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SOLUTION OF A GAS PIPELINE BURIED IN GROUND BY STOCHASTIC APPROACH

MIHELOVÁ Silvia[1], FRYDRÝŠEK Karel[2], JANČO Roland[1]

[1] *Faculty of Mechanical Engineering, Slovak University of Technology in Bratislava, Institute of Applied Mechanics and Mechatronics, Nám. Slobody 17,812 31 Bratislava, Slovakia, e-mail: silvia.mihelova@stuba.sk*
[2] *Faculty of Mechanical Engineering, VSB - Technical University of Ostrava, Institute of Applied Mechanics and Mechatronics, 17. listopadu 2172/15, 708 00 Ostrava-Poruba, Czech Republic*

Abstract: The paper deals with stress and reliability calculation of a gas pipeline straight section buried in a ground (elastic foundation) loaded by heating and internal overpressure. Mathematical derivation of relations necessary for reliability calculation of the pipeline section by stochastic approach is also included in the paper. Resulting values are obtained by software Anthill using the Monte Carlo method. Input random parameters of the examined pipeline section are shown by bounded histograms.

KEYWORDS: pipeline, internal overpressure, heating, stress, software Anthill, reliability, stochastic approach

1 Introduction

Oil, natural gas, and petroleum fuels can be distributed in various ways. A part of them are stored in tanks and transported by road or rail transport. The paper deals with the pipeline transport which is the most used method of gas transport worldwide. However, in comparison with other transport types, pipeline transport represents the greatest danger of fires or explosions due to the amount of transported substance. Despite stringent construction and operation requirements, pipeline systems may fail, in the worst case, cause a gas pipeline accident, resulting in numerous material and environmental damages.

The most common causes of gas pipeline failures are small fatigue cracks in the material microstructure that continue to grow until a pipeline rupture. Unexpected additional loads, material rusting, landslides, failure of welds and flanges, pump and compressor damages, a human factor, theft, and terrorist attacks can lead to a total pipeline failure which can be avoided by a regular pipeline assessment [1, 8].

Material non-isotropicity, manufacturing inaccuracies also the human intervention in a manufacture processing have an impact on the subsequent reliability assessment of the whole system. Constructions and mechanical systems are considered reliable if they can fulfil functions for which they were designed.

The reliability assessment cannot be achieved without a general knowledge of probability theory and statistics. Fully probabilistic methods for reliability assessment are still under development. Method called SBRA (Simulation Based Reliability Assessment) is one of the fully probabilistic methods based on the Monte Carlo method which is characterized by random in- puts and outputs defined at certain intervals. These methods provide an accurate description of the mechanical construction [2].

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2 Load analysis of the straight gas pipeline section buried in the ground

Sum of internal overpressure effects of the closed thin-walled pipeline buried in the ground and its relatively high axial compressive stress caused by a temperature change results in the result- ant stress of the high-pressure gas pipeline shown in the Fig.1 (principle of stress superposition).

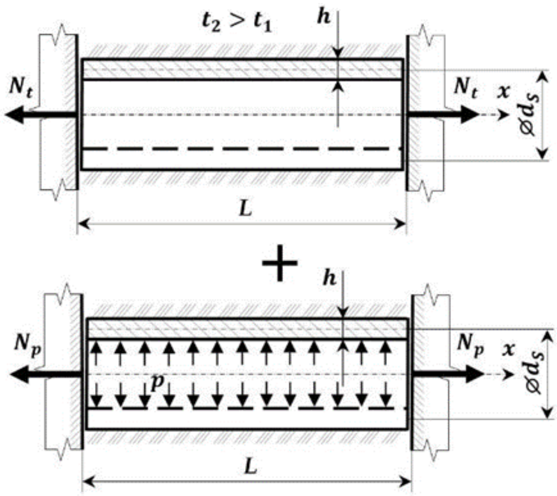

Fig. 1 Principle of superposition of the gas pipeline straight section stresses

The axial compressive stress value is high due to deformation restrain from the operating gas pressure and the temperature change which is negligible in the longitudinal direction. In the case of pipelines buried in the ground, according to the maximum shear stress hypothesis (stress intensity), compressive stress does not affect the equivalent stress. The maximum shear stress hypothesis defines inconvenient material state based on the maximum shear stress given by equation:

$$\tau_{max} \leq \tau_{allow} \, , \tag{1}$$

$$\tau_{allow} = \frac{\sigma_{allow}}{2} \, , \tag{2}$$

where

τ_{allow} [Pa] – allowable shear stress,
σ_{allow} [Pa] – allowable normal stress.

Relations (2), (3) are applicable when the general stress is considered. The maximum shear stress value can be determined by selecting one maximum value from the main stress differences divided by number 2 [3] which also represents the relation:

$$\tau_{max} = max \left\{ \begin{pmatrix} \left| \frac{\sigma_1 - \sigma_2}{2} \right| \\ \left| \frac{\sigma_1 - \sigma_3}{2} \right| \\ \left| \frac{\sigma_2 - \sigma_3}{2} \right| \end{pmatrix} \right\} . \tag{3}$$

Greek letter $\sigma_{1,2,3}$ [Pa] in the previous equation express the main stresses.

The subject of solution is a gas pipeline section DN 1200 with length L [m], mean diameter d_s [m], wall thickness h [m], and yield strength of structural steel R_p [MPa] buried in the ground consisted of limestone chippings. The pipeline section is affected by the internal gas overpressure p[Pa] and the pipe heating $\Delta_t = t_2 - t_1$ [°C]. The Δ_t represents difference between the reference temperature t_1 [°C] and the pipe temperature t_2 [°C]. For this case the heating does not reach the value causing significant material changes in the pipeline (no signs of creep or plasticity) [4].

Pipeline geometrical parameters and material properties are assumed as random. Much of these parameters are defined by the bounded normal distribution with the appropriate variance. Parameters figured in relations below, such as the bending stiffness EJ_{ZT} [Pa m^4], the coefficient of thermal expansion α_t [K^{-1}], or the Poisson material number μ [1] are not constant (usual cases of elasticity and strength), but their values are defined by closed intervals [2].

The following analysis of the selected gas pipeline part consider the theory of small deformations and Hook's law for an isotropic and homogeneous material.

3 Analytical solution of the straight gas pipeline section buried in the ground

The gas pipeline section that is subject of analysis is buried in the compacted soil. This situation leads to plane deformation (also called plane strain) in the pipeline section. The proportional elongation in the x-axis direction is equal to zero which means its components (the proportional elongation induced by the internal gas pressure p on pipeline walls and proportional elongation caused by the pipeline heating Δ_t) must be in balance with the complementary (normal) force (4) causing the proportional compression. The following equation represents resulting normal force obtained by applying the principle of superposition according to the Fig.1:

$$N = N_t + N_p \, , \tag{4}$$

where

N_t [N] – normal force induced by the gas pipeline heating,
N_p [N] – normal force induced by the internal overpressure.

The force N_t describing the gas pipeline heating effect can be derived from the next statically indeterminate tension/compression task with one degree of freedom shown in the Fig. 2 [4].

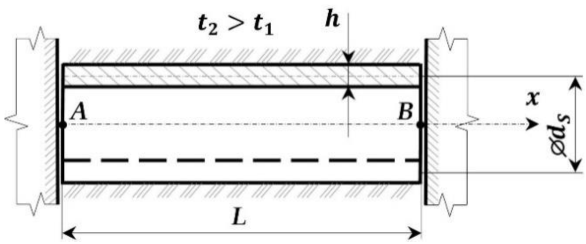

Fig. 2 Pipeline buried in the ground and uniformly loaded by the temperature difference

The pipeline in the Fig. 2 is placed between fixed constraints “A” and “B” (idealization). If temperature of the transported substance (gas) increases, the pipeline tends to expand resulting to reactions R_1 [N], R_2 [N] in constraints which leads to the uniform pressure stress in the pipeline section (Fig. 3 (*a*)). Static equilibrium of forces in the x-direction results in the relation:

$$R_1 + R_2 = 0 \, . \tag{5}$$

Values of both reactions in the previous equation (5) are unknown which leads to a statically indeterminate tension/compression task with one degree of freedom. The solution procedure is shown in the following figure [5].

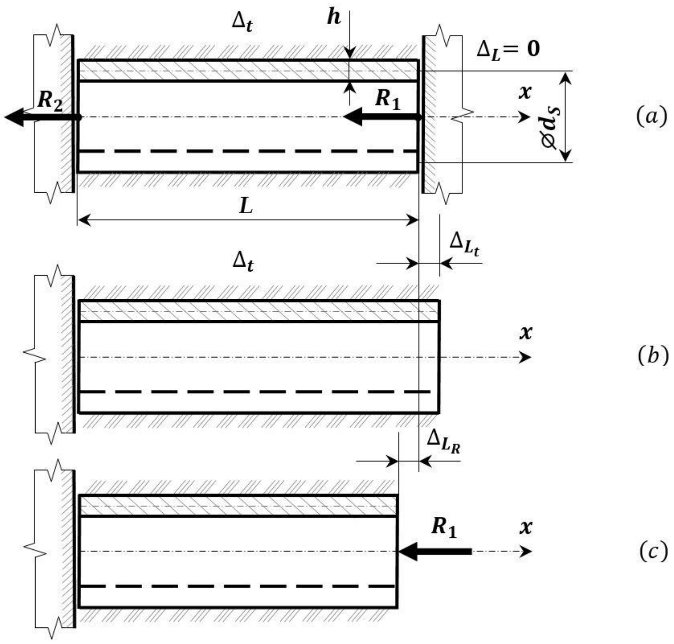

Fig. 3 Superposition applied to the pipeline loaded only by the temperature change

As already mentioned, the solution subject is the gas pipeline located between two rigid constraints (acceptable idealization) which makes the total pipe length change Δ_L [m] equals to zero (Fig. 3 (*a*)). Thus, the compatibility equation is:

$$\Delta_L = 0\,. \tag{6}$$

In order to obtain the total length change caused by the pipeline heating, one of the rigid constraints must be removed (Fig. 3 (*b*, *c*)). One side of the pipeline remains fixed and its other side is free to move. Therefore, if only the heating effect (Fig. 3 (*b*)) affects the pipeline, pipeline is extended by Δ_{L_t} [m]. If the gas pipeline section is loaded only by the normal pressure force $N_t = -R_1$, it is shortened by the dimension Δ_{L_R} [m] compared to its original length. The total length change Δ_L of the fixed pipeline subjected to stress is the superposition result of both cases shown in the Fig. 3 (*b*) and (*c*) [5]:

$$\Delta_L = \Delta_{L_t} + \Delta_{L_R}\,, \tag{7}$$

where

$$\Delta_{L_t} = \alpha_t L \Delta_t\,, \tag{8}$$

$$\Delta_{L_R} = \frac{N_t L}{EA} = \frac{-R_1 L}{EA}\,. \tag{9}$$

The relation (9) contains the Young's modulus of material elasticity in tension E [Pa] and the A [m^2] represents the pipeline cross-sectional area that is equal to the expression:

$$A = \pi d_s h\,. \tag{10}$$

The equation (11) is result of substitution of relations (8) and (9) into the equation (7). The complementary normal force value N_t prevented displacement of the pipeline, that is affected by the temperature change, can be obtained from this relation [4]:

$$\alpha_t L\Delta_t - \frac{R_1 L}{EA} = 0\,, \tag{11}$$

$$N_t = -R_1 = -EA\alpha_t\Delta_t\,. \tag{12}$$

The next step is to calculate the second axial force N_p representing the internal gas overpressure effect on pipeline walls. Its determination results from the shell theory without considering moments. It is necessary to determine normal stresses σ_m, σ_t [Pa] acting in meridian and tangential planes. Laplace equation for the selected pipeline section is in the form [6]:

$$\frac{\sigma_m}{\rho_m} + \frac{\sigma_t}{\rho_t} = \frac{p}{h}\,, \tag{13}$$

where ρ_m, ρ_t [m] are curvature radii of a centreline in meridian and tangential planes.

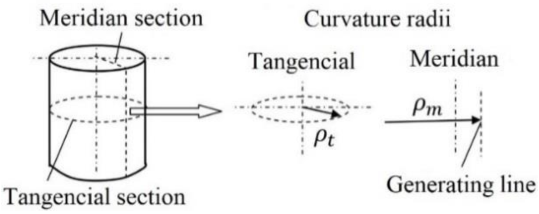

Fig. 4 Curvature radii of a centreline

Curvature radii values:

$$\rho_t = \frac{d_s}{2}, \qquad \rho_m = \infty\,. \tag{14}$$

The following relation expresses the circumferential stress value after the substitution curvature radii of the centreline into the Laplace relation (13):

$$\sigma_t = \frac{pd_s}{2h}. \tag{15}$$

The meridian stress value σ_m is determined from the equilibrium condition for the cut-off gas pipeline section (Fig. 5). It is the sum of all forces in the x-direction (17).

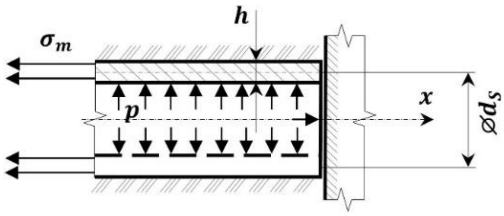

Fig. 5 Static equilibrium condition of the cut off pipeline for the meridian stress calculation

$$\sum F_{ix} = 0\,, \tag{16}$$

$$\sigma_m 2\pi \frac{d_s}{2} h - p\frac{\pi d_s^2}{4} = 0\,. \tag{17}$$

The resulting meridian stress is in the form:

$$\sigma_m = \frac{pd_s}{4h}. \tag{18}$$

The normal force value N_p describing the internal overpressure effect on pipeline walls is derived from the Hook´s law for the uniaxial tension/compression. The equation related to the relative deformation ε [1] is:

$$\varepsilon = \frac{\sigma}{E}\ , \tag{19}$$

$$\varepsilon = \frac{\Delta_L}{L}\ , \qquad \sigma = -\frac{N_p}{A}. \tag{20}$$

The proportional elongation is due to the substitution of relations (20) into the Hook´s equation in the form:

$$\varepsilon = \frac{-N_p}{EA}. \tag{21}$$

The total elongation in the x-direction is also derived from the Hook´s equation for plane stress:

$$\varepsilon = \frac{1}{E}(\sigma_m - \mu\sigma_t) = \frac{1}{E}\left(\frac{pd_s}{4h} - \mu\frac{pd_s}{2h}\right) = \frac{pd_s(1-2\mu)}{4Eh}\ . \tag{22}$$

The parameter μ [1] in the previous relation represents the Poisson material number. The normal force value N_p can be determined by comparing two previous equations (21) and (22) [4]:

$$N_p = \frac{pd_s(2\mu - 1)A}{4h}. \tag{23}$$

The resultant normal force N and its axial stress σ_N [Pa] are based on the equation (4) equal to the sum of obtained normal forces related to the heating and internal overpressure effects of the selected pipeline section:

$$N = -A\left[\frac{(1-2\mu)pd_s}{4h} + E\alpha_t\Delta_t\right], \tag{24}$$

$$\sigma_N = \frac{N}{A} = -\left[\frac{pd_s}{4h}(1-2\mu) + E\alpha_t\Delta_t\right]. \tag{25}$$

The resultant stress σ_N is therefore ratio of the normal force N and the pipeline cross-section A. This stress is often compared with the standardized stress value σ_{allow} [Pa] to determine the construction reliability. In the next chapter, the gas pipeline reliability is determined by the probabilistic approach using the Monte Carlo method and the Anthill calculation software considering the random character of individual input parameters.

4 Reliability solution of the pipeline section by probabilistic approach using the Monte Carlo method

There are no perfect isotropic materials, precisely manufactured components without little deviations from a production drawing also considerable number of factories are still not fully automated. Therefore, it is impossible to design a completely safe construction without any damages. It is caused by various input parameters with random distribution. This fact also affects reliability evaluation. Technical devices are considered reliable if they can fulfill the functions for which they were designed. The aim is to minimize errors in the process, from a component design to its manufacture, which can further reduce a potential failure occurrence.

Design still requires a more accurate reliability (durability) determination. However, this cannot happen without using statistical relations and knowledge of the probability theory. Fully probabilistic approaches for the reliability system evaluating are still in development process. Although, many of them are already a part of Czech or international standards. The beginnings of the reliability assessment and related standards or rules creation are mentioned in [7].

One of fully probabilistic methods for reliability evaluation is the Simulation-Based Reliability Assessment (SBRA) which is based on the Monte Carlo method. The method is used for solving random or deterministic tasks. In the case of deterministic problem, the task is converted to a stochastic character allowing the Monte Carlo method application. The required solution is the result of many observed occurrences that are further evaluated in a statistical file containing data for probability evaluation of random parameters. The number of random parameters repetitions is very large. Due to this reason, various computational software products are often used to facilitate random parameters generation and work with them [2]. The computational software named Anthill is used to apply the probability distribution of pipeline input parameters. The table below includes maximum and minimum input parameters also their median and mean.

Table 1 Input parameters of the pipeline section under their random character consideration

Inputs	Anthill inscription	Minimum	Median	Mean	Maximum	Fig.
d_s [mm]	1204*"n1-02.dis"	1179.92	1204.01	1204.01	1228.08	6.
h [mm]	13.5*"n1-02.dis"	13.23	13.50	13.50	13.77	7.
p [MPa]	"User defined 1"	0.00	6.60	6.62	29.99	8.
Δ_t [°C]	"User defined 2"	15.05	25.13	25.18	59.99	9.
E [MPa]	20800*"n1-03.dis"	201760.00	208007.91	208002.96	214240.00	10.
α_t [K^{-1}]	1.4E-5*"User defined 3"	0.000012	0.000014	0.000014	0.000016	11.
μ [1]	"Uniform"	0.30	0.32	0.32	0.34	12.
R_p [MPa]	"a36-mcont.dis"	248.00	338.34	339.18	499.99	13.

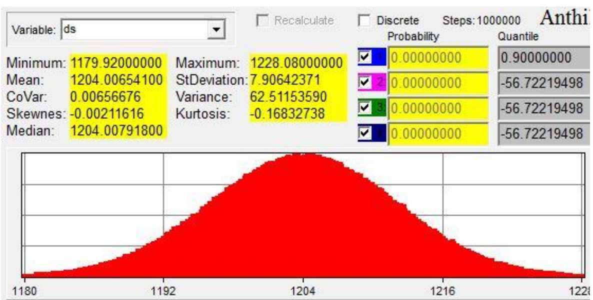

Fig. 6 Pipeline diameter histogram
$d_s = 1204.1 \pm {}^{26.06}_{24.07}$ [mm]

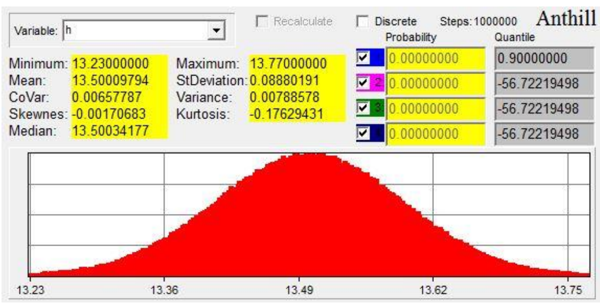

Fig. 7 Pipeline thickness histogram
$h = 13.5 \ \pm 0.27 \ [mm]$

Figures 8 and 9 consider the low probability of extreme fluctuations in the case of the internal gas overpressure and the pipeline temperature difference. This includes the possibility of potential explosions, drilling thefts, terrorist attacks or pipeline storage in extreme low or high temperature conditions.

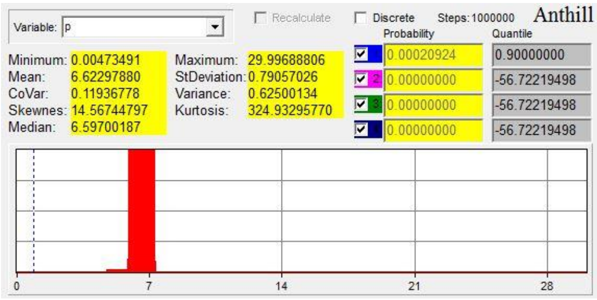

Fig. 8 Internal gas overpressure histogram $p = 6.60 \pm {}^{23.39}_{6.60}$ $[MPa]$

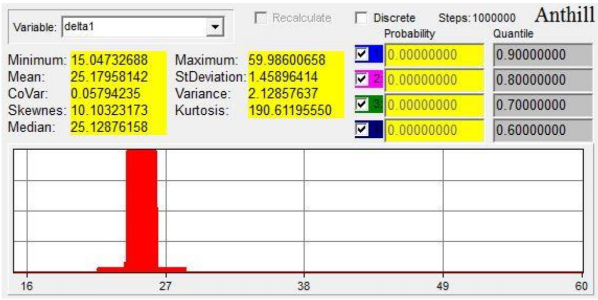

Fig. 9 Pipeline temperature difference histogram $\Delta_t = 25.13 \pm {}^{34.86}_{10.08}$ [°C]

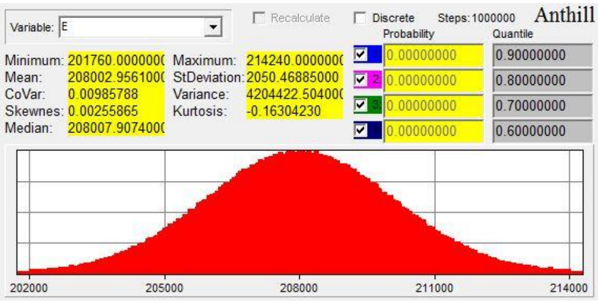

Fig .10 Young´s tensile elasticity modulus $E = 208007.91 \pm {}^{6247.91}_{6232.09}$ $[MPa]$

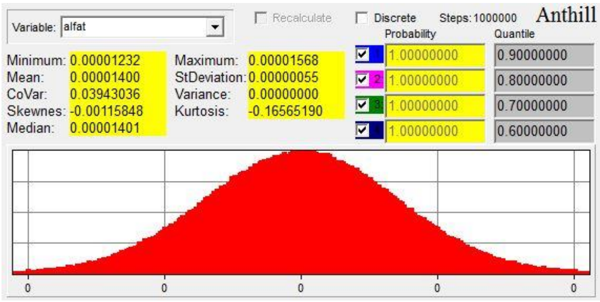

Fig. 11 Thermal expansion coefficient $\alpha_t = 0.000014 \pm 0.000002$ $[K^{-1}]$

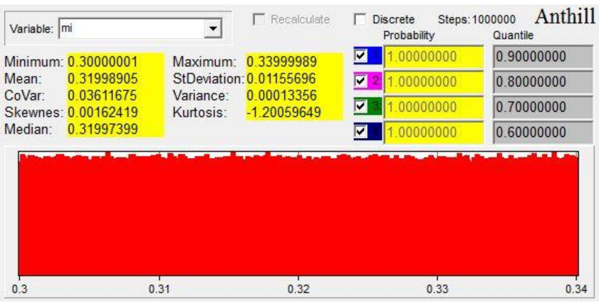

Fig. 12 Histogram of the Poisson pipeline material number $\mu = 0.32 \pm 0.02$ [1]

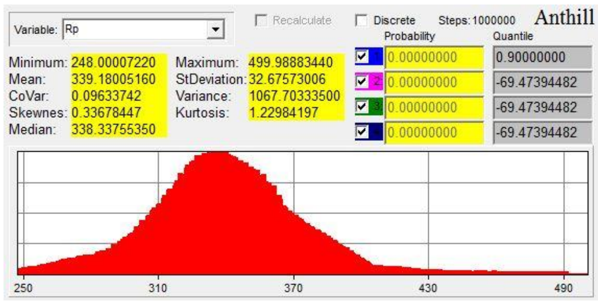

Fig. 13 Yield strength of the pipeline material (structural steel) $R_p = 338.34 \pm {}^{161.65}_{90.34}$ $[MPa]$

Input parameters from Table 1 are displayed in the histograms (Figure 6-13). Nature of individual parameters is random which better corresponds to real technical values. Types of histograms used for each input parameter those notations are also included in the Table 1:

- "n1-02.dis" = bounded normal distribution ± 2 % (SW Anthill standard),
- "n1-03.dis" = bounded normal distribution ± 5 % (SW Anthill standard),
- "Uniform" = bounded uniform distribution (SW Anthill standard),
- "User defined 1, 2, 3" = user defined,
- "a36-m-cont.dis" = yield strength of the carbon steel A36 (continuous histogram adjustment from discrete histogram "a36-m-dis" that is SW Anthill standard) [2].

Output parameter values of the gas pipeline section (Table 2), that are also random, can be determined based on relations from the previous chapter. The following table represents statistical characteristics of obtained parameters and their respective histogram numbers. Individual calculations are realized for 10^6 random simulations by the Monte Carlo method.

Table 2 Output parameters of the pipeline section under their random character consideration

Outputs	Relation	Minimum	Median	Mean	Maximum	Graph
A [mm^2]	(10)	49088.44	51064.13	51064.09	53044.73	14.
N_t [N]	(12)	-9668820.86	-3734439.75	-3744389.87	-2049692.18	15.
N_p [N]	(23)	-13637977.66	-2695992.54	-2714844.83	-2053.06	16.
N [N]	(24)	-17615595.31	-6436011.00	-6459234.70	-3279732.44	17.
σ_N [MPa]	(25)	-338.86	-126.05	-126.49	-63.84	18.

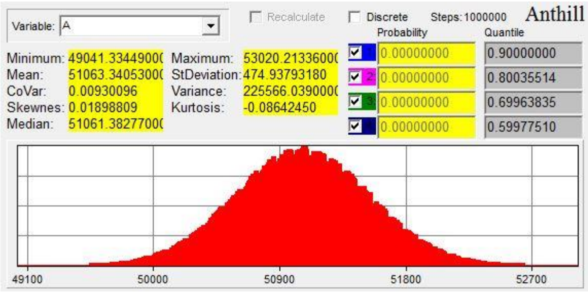

Fig. 14 Histogram of the pipeline cross-section $A = 51061.38 \pm {}^{2020.05}_{1958.83}$ $[mm^2]$

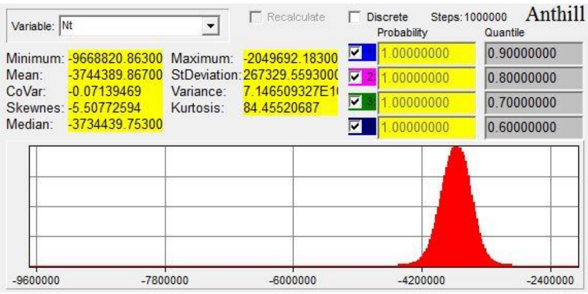

Fig. 15 Histogram of the normal force induced by the heating $N_t = -3734439.75 \pm {}^{5934381.11}_{1684747.57}$ $[N]$

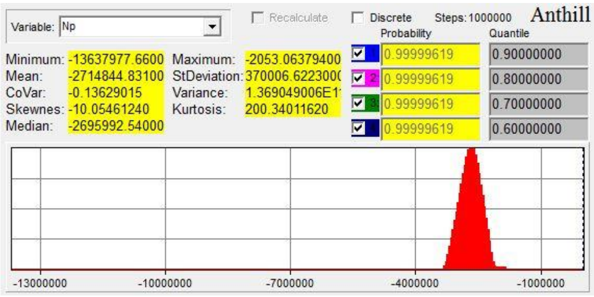

Fig. 16 Histogram of the normal force induced by the internal gas overpressure $N_p = -2695992.54 \pm {}^{10941958.48}_{2693939.48}$ $[N]$

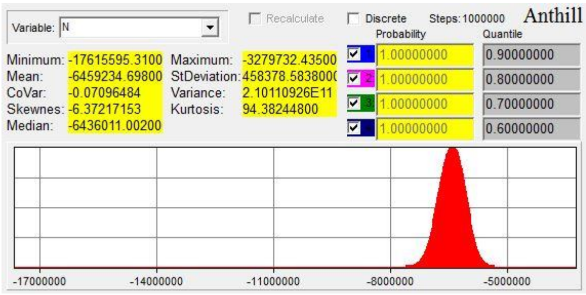

Fig. 17. Normal force histogram $N = -6436011.00 \pm {}^{11179584.31}_{3156278.56}$ $[N]$

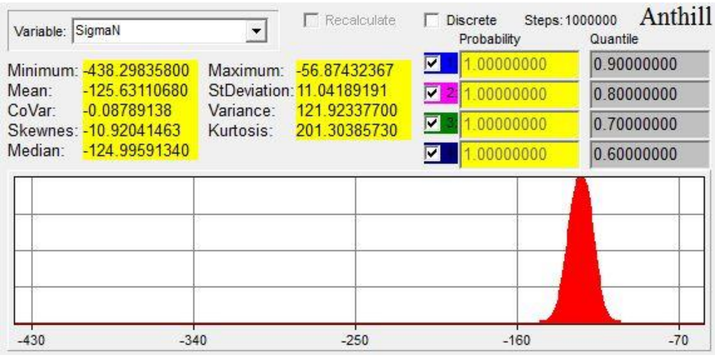

Fig. 18 Normal stress histogram $\sigma_N = -125.00 \pm {}^{313.30}_{68.13}$ $[MPa]$

Input or output parameters (Table 1 and 2) can also be written in the form:

$$x = x_{med}{}^{x_{max}-x_{med}}_{-(x_{min}-x_{med})}\,, \tag{26}$$

where x is value from interval $\langle x_{min};\ x_{max}\rangle$ and $x_{min,med,max}$ are its minimum, median, and maximum. This form is also used in the description of individual histograms (Fig. 6 - 18).

The paper aim is to determine the reliability of the gas pipeline straight section by the SBRA method using the software Anthill. It is the probability determination of an impermissible stress occurrence based on the yield material strength R_p. The allowable normal stress is possible to determine with respect to the strength hypothesis of maximum shear stress (1) to (3):

$$\sigma_{allow} = R_p\,. \tag{27}$$

An unsatisfactory condition is a situation when the calculated normal stress σ_N exceeds the yield straight value.

The gas pipeline section reliability is evaluated by the reliability function F_s that is defined by the difference in the yield straight histogram and the calculated normal stress:

$$F_s = R_p - |\sigma_N|\,. \tag{28}$$

Following Fig. 19 and 20 show the reliability function F_s (left) and the 2D normal stress histogram σ_N compared to the yield strength R_p. Two situations can occur in the case of the reliability assessment:

– $F_s \geq 0$ = construction is reliable, (29)
– $F_s < 0$ = construction is unreliable. (30)

The pipeline reliable area is shown on the Figure 19 and the reliability function given by (28) is represented by the green line in the 2D histogram.

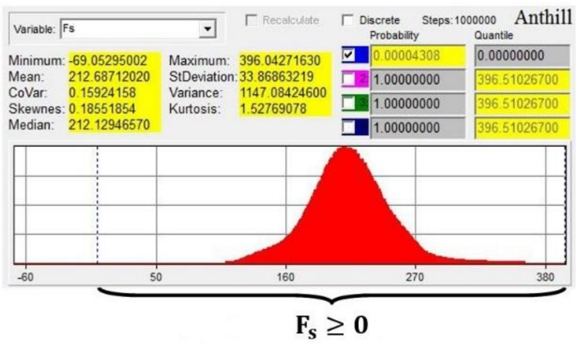

Fig. 19 Reliability function F_s

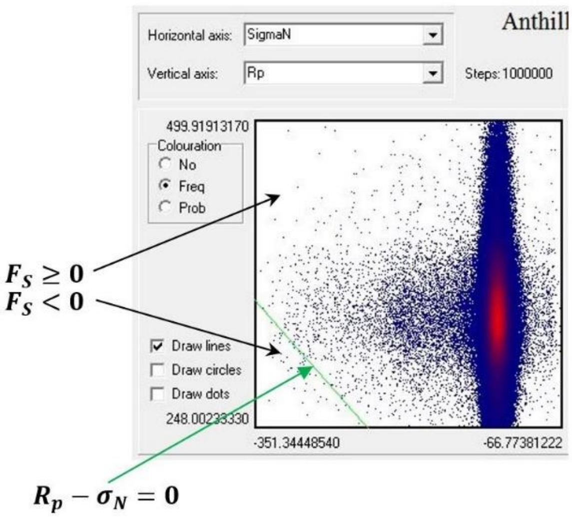

Fig. 20 2D histogram σ_N vs. R_p

The area to the left of the green line $R_p - \sigma_N\ (F_s)$ represents unsatisfactory reliability values (unfavourable conditions) (Fig. 20) indicating a certain pipeline failure probability P_f. This unfavourable condition is defined by the plastic deformation occurrence that does not always lead to serious pipeline wall damages. The probability value can be defined from the Fig. 19. If the quantile is equal to zero, the probability value is $P_f = 0.004$ %. This probability has the reliability function given by the relation (28) and is calculated by the software Anthill using the statistical relation:

$$P_f = \frac{N_f}{N_n} \,, \tag{31}$$

N_f – number of unfavorable conditions,
N_n – total simulation numbers.

The failure probability P_f is compared with the design probability P_{allow} based on standards or determined by a customer agreement. The failure probability value should be less than the allow probability:

$$P_f \leq P_{allow} \,. \tag{32}$$

Comparison of these two probabilities leads to reliability assessment of the construction (gas pipeline buried in the ground).

CONCLUSION

Equations for the determination of the stress-strain relation of the gas pipeline straight section buried in the ground were derived. The SBRA method based on the direct Monte Carlo method was used for the probability calculation. The probabilistic reliability assessment is further compared with the design probability P_{allow} obtained by the yield strength overcoming of the chosen material according to the low shear stress hypothesis.

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