ON SOME METHODS IN SAFETY EVALUATION IN GEOTECHNICS

WOJCIECH PUŁA, ŁUKASZ ZASKÓRSKI

Wrocław University of Technology, Faculty of Civil Engineering, Wrocław, Poland,
e-mail: wojciech.pula@pwr.edu.pl, lukasz.zaskorski@pwr.edu.pl

Abstract: The paper demonstrates how the reliability methods can be utilised in order to evaluate safety in geotechnics. Special attention is paid to the so-called reliability based design that can play a useful and complementary role to Eurocode 7. In the first part, a brief review of first- and second-order reliability methods is given. Next, two examples of reliability-based design are demonstrated. The first one is focussed on bearing capacity calculation and is dedicated to comparison with EC7 requirements. The second one analyses a rigid pile subjected to lateral load and is oriented towards working stress design method. In the second part, applications of random field to safety evaluations in geotechnics are addressed. After a short review of the theory a Random Finite Element algorithm to reliability based design of shallow strip foundation is given. Finally, two illustrative examples for cohesive and cohesionless soils are demonstrated.

Key words: *reliability index, random field, Eurocode 7*

1. INTRODUCTION

Safety concepts in geotechnics are based on an appropriate evaluation of soil properties as well as loads and then introducing them to a design process. Basic rules are given by Eurocodes in the European Union and the geotechnical Eurocode 7 [16] is one of them. However, one of the main concerns of geotechnics is the way of considering the uncertainty of soil properties during a designing process. The complexity of the issue is related to the various factors influencing this uncertainty. Sources of uncertainty of soil parameters are mostly associated with natural spatial variability of soil properties, measurement errors, insufficient number of in situ tests and uncertainty of transformation. The influence of the uncertainty connected with the three last sources can be decreased because human can affect them. In the case of spatial variability, it is unpredictable and not possible to control. Geotechnical designers in such circumstances can only try to describe the random character of shear strength parameters in the most effective (realistic) way. One of the existing approaches that gained recognition is an application of the random field theory.

The objective of this paper is to demonstrate the usefulness of safety evaluations based on probabilistic methods with reference to EC7 approaches. In the first part (Sections 2–4) the main emphasis is put on safety evaluations yielding from so-called first order reliability method (FORM) and second order reliability method (SORM). The second part shows the profitability of random fields theory in the evaluation of geotechnical safety.

2. DESIGN APPROACHES IN EC7 AND RELIABILITY BASE DESIGN

The limit state design approach (LSD) introduced by Eurocodes represents a sort of revolution with respect to the working stress design (WSD) approach which was earlier commonly used all over Europe. Two main methodological differences between LSD and WSD can be pointed out:
- The WSD employs global safety factors calibrated by the different design experiences performed throughout European countries whereas the LSD considers partial safety factors calibrated by statistical approaches and applied to action and strength distributions.
- The LSD introduces "characteristic values" concept for design values of loads and strengths while the WSD deals with "nominal values" for design parameters without taking care of how they are determined.

The relationships between characteristic and design values established by EC7 are based on three design

approaches DA1, DA2 and DA3 applied if ultimate limit states are under consideration. The suggested partial safety factors are collected into three groups, namely partial factors associated with actions or results of actions (γ_F), partial factors for soil properties (γ_M), partial factors dedicated to resistance (γ_R). Partial safety factor values are presented in Table 1. In the design approach DA1 two sets of partial factors are recommended: a combination DA1.C1 − A1+M1+R1 and a combination DA1.C2 − A2+M2+R1. Design approach DA2 contains sets A1, M1 and R2. Finally, the approach DA3 denotes partial safety factors: A1, M2 and R3. In the Polish National Annex [45] the DA2* is distinguished as a variation of DA2. In the case of design approach DA2* characteristic values of actions should be applied instead of design values when computing resistance.

Table 1. Partial safety factors established by EC7

Partial factors for permanent and variable actions γ_F				
Actions		Symbol	Set A1	Set A2
Permanent	Unfavorable	γ_G	1.35	1.0
	Favorable		1.0	1.0
Variable	Unfavorable	γ_Q	1.5	1.0
	Favorable		0	0
Partial factors for soil properties γ_M				
Soil parameters		Symbol	Set M1	Set M2
Friction angle		γ'_Φ	1.0	1.25
Cohesion		γ'_C	1.0	1.25
Soil unit weight		γ_γ	1.0	1.40
Partial factors for resistance γ_R				
Resistance	Symbol	Set R1	Set R2	Set R3
Bearing capacity	$\gamma_{R,v}$	1.0	1.4	1.0

No probabilistic methods are suggested by EC7 in order to calibrate partial safety factors presented in Table 1. As regards characteristic value, Clause 2.4.5.2(10) of EC7 defines it as being "selected as a cautious estimate of the value affecting the occurrence of the limit state". Furthermore, Clause 2.4.5.2(10) of EC7 states that statistical methods can be applied when selecting characteristic values, but they are not mandatory. On the other hand, Eurocode 0 (EC0) [17] gives general rules regarding safety requirements for various types of structures (including those considered in geotechnics). Its guidelines are formulated as minimal reliability indices depending on the type of structure and reference period. Clause 1.1.1(1) of EC7 states that "EC7 is intended to be used in conjunction with EC0, which establishes the principles and requirements for safety and serviceability, describes the basis of design and verification

and gives guidelines for related aspects of structural reliability".

It should be emphasised that reliability indices can be evaluated solely by applying probabilistic tools. The reliability index β is associated to probability of failure by the following relationship

$$p_F = \Phi_0(-\beta), \qquad (1)$$

where Φ_0 is one-dimensional standard Gaussian probability cumulative function. Therefore, designing a certain structure on selected reliability index means designing on equivalent "target" probability of failure. It is worth mentioning that in North America approach called as load and resistance factor design (LRFD) is used in practice. This approach, however, allows for more probabilistic evaluations than Eurocode 7.

3. METHODS OF RELIABILITY INDEX EVALUATION

Usually structural and geotechnical reliability problems are presented in the form of so-called limit state function $g(\mathbf{x})$. The argument $\mathbf{x}$ of the function g is a random vector $\mathbf{X} = (X_1, X_2, ..., X_n)$ consisting of basic random variables defining loads, material properties, geometrical quantities, etc., as well as some other properties considered as deterministic. The limit state function is defined such that it takes non-negative values if the argument $\mathbf{x}$ is in "a safe domain" and negative values otherwise ($\mathbf{x}$ in a failure domain). The first concepts of reliability index come from Cornell [11], [12]. However, most of the contemporary reliability evaluations are based on index proposed by Hasofer and Lind [32] that is defined as

$$\beta_{HL} = \min_{\mathbf{x} \in \{g(\mathbf{x})=0\}} \sqrt{(\mathbf{x} - \mathbf{E}\{\mathbf{X}\})^{\mathbf{T}} \mathbf{C}_\mathbf{X}^{-1} (\mathbf{x} - \mathbf{E}\{\mathbf{X}\})} \quad (2)$$

where $\mathbf{E}\{\mathbf{X}\}$ is the expected value of the vector $\mathbf{X}$, $\mathbf{C}$ is the covariance matrix and the hypersurface corresponding to the equation $g(\mathbf{x}) = 0$ is called the limit state surface. The most important step was done by conjunction of the Hasofer–Lind concept with some probability transformation that allowed the probability distribution of vector $\mathbf{X}$ to be taken into account. This was possible by utilizing the Rackwitz–Fiessler algorithm [50] and so-called normal tail approximation [14]. Consequently, these new approaches allow probability of failure evaluation. The next development enables elaboration of the First Order Reliability Method (FORM) and the Second Order Reliability Method (SORM) [34] that

are briefly outlined below. The probability of failure is used as a reliability measure

$$p_F = \int_{\{g(x)<0\}} f_{\mathbf{X}}(x)dx .$$ (3)

Here, $f_{\mathbf{X}}$ denotes a multidimensional joint probability density function (pdf) of the random vector $\mathbf{X}$. In the special case if $\mathbf{X}$ is a Gaussian random vector with uncorrelated components X_i, $i = 1, ..., n$, the linear transformation of the coordinate system, known as the standardisation, is convenient to use

$$y_i = \frac{x_i - \mathrm{E}(X_i)}{\sigma_{X_i}}, \quad i = 1, ..., n,$$ (4)

where $\mathrm{E}(X_i)$ is the expected value and σ_{X_i} denotes the standard deviation of the random variable X_i, respectively. The corresponding mapping of the limit state surface $g(\mathbf{x}) = 0$ is as follows

$$G(\mathbf{y}) \equiv g(\mathbf{x}(\mathbf{y})) = 0 .$$ (5)

If moreover, the limit state function g is a linear one, then G will remain linear. By utilizing the property that the family of Gaussian probability distributions is closed with respect to linear combinations, it could be easily demonstrated that [46],

$$p_F = \int_{\{G(\mathbf{y})<0\}} \phi_n(\mathbf{y})d\mathbf{y} = \Phi_0(-\beta) ,$$ (6)

(provided that $p_F < 0.5$) where ϕ_n is n-dimensional standard Gaussian probability density function, Φ_0 is one-dimensional standard Gaussian probability cumulative function, and β is the distance of the hyperplane $G(\mathbf{y}) = 0$ from the origin called the reliability index (see equation (1)). This result is schematically presented in Fig. 1.
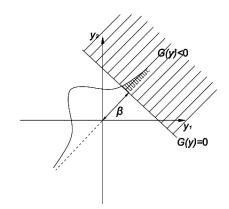
In the most practically interesting cases if either non-Gaussian probability densities or non-linear limit state functions appear the exact value of p_F is hardly obtainable. If distributions are non-Gaussian then a transformation to the standard normal space is necessary. As a result, a new Gaussian vector is obtained. Under some mathematical assumptions such a transformation always exists [52]. Next the point $\mathbf{y}^*$ that minimizes the distance of the limit state surface $G(\mathbf{y}) = 0$ from the origin in the standard normal space has to be found. This yields the following optimization problem

$$\text{minimize } \|y\|, \text{ subjected to } G(\mathbf{y}) = 0,$$ (7)

where $\|.\|$ denotes the Euclidean norm. Several sophisticated algorithms for this problem were developed. A critical review of them can be found in [37]. The point $\mathbf{y}^*$ is called the design point and its distance from the origin is denoted by β (therefore the point $\mathbf{y}^*$ is sometimes called the beta-point). Finally, the limit state surface in the standard normal space is replaced with the tangent hyperplane at the point $\mathbf{y}^*$ and the probability of failure is evaluated by the following approximate form

$$p_F \approx \int_{\{\nabla G(\mathbf{y}-\mathbf{y}^*)<0\}} \phi_n(\mathbf{y})d\mathbf{y} = \Phi_0(-\beta) .$$ (8)

The details can be found in [15], [34] and [46]. The idea of the above approximation is illustrated in Fig. 2. Comparison of equation (8) and equation (1) demonstrates the fundamental role of the reliability index β.
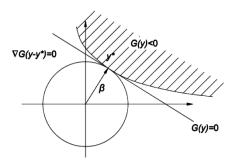

Fig. 2. Graphical presentation of limit state surface approximation according to the first-order reliability method

Having in mind the form on the n-dimensional normal pdf it is clear that point $\mathbf{y}^*$ is the most probable point in the transformed failure region and therefore it reflects the most probable failure combination of parametric values.

Fig. 1. Graphical representation of β geometrical meaning in two-dimensional case

In the SORM approximation, the limit state surface is fitted with a quadratic surface in the vicinity of the design point $\mathbf{y}^*$. The results were obtained by Tvedt [55] (exact result) and by Breitung [5]. Based on their results Hochenbichler and Rackwitz [35] proposed a correction factor to the failure probability, namely

$$p_F \approx \Phi_0(-\beta)\prod_{i=1}^{n-1}\left(1 - \frac{\phi_0(-\beta)}{\Phi_0(-\beta)}\kappa_i\right)^{\frac{1}{2}} = p_{F\,\mathrm{SORM}} \quad (9)$$

where κ_i are the main curvatures of the surface $G(\mathbf{y}) = 0$ at the design point $\mathbf{y}^*$ and ϕ_0 is the one-dimensional standard normal pdf. This gives the value of $p_{F\,\mathrm{SORM}}$. Next the reliability index β_{SORM} can be computed by inverting the following relationship

$$\beta_{\mathrm{SORM}} = -\Phi_0^{-1}(p_{F\,\mathrm{SORM}}). \quad (10)$$

The methods described in this section are classical methods in structural reliability. However, due to their mathematical complexity and requirement of specially dedicated software, through many years they were not very often used to safety assessments in geotechnics. The situation improved when FORM and SORM techniques had been coded as spreadsheet applications. The spreadsheet-automatic constrain optimization FORM computational approach was proposed by Low and Tang [38], [39]. More comments on FORM/SORM spreadsheet methods can be found in recent paper by Low and Phoon [40]. Some files illustrating the [38], [39] open-source approaches are available at http://alum.mit.edu/www/bklow.

4. EXAMPLES

In the present section, two examples of reliability based evaluation oriented toward design are presented. First example is compared with some EC7 results. The second one is confronted with WSD method.

4.1. EXAMPLE 1. RETAINING WALL

The example presented below in its deterministic part is based on the example given in Tomlinson's textbook [54]. The probabilistic approach follows the considerations given by Low and Phoon [40]. Tomlinson determines the factor of safety against bearing capacity failure of a retaining wall that is shown in Fig. 3.
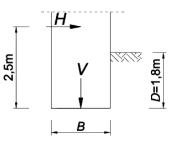

Fig. 3. The retaining wall analyzed in the example

The retaining wall is subjected to a horizontal load $H = 300$ kN/m applied at a point 2.5 m above the base and a centrally applied vertical load $V = 1100$ kN/m (see Fig. 3). The base ($B = 5$ m $\times$ $L = 25$ m) of the retaining wall is embedded at a depth of 1.8 m in a silty sand with an angle of internal friction $\varphi = 25°$, a cohesion $c = 15$ kN/m², and a unit weight $\gamma = 21$ kN/m³. For the bearing capacity case the limit state function is defined as

$$g = q_u - q \quad (11)$$

where

$$q_u = cN_c s_c d_c i_c + p_o N_q s_q d_q i_q + \frac{B'}{2}\gamma N_\gamma s_\gamma d_\gamma i_\gamma, \quad (12)$$

$$q = \frac{V}{B'}, \quad (13)$$

in which q_u is the ultimate bearing capacity, q the applied pressure, p_o the effective overburden pressure, B' the effective width of foundation ($B' = B - 2e$, where e is the eccentricity of the load), c is the cohesion of the subsoil, γ the unit weight of soil below the base of foundation. Bearing capacity factors N_c, N_q, N_γ that are functions of the angle of internal friction (φ) of soil are calculated according the same equations as given in EC7 (N_γ according to Vesic). The shape factors s_c, s_q, s_γ are defined by equation (14), the factors accounting for the depth effects of foundation d_c, d_q, d_γ are defined by equation (15) and the factors accounting for the inclination of the effect of the applied load i_c, i_q, i_γ are defined by equation (16), cf. Tables 4.5a and 4.5b in Bowles [3].

$$s_c = 1 + \frac{N_q}{N_c}\cdot\frac{B}{L}, \quad s_q = 1 + \frac{B}{L}\sin\phi, \quad s_\gamma = 1 - 0.4\frac{B}{L}, \quad (14)$$

$$d_c = 1 + 0.4k, \quad d_q = 1 + 2tg\phi(1-\sin\phi)^2 k, \quad d_\gamma = 1, \quad (15)$$

$$i_c = i_q - \frac{1 - i_q}{N_q - 1}, \quad i_q = \left[1 - \frac{0.5H}{V + A \cdot c \cdot \text{ctg}\phi}\right]^{\alpha_1}, \quad 2 \leq \alpha_1 \leq 5,$$

$$i_\gamma = \left[1 - \frac{0.7H}{V + A \cdot c \cdot \text{ctg}\phi}\right]^{\alpha_2}, \quad 2 \leq \alpha_2 \leq 5.$$

(16)

In order to illustrate reliability based design the example described above has been probabilistically elaborated. Soil strength parameters and both horizontal and vertical loads were treated as lognormally distributed random variables with mean values equal to corresponding values accepted in Tomlinson's example. All necessary probabilistic assumptions are given in Table 2.

The computations were carried out by means of the FORM method. The target reliability index against bearing capacity failure has been selected on the level equal to $\beta = 3.0$. The width of the foundation $B = 4.51$ m was obtained as the minimal satisfying the target reliability index. Detailed results are presented in Table 3 and Table 4.

Note that Table 3 contains the coordinates of design point that are understood as design values of cohesion, friction angle, horizontal load and vertical load, respectively. Values of the coefficients in equation (12) computed for the above design values of the parameters are presented in Table 4.

In geotechnical design according to EC7, the characteristic values of resistance parameters are divided by partial factors to obtain the design values, while the characteristic values of action parameters are multiplied by partial factors. The detailed values of partial factors depend on a design approach (see Table 1 in Section 2). Note that the partial factors in Table 1 do not contain any statistical information concerning coefficients of variations of neither loads nor parameters. As has already been mentioned in Section 2, the characteristic values of geotechnical parameters may be selected by applying some statistical tools, but statistical approach is not mandatory.

The role of a mean value, a standard deviation and a design point in the reliability based approach is analogical to the role of a characteristic value and a partial factor of the same parameter in the EC7 design approach. For example, for a given mean value and standard deviation of a friction angle φ one gets (by assuming at 10 percentile of the lognormal distribution) the characteristic value $\varphi_k = 21.9°$. Then the partial safety factor of φ^* (denoted by γ_φ) implied by the design point $\mathbf{y}^*$ is $\gamma_\varphi = \varphi_k/\varphi^* = 21.9°/20.77°$ $= 1.054$. In EC7, the partial safety factor is applied to $\tan\varphi$ instead of φ. However, assuming a mean value and a standard deviation of $\tan\varphi$ the partial factor $\gamma_{\tan\varphi}$ can be analogically evaluated using the FORM method. In the case of a horizontal load H assuming

Table 2. Probabilistic characteristics assumed in Example 1

Parameter	Unit	Mean value	Standard Deviation	Distribution	Correlation matrix R			
Cohesion c	[kPa]	15	3	lognormal	1	–0.5	0	0
Friction angle φ	[°]	25	2.5	lognormal	–0.5	1	0	0
Horizontal load H	[kN/m]	300	45	lognormal	0	0	1	0.5
Vertical lad V	[kN/m]	1100	110	lognormal	0	0	0.5	1

Table 3. Results of probabilistic computations

B	L	D	γ	e_B	e_L	B'	L'	q'	Design point				β
									c^*	φ	H^*	V^*	
[m]	[m]	[m]	[kN/m³]	[m]	[m]	[m]	[m]	[kPa]	[kPa]	[–]	[kN/m]	[kN/m]	[–]
4.51	25	1.8	21	0.87	0.0	2.77	25	37.8	15.20	20.77	412.63	1184.7	3.00

Table 4. Values of coefficients in equation (12) calculated at the design point

s_q	s_c	s_γ	i_q	i_c	i_γ	N_q	N_c	N_γ	d_q	d_c	d_γ
[–]	[–]	[–]	[–]	[–]	[–]	[–]	[–]	[–]	[–]	[–]	[–]
1.03	1.03	0.98	0.64	0.582	0.406	6.908	15.58	5.998	1.13	1.16	1.00

a characteristic value at 90 percentile of a lognormal distribution (with the mean value and the standard deviation as in Table 2) one gets $H_k = 359$ kN/m. Referring to the design point value the corresponding partial factor is $\gamma_{Hk} = H^*/H_k = 412.6/359 = 1.15$. In the case of cohesion the design value is slightly higher than the assumed mean value. This is the consequence of the negative correlation between strength parameters φ and c that was assumed in this example. Negatively correlated φ and c means low values of φ tend to occur with high values of c and vice versa.

In a reliability-based design the same target reliability index (equivalently the same probability of failure) can be utilised across different applications and different levels of parametric uncertainty and correlations. If the consequences of failure are more significant than the target reliability index should be raised to a higher level. Consequently, by means of the reliability computations proposed above, another set of partial factors can be obtained. Such a flexibility in parametric sensitivities and correlations are not found in EC7 based on code-recommended partial factors. Also characteristic values can be precisely defined if the reliability approach is used.

The EC0 code recommends a minimum reliability index for three reliability classes RC1, RC2, and RC3 for the ultimate limit state (Table B2 in EC0). The minimum reliability indices for 50 year reference period are 3.3, 3.8 and 4.3, respectively. In contrast, the EC7 established "Geotechnical categories". However, there is no link between RC classes and "Geotechnical categories" unless reliability-based calculations are carried out.

As has already been mentioned in the case of strength parameters mutual correlation of soil properties is reflected in the design values obtained by a reliability approach. It is worth mentioning that such a correlation cannot be considered in EC7.

4.2. EXAMPLE 2. PILE SUBJECTED TO LATERAL LOADS

The example presented below is based on results of a larger study published by Puła and Różański [48]. Let us now consider the problem of the evaluation of the bearing capacity of a rigid pile embedded in a homogeneous cohesionless soil deposit and subjected to lateral loads as in Fig. 4.

The mechanism associated with the failure assumes a rotation of the pile about the centre O due to external load as well as reaction of surrounding soil. Let H_u and M_u denote the ultimate lateral load and the ultimate moment. An ultimate ground resistance at a depth z is

denoted by $p_u(z)$. Treating the pile as a strip of width D (or diameter D in the case of a pile of circular cross-section) and length L the equilibrium equations can be written in the form

$$H_u = \int_0^{z_r} p_u(z)Ddz - \int_{z_r}^{L} p_u(z)Ddz, \quad (17)$$

$$M_u = H_u e = -\int_0^{z_r} p_u(z)Dzdz + \int_{z_r}^{L} p_u(z)Dzdz. \quad (18)$$
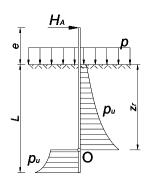

Fig. 4. A scheme of rigid pile embedded in soil
and subjected to lateral load

The parameters to be found in equation (17) and equation (18) are z_r and H_u. It is evident that the solution of those equations needs specifying the ground resistance distribution, $p_u(z)$ along the pile. Here the distribution proposed by Brinch Hansen [6] has been applied. This approach utilizes the limit state theory as well as rigid ideally-plastic model of subsoil. Using the equations derived by Brinch Hansen [6] it can be found (see Puła and Różański [48]) that the equations for the rotation centre z_r and the ultimate lateral load H_u can be written as follows

$$a_0 + a_1 z_r + a_2 z_r^2 + a_3 z_r^3 + b_1 \ln(D + a_q z_r) = 0, \quad (19)$$

$$H_u = a_0' + a_1' z_r + a_2' z_r^2 + b_1' \ln(D + a_q z_r) \quad (20)$$

where a_0, a_0', a_1, a_1', a_2, a_2', a_3, b_1, a_q are coefficients depending on soil properties, namely the friction angle φ and the unit weight γ, as well as load parameters like overburden pressure p and an eccentricity e (see Fig. 4). Due to a relatively complex character of Brinch Hansen solution also the coefficients specified above are expressed by rather complicated equations. Details can be found in a paper by Puła and Różański [48].

In deterministic computations as a measure of safety a total safety factor can be considered, which is defined as the ratio of the ultimate lateral force H_u and the applied lateral force H_a

$$F = \frac{H_u}{H_a}. \tag{21}$$

Some preliminary numerical studies have evidently showed high sensitivity of the ultimate loading force H_u to a value of the subsoil friction angle φ. Then investigation of random fluctuations due to inherent variability of φ seems to be a vital problem.

If some soil properties are assumed to be random variables then a natural question arises: how reliable the total safety factor is? Accordingly, appropriate reliability problem can be formulated as follows: find the probability p_F such that the applied loading H_a exceeds the ultimate lateral loading H_u

$$p_F = P\{H_a > H_u\} = P\left\{\frac{H_u}{H_a} < 1\right\} = P\{F < 1\}. \tag{22}$$

Due to complicated mathematical relationships between a soil friction angle and the total safety factor F it is reasonable to support FORM/SORM computations by some preliminary evaluations, namely by response surface method [4]. In general, this method consists in approximating an unknown by the known

function chosen appropriately. In the case of numerical computations, relationships between the model parameters $x_1, x_2, ..., x_n$, which are introduced as the input data (random parameters of the problem), and the values obtained as output data $y = f(x_1, x_2, ..., x_n)$ are defined. In the case considered in this example $y = H_u(\varphi, L)$. The fitting of the function f is done by means of the non-linear regression analysis. Having the dependence $H_u = f(x_1, x_2, ..., x_n)$ the SORM method can be utilized straightforward.

As an example let us consider a situation with three random variables (the scheme is presented in Fig. 4): the angle of internal friction of the soil φ, the pile length L and the external load H_a. The details are given in Table 5. In this case, the response function was established as

$$H_u = aL^b(c - \varphi)^d, \tag{23}$$

where $a = 0.17 \cdot 10^{20}$, $b = 2.868$, $c = 141.6$, $d = -9.411$, φ is introduced in degrees and H_u in kN.

Now, evaluation of reliability index that corresponds to a given value of the factor F can be performed according the following algorithm [48]:

Table 5. Parameter characteristics involved into the problem considered

No.	Parameter	Probability distribution	Expected value	Coefficient of variation cov[%]
1	Angle of internal friction φ	lognormal	33.6°	15%
2	External lateral load H_a	lognormal	8–23 kN	15%
3	Pile embedding L	normal	2.9 m	0%, 2%, 5%
4	Eccentricity e	constant (nonrandom)	8.64 m	
5	Unit weight γ	constant (nonrandom)	20.2 kNm^{-3}	
6	Pile diameter D	constant (nonrandom)	0.36 m.	
7	Overburden pressure p	constant (nonrandom)	8.8 kNm^{-2}	

Table 6. Reliability indices obtained in analysis with three random variables (φ, L, H_a)

Total safety factor F	Expected value of the applied lateral load H_a [kN]	Reliability index β cov$\{L\}$ = 0.0 L is non-random	Reliability index β cov$\{L\}$ = 0.02	Reliability index β cov$\{L\}$ = 0.05
1.2	21.92	0.38	0.37	0.35
1.4	18.79	0.75	0.73	0.70
1.6	16.44	1.09	1.07	1.01
1.8	14.61	1.41	1.38	1.30
2.0	13.15	1.70	1.67	1.56
2.2	11.95	1.97	1.94	1.81
2.4	10.96	2.22	2.19	2.04
2.6	10.12	2.46	2.44	2.26
2.8	9.39	2.69	2.67	2.46
3.0	8.77	2.91	2.90	2.66

1. Assume a value of F;
2. Replace all random variables by constants equal to their expectations and use them to evaluate the ultimate resistance H_u given by equation (20);
3. For a given total safety factor F (step 1) and obtained value of H_u (step 2) calculate the value of applied load H_a utilising equation (21);
4. Treat H_a and φ as random variables. Assign to H_a the expectation equal to the value obtained in step 3. Other probabilistic characteristics have to be additionally assumed;
5. Evaluate β and p_F by making use of equation (22);
6. All steps above have to be repeated for several different values of factor F.

The results of SORM computations for various values of safety factor F are summarised in Table 6.

Table 6 demonstrates how the total safety factor used in WSD method can be related to reliability index. Here, the values of factor F and index β are closed to each other. Consequently it is seen what range of lateral load H_a can be applied for a target level of safety in the sense of total factor as well as reliability index. It can also be seen that index β is sensitive with respect to possible fluctuations of the pile's length.

It should also be noted that the response surface method can be a useful tool that can support FORM and SORM methods in reliability-based design.

5. MODELLING BY RANDOM FIELDS

In previous sections the important role of reliability methods as a supporting tool in geotechnical design has been pointed out. However, both examples being discussed are based on quite a simple probabilistic modelling. Namely the modelling was conducted by assigning a single variable to a parameter treated as random. The parameters could be mutually correlated. This way of modelling suits well a lot of typical design problems as it takes in account mean values, standard deviations, probability distributions of individual parameters as well as a possible correlation between them. However, such an approach can be hardly applicable if the spatial variability of soil properties is under consideration, especially important if relatively large area can be involved in failure mechanism. Moreover, it could not be sufficient if the reliability evaluations are performed in conjunction with finite element computations. In both cases mentioned a probabilistic modelling by the theory of random fields seems to be adequate solution.

5.1. BASIC CONCEPTS OF RANDOM FIELDS THEORY

Random field (RF) is a generalisation of stochastic process. Given a probability space (Ω, S, P), by random field we understand a function $X: \Omega \times R^3 \to R$ such that for each $x \in R^3$, $X(\omega, x)$ is a random variable. The space R^3 means that the domain of the field is three-dimensional. However, it can be replaced by R^2 (two-dimensional random field) or by R (stochastic process).

Hence, to each point of a field (area) under consideration a random variable is assigned. Additionally, these random variables are related to each other by a certain correlation structure. A function $X(\omega_0, x)$, where ω_0 is fixed and x is variable is called a realization of the field X.

Random field theory has been implemented to geotechnical problem since the 1970s, when it was applied to examine variation of soil parameters with depth [2]. Important papers which contributed to the development of the application of random fields theory to description of soil properties were published by Vanmarcke [56], [57]. The modern approach to modelling of soil parameters by random fields was formulated by Rackwitz [51]. Worth mentioning is the fact that the development of computing power in the last decades is of great importance for effective application of random field theory for consideration of spatial variability of soil parameters in probabilistic analysis.

Assume that to a certain soil property a random field $X(x, y, z)$ is assigned. Then at each point (x, y, z) of the space (or plane in two-dimensional problems) the property is represented by a single variable. Usually it is convenient to express an RF as

$$X(x, y, z) = \mathrm{E}\{X(x, y, z) + \sigma_X(x, y, z)U(x, y, z)\},$$
(24)

where $E\{X(x, y, x)\}$ is a mean value, $\sigma_X(x, y, z)$ corresponds to a standard deviation and $U(x, y, z)$ is a random field with zero mean value and unit standard deviation. A correlation structure of RF is determined by covariance function which is defined as

$$C(x_1, x_2, y_1, y_2, z_1, z_2) = \mathrm{E}\{U(x_2, y_2, z_2)U(x_1, y_1, z_1)\},$$
(25)

$U(x, y, z)$ as in (24). If the mean value E is a constant for all (x, y, z) under consideration and covariance function depends solely on the lag vector between points, i.e.,

$$C(x_1, x_2, y_1, y_2, z_1, z_2) = C(x_2 - x_1, y_2 - y_1, z_2 - z_1),$$
(26)

then the RF is called weakly stationary (or wide-sense stationary). A weak stationary RF is called isotropic if its covariance function depends solely on the distance between points, i.e.,

$$C(x_1, x_2, y_1, y_2, z_1, z_2)$$

$$= C\left(\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\right). \quad (27)$$

It is clear that for isotropic RF the covariance function is a single variable function. Another important case of weakly stationary RF is a separable RF. For separable RF its covariance function can be factorised as follows

$$C(x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

$$= C(\Delta x, \Delta y, \Delta z) = C_1(\Delta x)C_2(\Delta y)C_3(\Delta z). \quad (28)$$

A feature of separability allows a correlation structure to be analyzed in each direction separately by three functions of single variable. In the sequel, only weakly stationary RF will be considered.

A suitable measure of the rate of variability of a random field is a scale fluctuation θ [56]. A value of a scale fluctuation (also referred as correlation length or correlation radius) specifies how two points in a field are correlated with each other. If two points are separated by a distance greater than θ, it means that these points are significantly uncorrelated. Precisely a scale of fluctuation can be defined (in one-dimensional case) as

$$\theta = \frac{2}{\sigma^2} \int_0^\infty C(\Delta z)d\Delta z = 2\int_0^\infty \rho(\Delta z)d\Delta z, \quad (29)$$

where

$$\rho(\Delta z) = \frac{C(\Delta z)}{\sigma^2} \quad (30)$$

is a correlation function. It can be proved that a covariance function must be positive-definite function.

Some examples of covariance functions (for one-dimensional RF) with corresponding correlation lengths are presented in Table 7.

For practical applications most important are Gaussian random fields. An RF is called Gaussian or normal if for any finite set of points $x_1, x_2, ..., x_n$ the random vector $(X_1, X_2, ..., X_n)$ is normally distributed.

In this case three things have to be defined to characterize a soil parameter by a weakly stationary and normally distributed random field:
1. the field mean μ_X,
2. the field variance σ_X^2,
3. the field correlation structure.

Table 7. Covariance functions and scales of fluctuation

Covariance function	Scale of fluctuation (correlation length)
$C(\Delta z) = \sigma^2 \cdot \exp(-a\,\lvert\Delta z\rvert)$	$\theta = \dfrac{2}{a}$
$C(\Delta z) = \sigma^2 \cdot \exp(-a\,\lvert\Delta z\rvert)\cos(b\Delta z)$	$\theta = \dfrac{2a}{a^2 + b^2}$
$C(\Delta z) = \sigma^2 \cdot \exp(-a\,\lvert\Delta z\rvert)J_0(b\Delta z)$	$\theta = \sqrt{\dfrac{\pi}{a}} \cdot \exp\left(-\dfrac{b^2}{8a}\right)I_0\left(\dfrac{b^2}{8a}\right)$
$C(\Delta z) = \sigma^2 \cdot \exp(-a(\Delta z)^2)$	$\theta = \sqrt{\dfrac{\pi}{a}}$

a and b are field parameters, J_0 is a Bessel function of the first kind, I_0 is a modified Bessel function of the first kind.

The correlation structure is defined by assigning a covariance (correlation function) and a fluctuation scale is included as a parameter of the covariance function. The application of RF theory in practical analysis requires some simplifications, because an implementation of a continuous random field in numerical analysis would be cumbersome. Hence a random field has to be discretized. In literature, various RF generators can be found [22], e.g., spectral random field generator (SRFFT), turning band method (TB), the matrix decomposition method (LU), the sequential Gaussian simulation method, the local average subdivision (LAS).

5.2. SPATIAL AVERAGING

An important step in utilizing random fields in geotechnics was applying spatial averaging procedure proposed by Vanmarcke [56]. Spatial averaging reflects the situation in which failure is caused by certain regions of the subsoil area, but is not a consequence of "point" behaviour. On the other hand, spatial averaging is the base of the local average subdivision method [18] that will be used in further part of this paper.

Assume now that a soil parameter X is described by a stationary random field $X(x, y, z)$ with a covariance function $C(\Delta x, \Delta y, \Delta z) = \sigma_X^2 \rho_X(\Delta x, \Delta y, \Delta z)$, where σ_X^2 is a variance of a random field X and ρ_X is

its correlation function. Let $V \subset R^3$ denote a certain area and $|V|$ the volume of V. Spatial (local) average, used by Vanmarcke [56], is based on an introduction of new field expressed by

$$X_V = \frac{1}{|V|} \iiint_V X(x, y, z) dx dy dz .\qquad (31)$$

It can be noticed that X_V is a random variable defining a certain mean of random field X in area V. By changing the area V another random field is created $X(V)$, for simplicity denoted X_V. Random fields X_V and X have the same mean value (that is a consequence of stationarity) but different variances. A variance of a random field X_V takes the form

$$VAR[X_V] = \sigma_X^2 = \gamma(V)\sigma_X^2 \qquad (32)$$

where $\gamma(V)$ is the variance function (called sometimes the variance reduction function). The variance function in one-dimensional case is defined as

$$\gamma(L) = \frac{2}{L}\int_0^L \left(1 - \frac{\Delta z}{L}\right)\rho(\Delta z)d\Delta z .\qquad (33)$$

According to many opinions (e.g., [8], [47]) the spatial averaging should be applied if reliability evaluations are carried out in geotechnics. Otherwise, one can obtain not realistic values of reliability measures. The example below demonstrates the effects of this procedure. Let us now come back to example 2 in Section 4 and assume that the friction angle of the soil surrounding the pile is characterised by one-dimensional isotropic lognormal RF (see Section 7) with constant mean value m_φ and constant point variance σ_φ^2. Additionally, assume that the covariance function in the field is given by Gaussian function (last row in Table 7). Consider now the spatial averaging of the φ along the pile's length

$$\varphi_L = \frac{1}{\bar{L}}\int_0^{\bar{L}} \varphi(z)dz \qquad (34)$$

where L is the pile length (in the cases where L is treated as random variable $\bar{L}$ is understood as the expected value of L). Under the above assumptions the variance function (33) takes the form

$$\gamma(\bar{L}) = \frac{\frac{\pi}{\delta}\bar{L}\,\mathrm{erf}\left(\frac{\sqrt{\pi}}{\delta}\bar{L}\right) - 1 + \exp\left(-\frac{\pi}{\delta^2}\bar{L}^2\right)}{\frac{\pi}{\delta^2}\bar{L}^2} \qquad (35)$$

where δ is a fluctuation scale. Values of function (35) for three different values of δ and resulting values of the variance function are shown in Table 8.

Table 8. Values of variance function for three different values of fluctuation scale

Averaging length $\bar{L}$ [m]	Values of the fluctuation scale		
	$\delta = 0.6$ [m]	$\delta = 0.8$ [m]	$\delta = 1.0$ [m]
	Values of the variance function		
2.9	0.1933	0.2516	0.3070

Next, the SORM computations were carried out with reduced variances yielding from equation (32). Comparison of results is presented in Fig. 5.
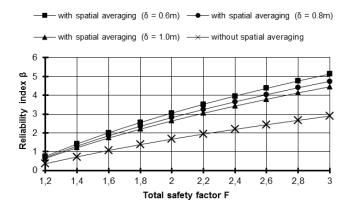

Fig. 5. The influence of spatial averaging on the values of reliability indices β

The results obtained evidently demonstrate high influence of spatial averaging on reliability measures β. Note that the total safety factor F of level 2.4 gives relatively high values of reliability indices (greater than 3.4), if the averaging is applied. On the other hand, it is worth mentioning that the effect of the value of fluctuation scale is remarkable.

6. APPLICATION OF RANDOM FIELDS IN CONJUNCTION WITH FEM

Local average subdivision method (LAS) has an advantage over other random field generators due to which it can be in an uncomplicated way applied to finite element method (FEM) which nowadays is commonly used by engineers during a designing process. LAS method allows a random field considered to be discretized on a finite number of elements which correspond to the number of elements in finite element grid in FEM [30]. Random field theory together with LAS method and Monte Carlo simulations was

used in an effective manner to the modeling of spatial variability of soil parameters in the random finite element method (RFEM). RFEM was introduced in a paper by Griffiths and Fenton [25]. Generally, RFEM is a specific type of earlier created stochastic finite element method SFEM [13], [24], [33]. RFEM is distinguished from other courses developed within SFEM in that the LAS method is consequently applied as a random field generator. RFEM is an useful tool, which can be used in reliability base design. More detailed description of RFEM can be found in a monograph by Fenton and Griffiths [22].

An effective application of RFEM was possible relatively late, namely after the development of computer technology, especially when personal computers gained high computing power. RFEM, as a probabilistic method, requires a great amount of results obtained through MC simulations to estimate statistical parameters such as a mean value, a standard deviation or a probability distribution. MC simulations together with random field generation and finite element computations are time-consuming. However, to this day it was successfully introduced to solve various geotechnical issues:

- A settlement of foundations [20], [31], [36];
- A bearing capacity of foundation on one layer soil [19], [27], [43], [44], [49];
- A slope stability [10], [29], [42];
- An earth pressure on retaining structures [28], [30];
- A flow of water in soil [1], [25], [26].

7. ALGORITHM OF RFEM TO RELIABILITY BASE DESIGN OF A SHALLOW STRIP FOUNDATION

At the beginning, as mentioned above, an application of random fields requires a determination of mean values, standard deviations and scales of fluctuations of soil parameters whose spatial variability has great influence on the safety assessment of a structure considered. These statistical parameters shall be obtained from in situ tests, existing knowledge and experience. A methodology for determination of statistical parameters of soil properties is not within the scope of the present study. Next information which has to be specified is probability distributions of soil properties being considered. In the case of cohesion or Young's modulus a lognormal distribution is commonly used. A lognormal random field Y can be obtained from normal random field G by the transformation $Y =$

$\exp\{G\}$. The probability density function of a lognormal distribution takes the form

$$f(x) = \frac{1}{x\sigma_{\ln Y}\sqrt{2\pi}} \cdot \exp\left\{-\frac{1}{2}\left(\frac{\ln x - \mu_{\ln Y}}{\sigma_{\ln Y}}\right)^2\right\}, \quad \text{for } x > 0,$$

(36)

where

$$\mu_{\ln Y} = \mathrm{E}[G], \quad \sigma_{\ln Y}^2 = \mathrm{Var}[G].$$

(37)

While a friction angle or Poisson's ratio can be described by a distribution of bounded support. Such a choice is supported by the fact that these soil parameters vary within bounded ranges. Hence an application of a normal or a lognormal distribution would not reflect the real nature of a friction angle or Poisson's ratio. Consequently, nonphysical values of these parameters could be obtained during a generation of random fields. A bounded random field Ω can be achieved by a hyperbolic tangent transformation from a normal random field G

$$\Omega = a + \frac{1}{2}(b-a)\left\{1 + \tan h\left(\frac{sG}{2\pi}\right)\right\},$$

(38)

where a and b correspond to the minimal and maximal values of a random field Ω considered. Parameter s denotes the scale factor related to a standard deviation of a random field Ω. The probability density function of the bounded tanh distribution takes the form

$$f(x) = \frac{\sqrt{\pi}(b-a)}{\sqrt{2}s(x-a)(b-x)} \cdot \exp\left\{-\frac{1}{2s^2}\left[\pi\ln\left(\frac{x-a}{b-x}\right) - m\right]^2\right\},$$

(39)

where m is a location parameter.

The last assumption concerns a correlation structure of random fields. In this paper, the anisotropic case is under consideration because a greater correlation radius is observed in a horizontal rather than vertical direction in natural conditions. This is obviously associated with geological processes that occurred in the past. It is assumed that a correlation function is described by a function of the form

$$\rho(\tau) = \exp\left(-\sqrt{\left(\frac{2|\tau_2|}{\theta_x}\right)^2 + \left(\frac{2|\tau_1|}{\theta_y}\right)^2}\right),$$

(40)

where $\tau_2 = x_2 - x_1$ and $\tau_1 = y_2 - y_1$. Furthermore, θ_x and θ_y are scales of fluctuation in horizontal and vertical directions, respectively.

In the next step, the geotechnical problem which we want to solve has to be transferred to FEM code. Based on specified characterization of random fields, the LAS method can be applied in order to generate random fields of soil parameters. Firstly, normal (Gaussian) random fields are created, and then using the above-mentioned transformations final random fields can be obtained. Figure 6 presents one realization of random field of a cohesion associated with FEM grid together with failure mechanism of shallow foundation. As can be noticed, each finite element is described by a single random variable.
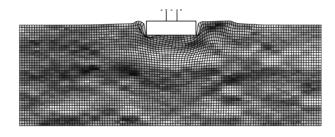

Fig. 6. A realisation of cohesion random field (lognormal)

The numerical computations can be performed after all assumptions have been made. A result of Monte Carlo simulation is a statistical set of results, e.g., a random bearing capacity of a foundation. The greater number of realizations N in MC simulation the more accurate the estimation of statistical parameters of a random bearing capacity will be. The accuracy in MC simulation can be approximately expressed by $\frac{1}{\sqrt{N}}$ (due to the Tschebyschev inequality). In consequence of numerical analysis a mean value, a standard deviation of a random bearing capacity can be evaluated. These statistical moments are crucial to estimate a probability distribution of a random bearing capacity whose application is one of the approaches to calculate a reliability index β. The reliability index β can also be evaluated based on distribution free methods such as Hasofer–Lind mentioned in Section 3.

A selection of theoretical pdf which will be well-fitted to the empirical pdf can be conducted by the Kolmogorov–Smirnov [23] goodness-of-fit test. This test is concentrated on the measurement of a distance D_N between empirical $S_N(x)$ and theoretical $F(x)$ probability distribution functions.

$$D_N = \sup_{-\infty < x < +\infty} |F(x) - S_N(x)|. \qquad (41)$$

There is no reason to reject a theoretical pdf if D_N is lower than assumed threshold value $\frac{\lambda_0}{\sqrt{N}}$. N denotes

the number of realisations in MC simulation, λ_0 is a value associated with a significance level α and can be found in the statistical table of Kolmogorov–Smirnov test [23].

A fitted theoretical pdf of random bearing capacity q_f can be applied to evaluate a reliability index β and equivalent failure probability p_F. For example, the design value Q_d of bearing capacity of foundation can be evaluated as the value that satisfies the following equation

$$p_F = P\,|\,q_f < Q_d\,|. \qquad (42)$$

8. EXAMPLES OF APPLICATION OF RFEM TO RELIABILITY BASE DESIGN

8.1. SAFETY ASSESSMENT OF A SHALLOW FOUNDATION ON COHESIONLESS SOIL

The authors of the present study conducted reliability analysis of an embedded shallow footing in cohesionless soil for various width of foundation [49]. The plane strain situation was considered because a foundation length was assumed as infinite. The bearing capacity analysis applied an elastic-perfectly plastic stress-strain law with a Mohr–Coulomb failure criterion. Soil parameters were investigated in the vicinity of Wrocław (see [53]) and are gathered in Table 9. Only a friction angle was described by a random field. Therefore, a random character of bearing capacity depended solely on spatial variability of a friction angle. Figure 7 presents a geometry of a foundation and soil.

Reliability indices β and corresponding design values of bearing capacity R_d for various width and a depth of embedment $h_z = 1.0$ m are shown in Fig. 8. The reliability base design was proceeded by RFEM according to the algorithm given in Section 7 on the basis of 2000 realizations. The Weibull distribution was adopted to characterize a bearing capacity of foundation by distribution fitting approach described in Section 7 (for details, see [49]). Next the design values of bearing capacity Q_d were found by equation (42). As can be expected, the higher the reliability index, the lower the design value of a bearing capacity regardless of the foundation width.

Table 9. Soil parameters used in numerical analysis
of the bearing capacity of cohesionless soil

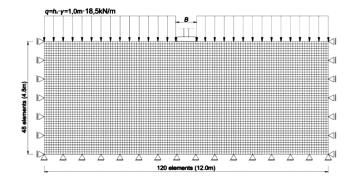| Soil parameter		Unit	Value	Distribution
Friction angle ϕ	μ_ϕ	[°]	30	bounded
	σ_ϕ	[°]	5.0	
	$\dfrac{\phi_{max}}{\phi_{min}}$	[°]	40/20	
	s	[–]	4.08	
Cohesion c		[kPa]	0	deterministic
Young's modulus E		[MPa]	93	deterministic
Poisson's ratio υ		[–]	0.3	deterministic
Soil unit weight γ		$\left[\dfrac{kN}{m^3}\right]$	18.5	deterministic
Dilation angle ψ		[°]	12	deterministic
Scale of fluctuation	$\theta_x \mathbf{x} \theta_y$	[m]	10.0×1.0	–

Fig. 7. The soil model used in numerical analysis
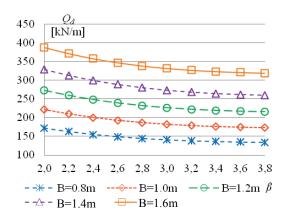of the bearing capacity of cohesionless soil

Fig. 8. Design values of the bearing capacity
depending on the footing width *B* and reliability index β

8.2. SAFETY ASSESSMENT
OF A SHALLOW FOUNDATION
ON COHESIVE SOIL

RFEM was applied in paper by Pieczyńska-Kozłow-ska et al. [44] to evaluate a random bearing capacity

of an embedded shallow foundation on cohesive soil from the vicinity of Taranto city in Italy. Reliability analysis was used to examine an influence of horizontal scale of fluctuation on reliability index β. As in example 8.1, the plane strain situation was considered. The bearing capacity analysis applied an elastic-perfectly plastic stress-strain law with a Mohr–Coulomb failure criterion. The width of the foundation was constant ($B = 1.0$ m) as the depth ($h_z = 1.0$ m). The soil parameters used in the analysis (Table 10) and correlation lengths concern Taranto Blue Clay were described by Cafaro [7] and Cherubini [9]. The soil FEM model is shown in Fig. 9.

Table. 10. Soil parameters used in numerical analysis
of the bearing capacity of cohesive soil

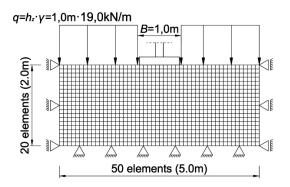| Soil parameter		Unit	Value	Distribution
Friction angle ϕ	μ_ϕ	[°]	20	bounded
	σ_ϕ	[°]	4.8	
	$\dfrac{\phi_{max}}{\phi_{min}}$	[°]	35/5	
	s	[–]	2.27	
Cohesion c	μ_c	[kPa]	36	lognormal
	σ_c	[kPa]	20	
Young's modulus E		[MPa]	36	deterministic
Poisson's ratio υ		[–]	0.29	deterministic
Soil unit weight γ		$\left[\dfrac{kN}{m^3}\right]$	19.0	deterministic
Scale of fluctuation	θ_x	[m]	1.0; 5.0; 10; 50; ∞	–
	θ_y	[m]	1.0	–

Fig. 9. The soil model used in numerical analysis
of the bearing capacity of cohesive soil

Figure 10 presents results of reliability analysis. The trend looks the same as in example 8.1. It can be observed that for $\theta_x/B \geq 10$ differences in Q_d for the same beta are negligible.
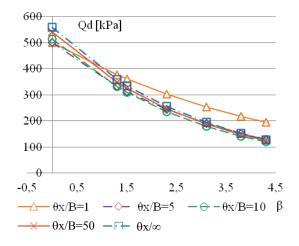
Fig. 10. Design values of the bearing capacity
versus different β and $\dfrac{\theta_x}{B}$ values

9. CONCLUSIONS

The paper demonstrates the way in which reliability method can support geotechnical design. The classical reliability methods like first- and second-order reliability methods can now be utilized in a more efficient way due to codding them as spreadsheet applications. The reliability-based design recently more frequently appears in geotechnics because some inconsistency between Eurocode 0 and Eurocode 7. EC0 establishes the principles and requirements for safety and serviceability that are based on typical reliability measure, that is, the reliability index β. On the other hand, partial safety factors recommended by EC7 do not allow for adopting any statistical information concerning uncertainty in soil properties. Moreover, the use of statistical methods is not mandatory when evaluating characteristic values. Applying FORM and SORM methods one finds the design point which is generated automatically without information of characteristic values and partial safety factors by means of reliability based design. The information is obtained by means of elaborating statistical data. Therefore the troubles in determining characteristic values of soil properties can be avoided. Additionally, information concerning correlation between soil properties can be included.

The second part of the paper is dedicated to applications of random fields theory. This theory is very useful when we deal with spatial variability of soil properties. In such a case, the probabilistic modeling by assigning to a soil property a single random variable may not be sufficient. The random field theory is

also a powerful tool when uncertainty of soil properties is considered in conjunction with finite element computations. Specially dedicated to these problems is the Random Finite Element Method.

An algorithm of Random Finite Element Method to reliability based design of shallow strip foundation is given and illustrated by two computational examples based on properties of natural soils.

A crucial point for random fields application is adequate selection of their correlation structure. It was demonstrated that the most important parameter is the scale of fluctuation. Determining the fluctuation scale usually requires a lot of results from in-situ investigations. But these results can be gained from SPT or SPTU testing.

The discussion given above shows the usefulness of FORM/SORM design as well as random field approach. Illustrative examples suggest that reliability-based design can provide additional to EC7 design or LRFD as well as working stress method when the statistical information (like mean values, standard deviations, correlations and probability distributions) for key parameters affecting the design are known. If the distributions are not obtainable the Hasofer–Lind approach can be applied.

Finally, it should be emphasized that reliability computations have to be supported by credible statistical data. In geotechnical practice quite often amount of this data coming from testing is not sufficient enough. In order to overcome this difficulty in several countries, data bases of soil properties are created. Also developing applications other mathematical theories like fuzzy sets theory or the theory of random sets seems to be permissible for future safety evaluations in geotechnics.

REFERENCES

[1] ASHRAF A.A., ABDALLAH S.B., *Three-Dimensional Analysis of Seepage below and around Hydraulic Structures*, J. Hydrologic Eng., 2009, 14(3), 243–247.

[2] ALONSO E., KRIZEK R.J., *Stochastic formulation of soil properties*, Proceedings of the 2nd International Conference on Applications of Statistics and Probability in Soil and Structural Engineering, Vol. II, Aachen, 1975, 9–32.

[3] BOWLES J.E., *Foundation analysis and design*, 5th ed. McGraw-Hill, 1996.

[4] BOX G.P., DRAPER N.R., *Empirical model-building and response surface*, J. Wiley & Sons, New York, 1996.

[5] BREITUNG K., *Asymptotic approximation for multinormal integrals*, J. Eng. Mech. ASCE., 1984, 110(3), 357–366.

[6] BRINCH HANSEN J., *The ultimate resistance of rigid piles against transversal force*, The Danish Geotechnical Institute, Bulletin, No. 12, Copenhagen, 1961.

[7] CAFARO F., CERUBINI C., COTECCHIA F., *Use of the scale of fluctuation to describe the geotechnical variability of an Italian clay*, Proceeding of the 8th International Conference on Applications of Statistics and Probability in Civil Engineering, eds. Melchers and Stewart, Rotterdam, 2000, 481–486.

[8] CHERUBINI C., *Probabilistic approach to the design of anchored sheet pile walls*, Computers and Geotechnics, 2000, 26(3–4), 309–330.

[9] CHERUBINI C., VESSIA G., PUŁA W., *Statistical soil characterization of Italian sites for reliability analysis*, Characterization and Engineering Properties of Natural Soil, eds. Tan W., Phoon K.K., Hight, Lerouell, Singapore, Taiwan, 2007, 2681–706.

[10] CHOK Y.H., JAKSA M.B., GRIFFITHS D.V., FENTON G.A., KAGGWA W.S., *A parametric study on reliability of spatially random cohesive slopes*, Australian Geomechanics, 2007, 42(2), 79–85.

[11] CORNELL C.A., *A first-order reliability theory for structural design*, Study No. 3, Structural Reliability and Codified Design, University of Waterloo, Ontario, 1969.

[12] CORNELL C.A., *A probability-based Structural Code*, ACI Journal, 1969, 66(12), 974–985.

[13] DER KIUREGHIAN A., KE J.-D., *The stochastic finite element method in structural reliability*, Probabilistic Engineering Mechanics, 1988, 3(2), 83–91.

[14] DITLEVSEN O., *Principle of Normal Tail Approximation*, Journal of the Engineering Mechanics Division, ASCE. 1981, 107(EM6), 1191–1208.

[15] DITLEVSEN O., MADSEN H.O., *Structural Reliability Methods*, John Wiley & Sons, Chichester, 1996.

[16] EN 1997-1:2004. *Eurocode 7. Geotechnical Design*, Part 1: *General Rules*. CEN, European Committee for Standardization, Brussels.

[17] EN 1990:2002. *Eurocode: Basis of structural design*. CEN, European Committee for Standardization, Brussels.

[18] FENTON G.A., VANMARCKE E., *Simulation of random fields via local average subdivision*, ASCE J. Geotech. Eng., 1990, 116(8), 1733–1749.

[19] FENTON G.A, GRIFFITHS D.V., *Bearing capacity prediction of spatially random c-φ soils*, Canadian Geotechnical Journal, 2003, 40(1), 54–65.

[20] FENTON G.A., GRIFFITHS D.V., CAVERS W., *Resistance factors for settlement design*, Canadian Geotechnical Journal, 2005, 42(5), 1422–1436.

[21] FENTON G.A., GRIFFITHS D.V., ZHANG X.Y., *Load and resistance factor design of shallow foundations against bearing failure*, Canadian Geotechnical Journal, 2008, 45(11), 1556–1571.

[22] FENTON G.A., GRIFFITHS D.V., *Risk assessment in geotechnical engineering*, John Wiley & Sons, New York, 2008.

[23] FISZ M. *Probability theory and mathematical statistics*, Wiley, New York, 1960.

[24] GHANEM R., SPANOS P., *Stochastic Finite Elements: A Spectral Approach*, Springer-Verlag, New York, 1991.

[25] GRIFFITHS D.V., FENTON G.A., *Seepage beneath water retaining structures founded on spatially random soil*, Geotechnique, 1993, 43(6), 577–587.

[26] GRIFFITHS D.V., FENTON G.A., *Probabilistic analysis of exit gradients due to steady seepage*, Journal of Geotechnical and Geoenvironmental Engineering, 1998, 124(9), 789–797.

[27] GRIFFITHS D.V., FENTON G.A., *Bearing capacity of spatially random soil: The undrained clay Prandtl problem revisited*, Geotechnique, 2001, 54(4), 351–359.

[28] GRIFFITHS D.V., FENTON G.A., TVETEN D.E., *Probabilistic earth pressure analysis by the Random Finite Element Method*, Proc. of the 11th International Conference on Computer Methods and Advances in Geomechanics (IACMAG 05), G. Barla and M. Barla (eds.), Turin, 2005, 4, 235–249.

[29] GRIFFITHS D.V., FENTON G.A., ZIEMANN H.R., *The influence of strength variability in the analysis of slope failure risk*, Proc. of the Second Japan-U.S. Workshop on Testing, Modeling and Simulation, Kyoto, P.V. Lade and T. Nakai (eds.), 2005, also in: Geotechnical Special Publication, 2006, 156, 113–123.

[30] GRIFFITHS D.V., FENTON G.A., ZIEMANN H.R., *Reliability of passive earth pressure*, Georisk: Assessment and Manag. of Risk for Engineered Systems and Geohazards, 2008, 2(2), 113–121.

[31] GRIFFITHS D.V., FENTON G.A., *Probabilistic settlement analysis by stochastic and random finite element methods*, Journal of Geotechnical and Geoenvironmental Engineering, 2009, 135(11), 1629–1637.

[32] HASOFER A.M., LIND N.C., *An Exact and Invariant First-Order Reliability Format*, Journal of Engineering Mechanics Division, ASCE, 1974, 100(EM1), 111–121.

[33] HISADA T., NAKAGIRI S., *Stochastic finite element developed for structural safety and reliability*, Proc. of the 4th International Conference on Structural Safety and Reliability ICOSSAR-3, 1981, 395–408.

[34] HOHENBICHLER M., GOLLWITZER S., KRUSE W., RACKWITZ R., *New light on first and second-order reliability methods*, Structural Safety, 1987, 4, 267–284.

[35] HOHENBICHLER M., RACKWITZ R., *Improvement of the second-order reliability estimates by importance sampling*, Journal Eng. Mech., ASCE, 1988, 114(12), 2195–2199.

[36] KUO Y.L., JAKSA M.B., KAGGWA W.S., FENTON G.A., GRIFFITHS D.V., GOLDSWORTHY J.S., *Probabilistic analysis of multi-layered soil effects on shallow foundation settlement*, Proc. of 9th Australia New Zealand Conference on Geomechanics, G. Farquhar et al. (eds.), University of Auckland, 2004, 2, 541–547.

[37] LIU P.L., DER KIUREGHIAN A., *Optimization algorithms for structural reliability*, Structural Safety, 1991, 9, 161–177.

[38] LOW B.K., TANG W.H., *Reliability analysis using object-oriented constrained optimization*, Structural Safety, 2004, 26(1), 69–89.

[39] LOW B.K., TANG W.H., *Efficient spreadsheet algorithm for first-order reliability method*, Journal Eng. Mech, ASCE, 2007, 133(12), 1378–1387.

[40] LOW B.K., PHOON K.K., *Reliability based design and its complementary role to Eurocode 7 design approach*, Computers and Geotechnics, 2015, 65, 30–44.

[41] LUMB P., *The variability of natural soils*, Canadian Geotechnical Journal, 1966, 3(2), 74–97.

[42] NUTTALL J.D., HICKS M.A., LLORET CABOT M., *Stochastic approach to slope stability analysis with in-situ data*, Industrial safety and life cycle engineering, 2013, 527–538.

[43] PIECZYŃSKA J., *Random finite element method in analysis of a bearing capacity of a shallow foundation*, PhD Thesis, Wrocław University of Technology, 2012, (in Polish).

[44] PIECZYŃSKA-KOZŁOWSKA J.M., PUŁA W., GRIFFITHS D.V., FENTON G.A., *Influence of embedment, self-weight and anisotropy on bearing capacity reliability using the random finite element method*, Computers and Geotechnics, 2015, 67, 229–238.

[45] PN-EN 1997-1:2008/Ap2. Polish annex to PN-EN 1997-1:2008.

[46] PUŁA W., *Applications of Structural Reliability Theory to safety evaluation of Foundations,* Wrocław University of Technology Press, 2004, (in Polish).

[47] PUŁA W., *Reliability of laterally loaded rigid piles*, [in:] *Probabilistic methods in geotechnical engineering*, CISM Courses and Lectures, Springer, 2007, No. 491, 169–183.

[48] PUŁA W., RÓŻAŃSKI A., *Reliability of rigid piles subjected to lateral loads*, ACME, 2012, 12, 205–218.

[49] PUŁA W., ZASKÓRSKI Ł., *Estimation of the probability distribution of the bearing capacity of cohesionless soil using the random finite element method*, Structure and Infrastructure Engineering, 2014, 11 (5), 707–720.

[50] RACKWITZ R., FIESSLER B., *Structural reliability under combined random loads sequences*, Computers and Structures, 1978, 9, 489–494.

[51] RACKWITZ R., *Reviewing probabilistic soils modeling*, Computers and Geotechnics, 2000, 26(3–4), 309–330.

[52] ROSENBLATT M., *Remarks on a multivariate transformation*, Annals of Mathematical Statistics, 1952, 23, 470–472.

[53] THAO N.T.P., *Geotechnical analysis of a chosen region of the Wrocław city by statistical method*, Wrocław University of Technology, Wrocław, 1984, (in Polish).

[54] TOMLINSON M.J., *Foundation design and construction*, 7th ed., Prentice Hall, 2001.

[55] TVEDT L., *Two second-order approximations to the failure probability*, Det Norske Veritas, RDIV/20-004-83, 1983.

[56] VANMARCKE E.H., *Probabilistic Modeling of Soil Profiles*, Journal of the Geotechnical Engineering Division, ASCE, 1977, 103(GT11), 1227–1246.

[57] VANMARCKE E.H., *Reliability of earth slopes*, J. Geotech. Eng. Div., 1977, 103(GT11), 1247–1265.

[58] VANMARCKE E.H., *Random Fields – Analysis and Synthesis.* MIT Press, 1983.