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STRESS-DILATANCY FOR SOILS. PART I: THE FRICTIONAL STATE THEORY

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Abstract: An unconventional subdivision of volumetric strains, the newly formulated frictional and critical frictional states and some of energetic and stress condition assumptions result in new stress-plastic dilatancy relationships. These new stress-plastic dilatancy relationships are functions of the deformation mode and drainage conditions. The critical frictional state presented in this paper is a special case of the classical critical state.

Key words: *soils, dilatancy, critical state, frictional state, critical frictional state*

NOTATION

A – slope of the stress-plastic dilatancy line in the $\eta - D^p$ plane
A^o – slope of the stress-natural dilatancy line in the $\eta - D^{pn}$ plane
D – dilatancy
D^p – plastic dilatancy
D^{pn} – natural dilatancy
e – void ratio
G – elastic shear modulus
J_2, J_3 – second and third invariants of the stress deviator
J_2^o, J_3^o – second and third invariants of the stress deviator at the reference state
$J_{\varepsilon 2}, J_{\varepsilon 3}$ – second and third invariants of the strain increment deviator
$J_{\varepsilon 2}^o, J_{\varepsilon 3}^o$ – second and third invariants of the strain increment deviator at the reference state
K – elastic bulk modulus
M^o – slope of frictional state line in the $q - p'$ plane at the critical frictional state
p' – mean normal effective stress
p^o – mean normal effective stress at the reference state
q – stress invariant
q^o – stress invariant at the reference state
Q – intercept of the stress-plastic dilatancy line in the $\eta - D^p$ plane at $D^p = 0$
M_c^o, M_e^o, M_b^o – slopes of the critical frictional state line in triaxial compression, triaxial extension and biaxial compression, respectively
α, β – soil parameters that characterise the stress-plastic dilatancy relationship
δ – increment
$\delta\varepsilon_\upsilon, \delta\varepsilon_\upsilon^e, \delta\varepsilon_\upsilon^p$ – total, elastic and plastic parts of the volumetric strain increment, respectively
$\delta\varepsilon_\upsilon^o = 0$ – volumetric strain increment at the reference state
$\delta\varepsilon_\upsilon^{pn}, \delta\varepsilon_\upsilon^{pa}$ – natural and additional plastic parts of the volumetric strain increment, respectively
$\delta\varepsilon_q, \delta\varepsilon_q^e, \delta\varepsilon_q^p$ – invariants of the total, elastic and plastic parts of the strain increment, respectively
$\delta\varepsilon_q^o$ – invariant of the strain increment at the reference state
$\delta\varepsilon_k, \delta\varepsilon_k^p$ $(k = 1, 2, 3)$ – principal total and plastic parts of the strain increment, respectively
$\delta\varepsilon_k^o$ $(k = 1, 2, 3)$ – principal strain increment at the reference state
η – stress ratio
θ – Lode angle for stress
θ^o – Lode angle for stress at the reference state
θ_ε – Lode angle for the strain increment
θ_ε^o – Lode angle for the strain increment at the reference state
κ – slope of the unloading/reloading line in the $e - \ln p'$ plane
ν – Poisson's ratio
σ_k' $(k = 1, 2, 3)$ – principal effective stress
σ_k^o $(k = 1, 2, 3)$ – principal effective stress at the reference state
ϑ – specific volume
Φ' – effective angle of shearing resistance
Φ'_{cv} – critical state angle of shearing resistance
Φ'_r – residual state angle of shearing resistance
Φ^o – critical frictional state angle of shearing resistance
χ_1, χ_2 – parameters of frictional state theory
$\delta\varepsilon_{ij}, \delta\varepsilon_{ij}^e, \delta\varepsilon_{ij}^p$ – components of the total, elastic and plastic parts of the strain increment, respectively
$\delta\varepsilon_{ij}^o$ – components of the strain increment at the reference state
$\delta e_{ij}, \delta e_{ij}^p$ – components of the total and plastic parts of the strain increment deviator
δe_{ij}^o – components of the plastic strain increment deviator at the reference state

σ'_{ij} – components of the effective stress

σ^o_{ij} – components of the effective stress at the reference state

s_{ij} – components of the stress deviator

s^o_{ij} – components of the stress deviator at the reference state

1. INTRODUCTION

Changes in soil volume may be generated by changes in the stress and shear strain (mechanical effects) and changes in the water content, the temperature and other factors (non-mechanical effects). The soil phenomenon in which shear deformation results in volume changes is called dilatancy, which was previously investigated by Reynolds, Casagrande and Taylor [32].

Based on energy considerations, Taylor [32] found a stress-dilatancy relationship for non-cohesive soils in the simple shear condition. The critical state concept and similar energetic considerations [27] resulted in a simple stress-dilatancy relationship for cohesive and non-cohesive soils in triaxial compression conditions; this was the basis of the original Cam clay model. The special energetic consideration investigated by Rowe [26] resulted in stress-dilatancy relationships for granular materials in triaxial compression, triaxial extension and biaxial compression conditions. Rowe's stress-dilatancy theory for an irregular assembly was examined by Horne [15], [16] and by De Josselin de Jong [9]. Gutierrez and Wang [13] developed a non-coaxial version of Rowe's stress-dilatancy relation. In the past, the stress-dilatancy relationship was broadly examined by many researchers, including Matsuoka [19], Tatsuoka [28], [29], Cornforth [6], Bolton [2], De Josselin de Jong [9], Coop [3], Cuccovillo and Coop [8], Coop and Wilson [4], Wanatowski [33], and others [1], [11], [14], [30], [31]. Cotecchia and Chandler [7] showed that the stress-dilatancy relationship for triaxial compression tests is different for drained and undrained conditions; this phenomenon has not been investigated until now.

The stress-dilatancy relationship is the basis of many soil models. The best-known models include Cam clay [25], [27], Modified Cam clay [24], Nova [21], Li and Dafalias [17], McDowell [20], and DeSimone and Tamagnini [10].

In this paper, the strain increment is classically divided into elastic and plastic parts based on the elasto-plasticity consideration. Furthermore, the volumetric plastic strain increment is arbitrarily divided into natural and additional parts. The natural volumetric strain increment is treated as (directly) accompanied by a plastic shear strain increment due to the granular nature of soil. The additional volumetric plastic strain increment is the difference between the total and natural plastic volumetric strain increments and depends on the initial structure and the stress and strain paths during shearing.

This assumption is necessary for describing the complex stress-strain behaviour of soils caused by soil structure degradation, breakage, rolling and sliding of soil grains and other effects during the deformation process.

Based on this assumption, the *frictional state* and *critical frictional state* are defined. The *critical frictional state* is a special case of the *critical state*. Treating the critical frictional state as a reference state and assuming some additional stress and energetic conditions, a general stress-plastic dilatancy relationship is established. The most widely known stress-dilatancy relationships are special cases of the derived stress-plastic dilatancy relationships. The full validation of the derived stress-dilatancy relationship based on experimental data is presented in Part II for triaxial conditions, Part III for plane strain conditions and Part IV for simple shear conditions.

In this paper, a positive contraction and compression convention is assumed.

2. GENERAL ASSUMPTIONS

Soil is treated as an isotropic continuum in this paper, and only monotonic loading is considered. Breakage, swelling, shrinkage, viscosity, temperature changes and other non-mechanical effects are neglected.

As in classical elasto-plasticity, the strain increment is divided into elastic and plastic components

$$\delta\varepsilon_{ij} = \delta\varepsilon^e_{ij} + \delta\varepsilon^p_{ij} \tag{1}$$

where the superscripts e and p refer to the elastic and plastic parts, respectively.

The behaviour in the elastic range is governed by Hooke's law

$$\delta\varepsilon^e_{\upsilon} = \frac{\delta p'}{K}, \tag{2}$$

$$\delta e^e_{ij} = \frac{\delta s_{ij}}{2G}, \tag{3}$$

where

$$\delta\varepsilon^e_{\upsilon} = \delta\varepsilon^e_{ii}, \tag{4}$$

$$\delta e_{ij}^{e} = \delta \varepsilon_{ij}^{e} - \frac{1}{3} \delta \varepsilon_{\upsilon}^{e} \delta_{ij} , \tag{5}$$

and G and K are the shear and bulk elastic moduli, respectively, which are generally functions of the void ratio, effective pressure and stress history.

Similar to critical state soil mechanics, it is assumed that

$$K = \frac{1+e}{\kappa} p' , \tag{6}$$

$$G = \frac{1-2\nu}{2(1+\nu)} K , \tag{7}$$

where ν is Poisson's ratio and κ is the slope of the unloading/reloading line in the $e - \ln p'$ plane.

In this paper, the volumetric part of the plastic strain increment $(\delta \varepsilon_{\upsilon}^{p})$ is divided into two parts: the natural part $(\delta \varepsilon_{\upsilon}^{pn})$ and the additional $(\delta \varepsilon_{\upsilon}^{pa})$ part

$$\delta \varepsilon_{\upsilon}^{p} = \delta \varepsilon_{\upsilon}^{pn} + \delta \varepsilon_{\upsilon}^{pa} . \tag{8}$$

The natural part of the volumetric strain increment is essentially connected with the rearrangement of soil grains during shearing that is specific to the soil. The volumetric strain increment due to debonding, breakage and other effects not essentially connected with shearing are treated as the additional part.

The theory presented in this paper is for medium and large strains in which the plastic part of the total strain increment is substantial.

This paper assumes the following definitions for dilatancy

$$D = \frac{\delta \varepsilon_{\upsilon}}{\delta \varepsilon_{q}} , \tag{9}$$

plastic dilatancy

$$D^{p} = \frac{\delta \varepsilon_{\upsilon}^{p}}{\delta \varepsilon_{q}^{p}} , \tag{10}$$

and natural dilatancy

$$D^{pn} = \frac{\delta \varepsilon_{\upsilon}^{pn}}{\delta \varepsilon_{q}^{p}} . \tag{11}$$

For the simplicity of further considerations, a linear relationship between D^{pn} and D^{p} is assumed

$$D^{pn} = \alpha + \beta D^{p} \tag{12}$$

where α and β are new soil parameters.

3. FRICTIONAL AND CRITICAL FRICTIONAL STATES

The *frictional state* is the state of shear deformation at which only natural volumetric strain exists ($\delta \varepsilon_{\upsilon}^{pa} = 0$, $\delta \varepsilon_{\upsilon}^{pn} \neq 0$, $D^{pn} = D^{p} \neq 0$) and the stress ratio $\eta = q/p'$ is a linear function of the natural dilatancy (Fig. 1)

$$\eta = M^{o} - A^{o} D^{pn} . \tag{13}$$

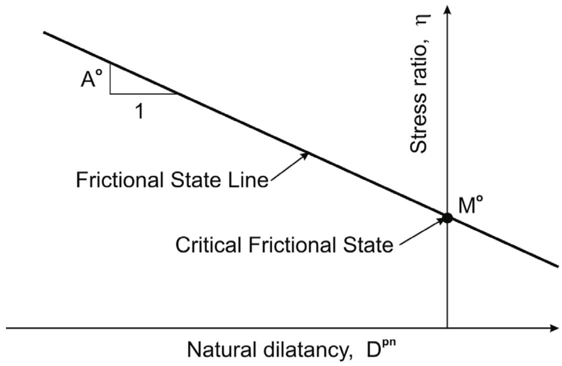

Fig. 1. Stress ratio and natural dilatancy relationship for the *frictional state*

In the $\eta - D^{pn}$ plane, the *frictional state* for constant A^{o} is represented by a straight line that is named the *frictional state line*, as shown in Fig. 1; Coop and Wilson [4] named this line the *frictional trend line*.

The parameters M^{o} and A^{o} are characteristic values for a soil's dependence on the mode of deformation and the drainage condition considered later in this paper.

The *critical frictional state* is the state at which the soil deforms with a stable structure without any change in the stress or in any component of the volumetric strain. The soil structure is fully erased during the previous deformation process, so the *critical frictional state* can be treated as a special case of the *critical state*. At the *critical state*, the soil deforms at a constant stress and volume. At this state, some components of the volumetric strain increment may not vanish but instead be counter-balanced, resulting in a constant volume. Similarly, at the residual state, the stress is constant, but some volumetric deformation may be observed [5], [18].

Similar to the critical state in the q–p' plane, the *critical frictional state* is represented by a straight line with a slope of M^{o} to the horizontal p' axis (Fig. 2). The existence of the *critical frictional state line* in the $e - \ln p'$ plane is not considered in this paper.

4. CURRENT AND REFERENCE STATES

The current state of plastic flow is defined by the current effective stress tensor (σ'_{ij}) and the plastic strain increment tensor $(\delta\varepsilon^p_{ij})$. Similarly, the reference state of plastic flow is defined by the stress tensor (σ^o_{ij}) and the strain increment tensor $(\delta\varepsilon^o_{ij})$, where the superscript ^{o} indicates the reference state.

For isotropic soils, it is convenient to use the principal values of the stress and plastic strain increment tensors, σ'_k, σ^o_k, $\delta\varepsilon^p_k$, and $\delta\varepsilon^o_k$ ($k = 1, 2, 3$), and their invariants, p', q, θ, p^o, q^o, θ^o, $\delta\varepsilon^p_\upsilon$, $\delta\varepsilon^p_q$, θ_ε, $\delta\varepsilon^o_\upsilon$, $\delta\varepsilon^o_q$ and θ^o_ε.

The principal values of the stress and plastic strain increments [12] are

$$\sigma'_k = p' - \frac{2}{3}q\sin\left\{\theta + \frac{2}{3}(k-2)\pi\right\}, \qquad (14)$$

$$\sigma^o_k = p^o - \frac{2}{3}q^o\sin\left\{\theta^o + \frac{2}{3}(k-2)\pi\right\}, \qquad (15)$$

$$\delta\varepsilon^p_k = \frac{1}{3}\delta\varepsilon^p_\upsilon - \delta\varepsilon^p_q\sin\left\{\theta_\varepsilon + \frac{2}{3}(k-2)\pi\right\}, \qquad (16)$$

$$\delta\varepsilon^o_k = \frac{1}{3}\delta\varepsilon^o_\upsilon - \delta\varepsilon^o_q\sin\left\{\theta^o_\varepsilon + \frac{2}{3}(k-2)\pi\right\} \qquad (17)$$

where $k = 1, 2, 3$

$$p' = \frac{1}{3}\sigma'_{kk},\quad q = \sqrt{3J_2},\quad \theta = \frac{1}{3}\sin^{-1}\left(-\frac{3\sqrt{3}\,J_3}{2\,J_2^{3/2}}\right),$$

$$p^o = \frac{1}{3}\sigma^o_{kk},\quad q^o = \sqrt{3J^o_2},\quad \theta^o = \frac{1}{3}\sin^{-1}\left(-\frac{3\sqrt{3}J^o_3}{2\,J_2^{o\,3/2}}\right),$$

$$\delta\varepsilon^p_\upsilon = \delta\varepsilon^p_{kk},\quad \delta\varepsilon^p_q = \sqrt{\frac{4}{3}J_{\varepsilon 2}},\quad \theta_\varepsilon = \frac{1}{3}\sin^{-1}\left(-\frac{3\sqrt{3}\,J_{\varepsilon 3}}{2\,J_{\varepsilon 2}^{\;3/2}}\right),$$

$$\delta\varepsilon^o_\upsilon = \delta\varepsilon^o_{kk},\quad \delta\varepsilon^o_q = \sqrt{\frac{4}{3}J^o_{\varepsilon 2}},\quad \theta^o_\varepsilon = \frac{1}{3}\sin^{-1}\left(-\frac{3\sqrt{3}J^o_{\varepsilon 3}}{2\,J^{o\;3/2}_{\varepsilon 2}}\right),$$

and

$$J_2 = \frac{1}{2}s_{ij}s_{ij},\quad J_3 = \frac{1}{3}s_{ij}s_{jk}s_{ki},\quad s_{ij} = \sigma'_{ij} - p'\delta_{ij},$$

$$J^o_2 = \frac{1}{2}s^o_{ij}s^o_{ij},\quad J^o_3 = \frac{1}{3}s^o_{ij}s^o_{jk}s^o_{ki},\quad s^o_{ij} = \sigma^o_{ij} - p^o\delta_{ij},$$

$$J_{\varepsilon 2} = \frac{1}{2}\delta e^p_{ij}\delta e^p_{ij},\quad J_{\varepsilon 3} = \delta e^p_{ij}\delta e^p_{jk}\delta e^p_{ki},$$

$$\delta e^p_{ij} = \delta\varepsilon^p_{ij} - \frac{1}{3}\delta\varepsilon^p_\upsilon\delta_{ij},$$

$$J^o_{\varepsilon 2} = \frac{1}{2}\delta e^o_{ij}\delta e^o_{ij},\quad J^o_{\varepsilon 3} = \delta e^o_{ij}\delta e^o_{jk}\delta e^o_{ki},$$

$$\delta e^o_{ij} = \delta\varepsilon^o_{ij} - \frac{1}{3}\delta\varepsilon^o_\upsilon\delta_{ij}.$$

Only the deformation processes for which $\sigma'_1 \ge \sigma'_2 \ge \sigma'_3$ ($-\pi/6 \le \theta \le \pi/6$) and $\delta\varepsilon^p_1 \ge \delta\varepsilon^p_2 \ge \delta\varepsilon^p_3$ ($-\pi/6 \le \theta_\varepsilon \le \pi/6$) are considered.

In this paper, it is assumed that the current and reference states of plastic flow fulfil the four conditions listed below:

(C1) *The reference state is a critical frictional state.*

(C2) *The stress tensors* σ'_{ij} *and* σ^o_{ij} *are coplanar.*

(C3) *One of the current principal stresses is equal to the appropriate principal reference stress* $(\sigma'_k = \sigma^o_k)$.

(C4) *The natural dilatancy part of the work done in the current plastic flow state is equal to the work done in the appropriate reference state.*

Condition (C1) states that

$$\delta\varepsilon^o_\upsilon = 0,\quad q^o/p^o = M^o. \qquad (18)$$

It is also postulated that the reference state tensor (σ^o_{ij}) fulfils the Mohr–Coulomb criterion

$$M^o = g(\theta)M^o_c \qquad (19)$$

where

$$M^o_c = \frac{6\sin\Phi^o}{3-\sin\Phi^o}, \qquad (20)$$

is the value of M^o for triaxial compression ($\theta = \pi/6$) and

$$g(\theta) = \frac{3-\sin\Phi^o}{2\{\sqrt{3}\cos\theta - \sin\Phi^o\sin\theta\}} \qquad (21)$$

where Φ^o is the angle of shearing resistance at the *critical frictional state*. The experimental [4] and theoretical micromechanical considerations prove that for some granular soils, an unexpectedly small value for the angle of shear resistance may be observed [22], [23]. Generally, this paper assumes that Φ^o may differ from Φ'_{cv} or Φ'_r.

For triaxial extension conditions at the *critical frictional state* ($\theta = -\pi/6$)

$$M^o = M_e^o = \frac{6\sin\Phi^o}{3+\sin\Phi^o}. \quad (22)$$

Condition (C2) is equivalent to

$$p' = (1+\chi_1)\, p^o, \quad (23)$$

$$q = (1+\chi_2)\, q^o, \quad (24)$$

$$\theta = \theta^o. \quad (25)$$

In the q–p' plane, the reference and current states are schematically shown in Fig. 2.

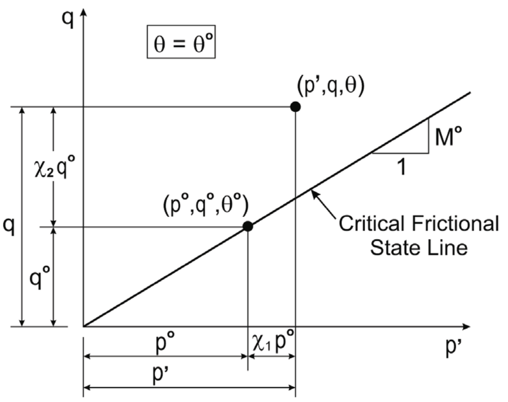

Fig. 2. Reference and current states in the q–p' plane

The two scalar coefficients χ_1 and χ_2 describe the difference between the current and reference stresses.

Condition (C3), after combining with conditions (C1) and (C2) and applying some simple algebra, may be written in the form

$$\chi_1 = \frac{2}{3}\chi_2 M^o \sin\left\{\theta + \frac{2}{3}(k-2)\pi\right\}. \quad (26)$$

Thus, for

$$k=1, \quad \sigma'_1 = \sigma_1^o, \quad \chi_1 = \frac{2}{3}\chi_2 M^o \sin\left(\theta - \frac{2}{3}\pi\right), \quad (27)$$

$$k=2, \quad \sigma'_2 = \sigma_2^o, \quad \chi_1 = \frac{2}{3}\chi_2 M^o \sin\theta, \quad (28)$$

$$k=3, \quad \sigma'_3 = \sigma_3^o, \quad \chi_1 = \frac{2}{3}\chi_2 M^o \sin\left(\theta + \frac{2}{3}\pi\right). \quad (29)$$

Condition (C4) may be written in the form

$$p'\delta\varepsilon_{\upsilon}^{pn} + q\delta\varepsilon_q^p \cos(\theta-\theta_\varepsilon) = q^o \delta\varepsilon_q^p \cos(\theta-\theta_\varepsilon), \quad (30)$$

because $\delta\varepsilon_{\upsilon}^o = 0$, $\delta\varepsilon_q^o = \delta\varepsilon_q^p$, and $\theta_\varepsilon^o = \theta_\varepsilon$.

Energetic equation (30) is similar to the Cam clay model energetic equation [27]. Comparing equations (26) and (30), the coefficients χ_1 and χ_2 have the forms

$$\chi_1 = -\frac{2}{3}\frac{\sin\left[\theta + \frac{2}{3}(k-2)\pi\right] D^{pn}}{\cos(\theta-\theta_\varepsilon) + \frac{2}{3}\sin\left[\theta + \frac{2}{3}(k-2)\pi\right] D^{pn}}, \quad (31)$$

$$\chi_2 = -\frac{D^{pn}}{M^o\left\{\cos(\theta-\theta_\varepsilon) + \frac{2}{3}\sin\left[\theta + \frac{2}{3}(k-2)\pi\right] D^{pn}\right\}}. \quad (32)$$

Thus, parameters χ_1 and χ_2 are functions of k, the natural dilatancy D^{pn}, the value of M^o and the non-coaxiality angle $(\theta-\theta_\varepsilon)$ defined by Gutierrez and Ishihara [12].

5. GENERAL STRESS-PLASTIC DILATANCY EQUATION

The stress ratio takes the form

$$\eta = \frac{q}{p'} = \frac{(1+\chi_2)q^o}{(1+\chi_1)p^o} = \frac{1+\chi_2}{1+\chi_1}M^o. \quad (33)$$

Introducing χ_1 and χ_2, as defined by equations (31) and (32), into equation (33), we obtain the stress-natural dilatancy relationship shown in equation (13), with M^o defined by equation (19) and

$$A^o = \frac{1}{\cos(\theta-\theta_\varepsilon)}\left\{1 - \frac{2}{3}M^o \sin\left[\theta + \frac{2}{3}(k-2)\pi\right]\right\}. \quad (34)$$

Additionally, this paper assumes that $\sigma'_3 = \sigma_3^o$ ($k = 3$) for the drained condition and that $\sigma'_1 = \sigma_1^o$ ($k = 1$) for the undrained condition.

The characteristic values of the stress-natural dilatancy relationships, i.e., θ, θ_ε, M^o and A^o, for the triaxial and biaxial compression and triaxial extension tests are summarized in Table 1.

Table 1. Characteristic values of the stress-natural dilatancy relationship

<table>
<tr><th rowspan="3" colspan="2">Values</th><th colspan="6">Test</th></tr>
<tr><th colspan="2">Triaxial compression</th><th colspan="2">Triaxial extension</th><th colspan="2">Biaxial compression</th></tr>
<tr><th>Drained</th><th>Undrained</th><th>Drained</th><th>Undrained</th><th>Drained</th><th>Undrained</th></tr>
<tr><td colspan="2">θ</td><td colspan="2">π/6</td><td colspan="2">–π/6</td><td colspan="2">θ_b (*)</td></tr>
<tr><td colspan="2">θ_ε</td><td colspan="2">π/6</td><td colspan="2">–π/6</td><td colspan="2">$\theta_{\varepsilon b}$ (*)</td></tr>
<tr><td colspan="2">M^o</td><td colspan="2">$M_c^o = \frac{6\sin\Phi^o}{3-\sin\Phi^o}$</td><td colspan="2">$M_e^o = \frac{6\sin\Phi^o}{3+\sin\Phi^o}$</td><td colspan="2">$M_b^o = \frac{3\sin\Phi^o}{\sqrt{3}\cos\theta_b - \sin\Phi^o \sin\theta_b}$</td></tr>
<tr><td rowspan="3">A^o</td><td>$k=1$</td><td></td><td>$A_c^o = 1 + \frac{2}{3}M_c^o$</td><td rowspan="2"></td><td rowspan="2">$A_e^o = 1 + \frac{1}{3}M_e^o$</td><td></td><td>$A_b^o = \frac{\left\{1 - \frac{2}{3}M_b^o \sin\left(\theta_b - \frac{2}{3}\pi\right)\right\}}{\cos(\theta_b - \theta_{\varepsilon b})}$</td></tr>
<tr><td>$k=2$</td><td rowspan="2">$A_c^o = 1 - \frac{1}{3}M_c^o$</td><td rowspan="2"></td><td></td><td></td></tr>
<tr><td>$k=3$</td><td>$A_e^o = 1 - \frac{2}{3}M_e^o$</td><td></td><td>$A_b^o = \frac{\left\{1 - \frac{2}{3}M_b^o \sin\left(\theta_b + \frac{2}{3}\pi\right)\right\}}{\cos(\theta_b - \theta_{\varepsilon b})}$</td><td></td></tr>
</table>

Note: (*) – angles θ_b and $\theta_{\varepsilon b}$ change during shearing.

Assuming a linear relation between D^{pn} and D^p, as shown in equation (12), equation (13) as the form [7]

$$\eta = Q - AD^p \tag{35}$$

where

$$Q = M^o - \alpha A^o, \tag{36}$$

$$A = \beta A^o. \tag{37}$$

Equation (35) is the general stress-plastic dilatancy relationship for soils.

The family of straight lines in the $\eta - D^p$ plane represents the stress-plastic dilatancy relationships generated by different sets of α and β parameters (Fig. 3) for a constant non-coaxiality angle $(\theta - \theta_\varepsilon)$.

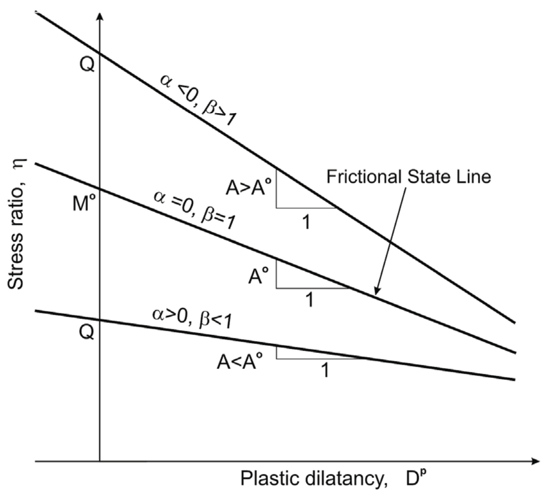

Fig. 3. Stress-plastic dilatancy relationships for different α and β parameters

Cotecchia and Chandler [7] showed that $A < 1$ for the drained and $A > 1$ for the undrained triaxial compression condition. According to the theory of the *frictional state* presented in this paper, $A^o < 1$ for the drained condition, and $A^o > 1$ for the undrained condition.

For the drained condition, the elastic parts of the strains are usually neglected, and the stress-dilatancy relationship ($\eta - D$) is presented in the literature instead of the stress-plastic dilatancy relationship ($\eta - D^p$).

The validation of the stress-plastic dilatancy relationship, as shown in equation (35), based on experimental data presented in the literature and a comparison with the best known relationships will be presented later in Parts II, III and IV of this paper.

6. CONCLUSIONS

The subdivision of the plastic part of the volumetric strain increment into the natural and the additional parts offers the possibility of finding new stress-plastic dilatancy relationships for soils.

In both the *critical state* and the *critical frictional state*, soil can deform at constant stress and constant volume. However, only in the *critical frictional state* does soil deform with a stable structure.

Based on some energetic and stress state assumptions, new stress-plastic dilatancy relationships are developed. The new stress-plastic dilatancy relationships are functions of the deformation mode and the drainage conditions. The influence of the drainage conditions on the stress-dilatancy relationship has not been theoretically considered until now.

The stress-plastic dilatancy relationships developed in this paper will be validated based on experimental data presented in the literature in the next parts of this paper.

Specially designed experiments and deeper theoretical considerations must be conducted in the future to fully validate the results presented in this paper.

The newly formulated stress-plastic dilatancy relationships offer new possibilities for soil modelling.

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