kurt Gödel in 1930 in his paper about the completeness of FOL. One consequence of compactness is what is often called the upwards Löwenheim-Skolem theorem: *If T has an infinite model, then T has an uncountable model*.

The following property of FOL was proved earlier.

The (downward) **Löwenheim–Skolem theorem**. *If T has a model, then T has a countable model, too*.

Much later Lindström in [74] showed that these properties are characteristic for FOL in the following sense:

Lindström's theorem. The first-order logic is a maximal logic (closed under $\wedge$, $\neg$, $\exists$) which satisfies the compactness theorem and the (downward) Löwenheim-Skolem's theorem.

Lindström's paper became paradigmatic for the major researches in logic of the last quarter of the 20th century. In essence, Lindström's theorem defines FOL, more precisely FOL(=), in terms of its global properties. But a serious limitation on expressive means of FOL follows from these properties. The simplest infinite mathematical structure is constituted by natural numbers and the most fundamental mathematical concept is the concept of finiteness. However, from the theorem of compactness it follows that central concepts such as finiteness, countability, well-orderedness, etc. cannot be defined in first-order logic. Actually, the finiteness is not distinctive from the infiniteness. In turn, from Löwenheim-Skolem's theorems it follows that the first-order logic does not distinguish the countability from the uncountability and, hence, no infinite structure can be described up to isomorphism. Moreover, many linguistic concepts, distinctions and constructions are beyond applications of FOL (see [43][2] and [75]). Of course, FOL possesses such attractive properties as *soundness* (a soundness property assures us that a formal system is consistent) and *completeness* (a formal system is 'semantically' complete when all its valid formulas are theorems), but our knowledge is often inconsistent, incomplete, and nonmonotonic[3].

There is a lot of interesting logics, which are richer than the first-order logic such as the weak logic of the second order which tries to construct the concept of finiteness in logic in the natural way (it allows to quantify over finite sets); logics with various extra-quantifiers such as there exists finitely many', 'there exists infinitely many', 'majority', etc.; logics with formulas of infinite length; logics of the higher-order (see [11]). However, it doesn't matter how we extend FOL – in any case we lose either the property of compactness, or Löwenheim-Skolem's property, or both as well as we lose the interpolation property and in most cases completeness. However, Boolos (see [17]) protecting the second-order logic, asks: Why the logic should necessarily have the property of compactness? It is interesting that we find a similar question in 1994 on pages of 'The New Encyclopedia Britannica': Why Löwenheim-Skolem's property should correspond to the internal nature of logic? (Vol. 23, p. 250). In [99, p. 304] the author argues that "the lack of completeness theorem, despite being an interesting result, cannot be held against the status of SOL as a proper logic."

The construction of various extensions of FOL, especially logics with the generalized quantifiers, drew big attention of linguists, mathematicians, philosophers, cognitivists. A total of development of this direction is reflected in the fundamental work 'Model-Theoretic Logics' (see [2]), where Barwise comes to the following conclusion: "Mathematicians often lose patience with logic simply because so many notions from mathematics lie outside the scope of first-order logic, and they have been told that that *is* logic […] There is no going back to the view that logic is first-order logic." (p. 23). Shapiro in [107] is of the same opinion too. His book presents a formal development of second- and higher-order logic and an extended argument that higher-order formal systems have an important role to play in the philosophy and foundations of mathematics.

However, SOL is too complicated. Incompleteness of SOL means that this formal system does not properly capture logical consequence. The basic problems arise with logical truths. For example, there are statements which are logically true if and only if the generalized continuum hypothesis holds. All these difficulties and many other are an inevitable corollary of a huge potency of expressive means of second-order languages.

Probably, one of the most interesting extensions of FOL belongs to Hintikka [54]. He enveloped *independence friendly logic* (IF logic) which is an extension of FOL with existential quantifiers $\exists x/y$, meaning that a value for x is chosen independently of what has been chosen for y. IF logic has the same expressive power as existential second-order logic. Although IF logic shares a number of metalogical properties with FOL (among them Lindström's theorem), there are some important differences. Due to its greater expressive power, IF logic is not axiomatizable. It means that IF logic is semantically incomplete. On the other hand, IF logic admits a self-applied truth-predicate and possesses many other interesting properties (see [123], [80]. Pay attention to the papers with the title 'A Revolution in Logic?' (see [57]) and 'A Revolution in the Foundations of Mathematics?' (see [55]). However, Hintikka's proposal that IF logic and its extended version be used as new foundation of mathematics has been met with skepticism by some mathematicians, including Feferman [33].

6. Apparently, we should agree with Benthem and Doets (see [11, p. 235]) that "No specific theory is sacrosanct in contemporary logic." This point of view, the authors add, applies also to alternatives to classical logic (such as intuitionistic logic). In general, it is possible to consider it as the answer to Tharp's article 'Which logic is the right logic?" (see [122]).

It's worth stressing that the traditional approach to the understanding what logic is seems to be very attractive in respect to the possibility to define logic by means of its basic laws. As Frege wrote in 1893: "Laws of logic … are the most general laws, which prescribe universally the way in which one ought to think if one is to think at all" (see [38, p. 12]). Then one of the philosophical problem in foundation of logic is the critics of basic logical laws undertaken early in the beginning of the twentieth century by L. Brower (Law of the Excluded Middle), Vasilyev and Łukasiewicz (Law of Non-Contradiction). Different systems of intuitionistic and paraconsistent logics first appeared as the result of this process. Later Lewis criticized the main properties of material (classical) implication in 1912, and Ackermann rejected the properties of strict implication in 1956. Thus multiple systems of modal and relevant logic appeared. Subsequently, criticism of basic logical laws became total, and it is worth to say that by the 20th century none of the ever known classical laws remained undoubted. Even the implicational law of identity $A \rightarrow A$ does not bear the test of time. Since, according to E. Schroedinger, generally it has no place for microscopic objects. Such logics are called 'Schrödinger logics' (see [26]).

Eventually this led to the extreme diversity of non-classical directions in logic (see [40][4], ([49], [117], and [94]). Unexpected result of this process was the appearance of huge classes of new logical systems. Moreover, in the most cases cardinality of these classes equals to continuum. The first outcome of a similar sort belongs to [64] and concerns a cardinality of the class of extensions of intuitionistic logic. Also there are continual classes of Lewis' modal systems, relevant systems, paraconsistent systems and so on. Now the discovery of the continual classes of logics is the most ordinary thing (see [44]). In this work it is shown how continual families of logics are *normally* built and what corollaries can be obtained from the corresponding construction.

Recently discussion about the nature of logical consequence and the view that there is more than one 'correct' conception of logical consequence has given new impetus to the development of the idea of 'logical pluralism' (see [6]). In the paper [35] the attempt is made to maximally limit the scope of logical pluralism. As noted in [100], "historical discussions have usually presupposed that if one of the logics is correct, then that it is correct for all and everyone".

The unusual variety of logical systems and logical tools for proving theorems, the possibility of representation of the same system in different ways (Gilbert's style, natural deduction, sequent calculus, analytic tableaux, etc.)[5], and the fact that logic becomes more vital in the computer science, artificial intelligence, and programming led to the publication of the collected works (in England and in one year in the USA) with the title 'What is a logical system?' (see [39]). Generally speaking, the problem is formulated as follows: whether there is the one "true" logic and in the case if not, how we can limit our notion of logic or, more precisely, of a logical system?

In fact, everything looks much more difficult. On one hand, deadly criticism of "basic" laws of classical logic, on the other hand, almost unlimited extensions of the concept of the logical truth (in essence this process is inverse to the first), various specifications of the concept of logical consequence, and the same is about logical notions, evident inadequacy of formal-logical constructs in relation to the way in which the actual process of human reasoning takes place, serious problems (hardly explainable) that appear intuition of logic (see [126], [79]), the development of computer science and artificial intelligence – all of that points at the *global crisis* in the foundations of logic and clearly raises the question '***What is logic?***'

7. Exactly in hundred years after the appearance of Frege's well-known work '*Begriffsschrift*' (see [37]), in which predicates, negation, conditional, and quantifiers are introduced as the basis of logic, and also the idea of formal system is introduced, in which demonstration should be carried out by means of obviously formulated syntactic rules, – after hundred years of the triumphal development of logic as the independent science calling the worship, surprise, and occasionally bitter dismissal and even revenge for its former adherents and the mystical fear for the majority of others, suddenly there is Hacking's article under the title 'What is a logic?' (see [50]). Hacking highly evaluates Gentzen's introduction of structural rules, because the operation with them allows us to express the aspects of logical systems in which the role of constants is entirely given by their elimination and introduction rules, without any appeal to semantic notions. This important discovery is made by Gentzen in 1934. The presentation and development of logic by the way of sequent calculus, where the principles of deduction are set by the rules, permitting to pass from one statements about the deducibility to others, allowed Hacking to define logic as science about deduction. Therefore there are some reasons why Hacking's article is in the beginning of the above mentioned collected works [39].

Let us note that under the same title as Hacking's paper the works by several outstanding logicians have emerged (see Wang [124], Hodges [61], Hintikka and Sandu [58]). In these papers is gathered the big amount of historical, factual and analytical material concerning the great science aspiring to study the principles of correct reasoning. Of course, it is necessary to discuss the sphere of application and the limits of logic (see [65], [105], [127], [62], [56].

Not many working, qualified logicians think that logic is related to the laws of thought. In the second edition of HPL Hodges expands his paper devoted to elementary predicate logic from the first edition of HPL with the section 'Laws of Thought?', at the beginning of which he writes: "The question whether the sequent $p \wedge q \mid\!- q$ is valid has nothing more to do with mind than it has to do with the virginity of Artemis or the war in Indonesia" (see [60 p. 100]). Complete disappointment with the current state of logic is expressed in [9], when Benthem writes about himself: "who has taken a vow to study methods per se, chastely staying away from the wear and tear of the realities of reasoning." The prospects of the development of logic are also sketched there. Although, most logicians would agree with Van Benthem "if logical theory were *totally disjoint* from actual reasoning, it would be no use at all" [10, p. 69]. The same majority, let less emphatically, would agree with the normative role of logic, which, in the words of Feferman (see [32, p. 32), deals not with "how men actually reason but how they *strive* to do so" (italics mine).

To defend logic from accusations in psychologism, the logicians, starting from Charles Peirce and especially Gottlob Frege, have declared logic a normative discipline. This means that logic tells us how we *ought* to reason if we want to reason correctly. In the much talked-of book (see [52]) it is stated that logic is neither a normative nor a psychological theory. In other words, he has argued that actual reasoning, as “reasoned change in view”, has nothing to do with logic. To the criticism of Harman’s statements is devoted the paper [36], where it is explained why logic should be tied to norms of rationality. But the publication of an even more critical work (see [53] is already taken as a given if we consider where it is published.

Note the nice article [29][6], where the following questions are discussed:

(a) how do we reason?
(b) how ought we to reason?
(c) what justifies the way we ought to reason?

In the latter case the major role is given to the rules of introduction and elimination of logical constants as logical norms. Although in the context of very well founded concept of pluralism in logic, a serious problem arises. If logic is a normative discipline, then too many logical norms emerge. Engel’s paper ends with the following notable words: “The gap between logic and the psychology of reasoning is not, on my view, as large as it is often claimed to be” (p. 234).

The return of psychologism to logic is one of the most significant tendencies of the development of modern logic. Surprisingly, to this question is devoted the Special Issue of one of the world’s strictest logical journals (see [72]). Let us just reference the paper [10, p. 67], where Benthem talks about “understanding of ‘psychologism’ as a friend rather than an enemy of logical theory”.

The return to psychologism is not accidental. Recently an exceptional development was obtained in *informal logic*, the movement that was born in North America in the 1970s. Informal logic is usually associated with everyday discourse, critical thinking, reasoning in ordinary language, studying of informal inference, and so on (see survey [48] and book [103]). Apparently, the case is that it has always been implicitly assumed that logic studies not all reasonings indiscriminately, but only the reasonings related to logic, i.e. it studies the logical reasonings. But in that case a pure tautology comes out: logic studies logic. In summary, it is the time to ultimately dismantle this tautology.

Interesting are also the tendencies that arise within mathematical logic itself. In the first place, it is the extraction of the necessary minimum of logical means, which leads to maximal generalization and abstraction of logic itself. In the work [18] a notion of ‘abstract logic’ was put into use, where an abstract logic is defined as a pair <*A*, *Cn*> such, that *A* is a universal algebra and *Cn* is a consequence (alias ‘closure') operation on the carrier of A. A consequence operation *Cn* was introduced by Alfred Tarsky early in 1930.[7] In other terms, a Tarskian consequence relation is a binary relation (between sets of *L*-formulas and *L*-formulas), that satisfies the following conditions: *reflexivity*, *transitivity* and *monotonicity*. But nobody even tried to explain, why this topological closure operator’s properties should determine some “kernel” of human reasoning.

Due to the fact that monotonicity property is counter-intuitive it has to be abandoned (or discarded at all), if we want to give a formal account of *defeasible reasoning* (see [70]). Concerning that it is difficult to find solid arguments against the properties of reflexivity and transitivity (although possible, if desired), the following definition of logic is not surprising (see [116, p. 136]): an abstract logic is defined as a pair <*A*, *Cn*> such, that a consequence operation satisfies only reflexivity and transitivity, in other words, “a logic is simply a *preorder*” (italics mine).

Definition of abstract logic suggested by Suszko has received further generalization and led to the notion of ‘universal logic’ (see [12], [13]). A universal logic is defined as a pair <*S*, |–> where *S* is some structure without any specification, and |– is a relation on *S*. Notice, that unlike *Cn* operation |– is not constrained, i.e. no axioms are stated for the consequence relation |–. Béziau’s idea is that the relation of universal logic to all concrete logics is the same as of universal algebra to concrete algebras. Of course, the field of universal logic has arguably existed for many decades.

The term ‘abstract logic’ is also used in another sense, even contrary, maximally extending the notion of ‘logic’. Such are the ‘model-theoretic logics’ (see [2]) which consist of a collection of mathematical structures, a collection of formal expressions of a language used to describe properties of such structures, and a relation of satisfaction between the two. The basic notion is that of satisfaction: $M \models \varphi$ if the expression φ is *true of*, or satisfied by, the structure M. The rigorous definition of abstract logic under the name ‘general logics’ is given in [27, pp. 27-28]. The structures can be very rich and so the construction of expressions, describing the properties of said structures, has much more expressive power than language of first-order logic. Hence, the problem of logical constants is not significant here. As stressed by Barwise: “We are primarily interested in logics where the class of structures are those where some important mathematical property is built in, and where the language gives us a convenient way of formalizing the mathematician’s talk about the property” (see [3, pp. 4-5]). Note that the starting point for the study of abstract logics was Lindström’s theorem (see above), and FOL itself is its simplest example. It is interesting that in the third edition of the famous book (see [23]) the new section is introduced under the title ‘Lindström’s Chatracterization of First-order Logic’, which contains a definition of *abstract logic*, but more narrow than in [27].

As a whole, abstract logic associated with specific model-theoretic languages, aspires to overview the entire spectrum of logics. However, this tendency is also observed at the propositional level, where the main goal is set not as investigation of properties of a specific logic, no matter how interesting it is, but the whole classes of logics. The fourth chapter of the book ([22] is entitled ‘From Logic to Classes of Logics’, in it the classes of extensions of modal logics are regarded as lattices, and now the most important is the study of the properties of these lattices and various classes and subclasses of the elements of a given lattice, where the elements are logics themselves.

Currently the most impressive tendency of the development of modern logic is its intention towards unification of different logical systems and even whole movements. This phenomenon has received the name ‘combining logics’. In the first book on this topic (see [19]) are presented general methods for combining logics, lots of examples and some suggested applications, including ones in Computer Science, where knowledge representation frequently requires the integration of several logical systems into a homogeneous environment. See also overview [20][8].

The latter of pointed out tendencies allows us to make the assumption that if logic has any relation to human thought process, then the level of human formal logicality lays hidden behind the ‘functioning’ of infinite classes of different logical systems. Or, in other words, we are on our way to combined reasoning. However, one thing may be absolutely positively stated: various discussions concerning the status and basic principles of logic, its current tendencies of development, tell us that logic stands in the face of grandiose changes and fundamental discoveries await us.

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Notes

1. It is English translation of the German version by J. H. Woodger. English translation of the Polish version by M. Stroińska and D. Hitchcock see in [121].
2. L. T. F. Gamut was a collective pseudonym for the Dutch logicians Johan van Benthem, Jeroen Groenendijk, Dick de Jongh, Martin Stokhof and Henk Verkuyl: “Any logical system which is appropriate as an instrument for the analysis of natural language needs a much richer structure than first-order predicate logic” (see [43, p. 75]).
3. The property of monotonicity states that if sentence A is a consequence of the set Γ then it is also a consequence of any set containing Γ as a subset (see [1]. Meaningfully, monotonicity indicates that learning a new piece of information cannot reduce the set of what is known. Classical first-order logic and many non-classical logics are monotony.
4. 2nd and 3rd volumes of ‘Handbook of Philosophical Logic’ (HPL) are nothing else but the overview of various non-classical logics: in the 2nd volume are considered the extension of classical propositional logic, for example, such as modal, temporal, deontic logic and others, and in the 3rd volume – the alternatives to classical logic, for example, such as multi-valued, intuitionistic, relevant logic and others. In the second edition of HPL (see [41]) this division is removed and a lot of other lines of non-classical logics is added. On completely new tendency in philosophical logic see [113].

5. See also [71] where the author considers five styles of deductive systems.
6. The paper was firs published in Italian in 2001.
7. See [118]. Here Tarski find the unexpected use for the closure operator to study abstract consequence relation. This work is preceded by his papers on logical consequence (see above), which is not always acknowledged.
8. However, let us note that first examples of combined logical systems appeared in middle the 1950s, when Rasiowa in [97] has obtained a product of two-element matrix for classical equivalence and three-valued matrix for Łukasiewicz's implication and has given the axiomatization for the resulting new six-valued matrix. In turn, Prior in [95] gives the first examples of combined modal-temporal logics.