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USING FLOQUET THEORY IN THE PROCEDURE FOR INVESTIGATION OF THE MOTION STABILITY OF A ROTOR SYSTEM EXHIBITING PARAMETRIC AND SELF-EXCITED VIBRATION

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Abstract: The computational procedure for investigation vibration stability of a flexible rotor consisting of an asymmetric shaft, one disc, and supported by ball bearings is developed in this work. Lagrange equations of the second kind were used for derivation of the motion equation. The vibration response stability of the Jeffcott-like rotor was studied by means of eigenvalues of a transition matrix. Three different methods for approximation of the transition matrix have been investigated. The presented simulations are focused on studying the influence of parametric excitation produced by the shaft asymmetry and self-excitation vibration caused by the shaft material damping. The numerical results proved the applicability of the developed procedure, which has been verified by the direct integration of the motion equation.

KEYWORDS: Floquet theory, parametric excitation, self-excitation, asymmetric shaft, direct integration

1 Introduction

In rotating machinery with parametric excitation, large growth of a vibration response amplitude is observed at certain intervals of operating speed. Parametric excitation occurring at rotor systems is generated by different causes such as: (i) asymmetric cross-section of a rotor, (ii) rotor with a cracked shaft, (iii) geared rotor-bearing system, (iv) technological processes, etc. The coefficient matrices in a motion equation of the system with parametric excitation are periodically time-dependent [1, 2]. Another undesirable and sometimes dangerous vibration in the rotor systems can be caused by self-excitation vibration initiated by various sources such as: (i) material damping of rotors, (ii) dry friction, (iii) elements such as journal bearings, seals, etc., (iv) flow-induced vibration of elastic bodies, etc. The linearized motion equation of the system with self-excitation is characterised by a circulatory matrix, which other authors also call a non-conservative positional force [3-8].

Only in some cases it is possible to supress effectively the vibration response of a large amplitude or undesirable vibrations, for example using the concept of active vibration control [9] or by semi-active damping devices [10-13].

Nevertheless, for reliable operation of the rotor system, it is necessary to precisely know the regions of stable/unstable vibration response caused by the parametric excitation and self-excitation. Srinath, et al. [14] presented an approach to derive generalized criteria for determining the stability/instability regions of any asymmetric rotating shaft system and showed its applicability for comparison with experimental results and other practical applications. In [15], the stability analysis of an asymmetric shaft discretized by finite

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elements is evaluated using Floquet theory. The stability of the rotor system is improved here by tuning of radial active magnetic bearing parameters.

Also, the lateral stability of spherical parallel manipulators [16] used as robotic joint and the stability of the feedback controller for the planar robot [17] is investigated by means of an approach based on the utilization of the Floquet transition matrix method.

In the presented work, the effect of an asymmetric cross-section of a rotor shaft and a material damping is investigated, with regard to vibration response stability. To study the vibration stability of the motion equation, the coefficient matrices of which are periodic functions of time, the Floquet theory [18-20] is used. In this work, a computational procedure for investigation of the vibration stability of a Jeffcott-like rotor was developed and its results were verified by means of a direct integration of the motion equation.

2 Floquet theory and approaches for assembly of the transition matrix in rotor dynamics

Investigation of the vibration stability in rotor dynamics is characterized by solving a homogeneous differential equation of the second order with variable coefficients, describing time history of deviations of a disturbed motion

$$\mathbf{M}(t)\Delta\ddot{\mathbf{x}} + \mathbf{B}(t)\Delta\dot{\mathbf{x}} + \mathbf{K}(t)\Delta\mathbf{x} = \mathbf{0}. \tag{1}$$

$\mathbf{M}(t)$, $\mathbf{B}(t)$, $\mathbf{K}(t)$ denote the matrix of mass, damping, and stiffness, respectively, which are square of order n and dependent on time t, $\Delta\ddot{\mathbf{x}}$, $\Delta\dot{\mathbf{x}}$, and $\Delta\mathbf{x}$ are the vectors of deviations of generalized accelerations, velocities, and displacements of the disturbed motion, respectively, $\mathbf{0}$ is zero vector, and $(\dot{\cdot})$ denotes the first derivative with respect to time.

The disturbed motion equation is transformed into the state space

$$\dot{\mathbf{y}} = \mathbf{A}(t)\mathbf{y}, \quad \mathbf{A}(t) = \begin{bmatrix} -\mathbf{M}^{-1}(t)\mathbf{B}(t) & -\mathbf{M}^{-1}(t)\mathbf{K}(t) \\ \mathbf{I} & \mathbf{O} \end{bmatrix}, \quad \mathbf{y} = [\Delta\dot{\mathbf{x}} \quad \Delta\mathbf{x}]^{\mathrm{T}}. \tag{2}$$

$\mathbf{I}$ is a unit matrix, $\mathbf{O}$ is a zero matrix, and matrix $\mathbf{A}(t)$ is a continuous function of time with the period of T.

The fundamental matrix [18] of solution of Eq. (2) can be expressed as

$$\mathbf{F}(t) = \mathbf{Z}(t)\mathrm{e}^{t\mathbf{R}}. \tag{3}$$

$\mathbf{F}(t)$ denotes the fundamental matrix, $\mathbf{Z}(t)$ is a regular periodic matrix with the period of T, and $\mathbf{R}$ is a constant matrix. Matrix $\mathrm{e}^{t\mathbf{R}}$ is called the monodromy matrix [18], and its eigenvalues are called characteristic multipliers of the system. The eigenvalues of matrix $\mathbf{R}$ are called the characteristic exponents (sometimes called Floquet exponents).

Solution of the linear differential Eq. (2) with the initial condition $\mathbf{y}(t_0), t_0 = 0$ s is

$$\mathbf{y}(t) = \mathbf{H}(t,t_0)\mathbf{y}(t_0), \qquad \mathbf{H}(t,t_0) = \mathbf{F}(t)\mathbf{F}^{-1}(t_0), \tag{4}$$

where $\mathbf{H}(t,t_0)$ is the transition matrix that maps initial condition at time t_0 to the state of the system at time t.

After substituting (3) into (4) for the time of one period ($t_0 = 0$ s, $t = T$) and considering condition $\mathbf{Z}(T) = \mathbf{Z}(0)$ due to the periodicity of matrix $\mathbf{Z}(t)$, the transition matrix of the solution of Eqs. (1) and (2) has the following form

$$\mathbf{H}(T,0) = \mathbf{Z}(0)\mathrm{e}^{T\mathbf{R}}\mathbf{Z}^{-1}(0). \tag{5}$$

All eigenvalues of the monodromy matrix $\mathrm{e}^{T\mathbf{R}}$ and the transition matrix $\mathbf{H}(T,0)$ are the same because relation (5) represents a similarity transformation.

The stability of solution of the Eqs. (1) and (2) can thus be judged by the eigenvalues of the transition matrix (5), assembled over the span of time of one period. If the modulus of eigenvalue $|\lambda_{\mathrm{i}}| < 1$ for $i = 1, 2, \dots, 2n$, then the solution is asymptotically stable. However, if the modulus of eigenvalue $|\lambda_{\mathrm{i}}| \leq 1$ for $i = 1, 2, \dots, 2n$, then the solution is only Lyapunov stable. In all other cases, though, the solution is unstable. Therefore, the stability is evaluated by the locations of the eigenvalues on unit circle in the complex plane [18]. According to the point, at which the largest eigenvalue of the transition matrix crosses the unit circle, it is possible to determine the type of instability [19].

There exists a number of methods [18-20] for the construction of the transition matrix (5). One of them is to obtain the transition matrix by a repeated solution for differently chosen initial conditions. Then the transition matrix has the following representation

$$\mathbf{H}(T,0) = \mathbf{Y}(T)\mathbf{Y}^{-1}(0). \tag{6}$$

$\mathbf{Y}(0)$ is the fundamental matrix, the columns of which are vectors of initial conditions, and $\mathbf{Y}(T)$ is a matrix, the columns of which are vectors of solutions at time T.

The second method represents an approximation of the transition matrix and requires repeated computation of exponential matrices. The time interval of period T is divided into N time subintervals, and it is assumed that matrix $\mathbf{A}(t)$ of the system is constant in each of them and therefore, the transition matrix can be expressed as a product of exponential matrices

$$\mathbf{H}(T,0) = \mathrm{e}^{(T-t_{\mathrm{N}-1})\mathbf{A}_{\mathrm{N}}} \dots \mathrm{e}^{(t_{\mathrm{i}}-t_{\mathrm{i}-1})\mathbf{A}_{\mathrm{i}}} \dots \mathrm{e}^{(t_1-t_0)\mathbf{A}_1}, \text{ for } i = 1, 2, \dots, N. \tag{7}$$

In this study, the last method for the approximation of the transition matrix is based on kinematic relations of the Newmark integration technique. As in the previous case, the time interval of T is divided into N time subintervals, see [18].

3 The studied rotor system and its equations of motion

The Jeffcott-like rotor has been investigated and its arrangement can be seen in Fig. 1.

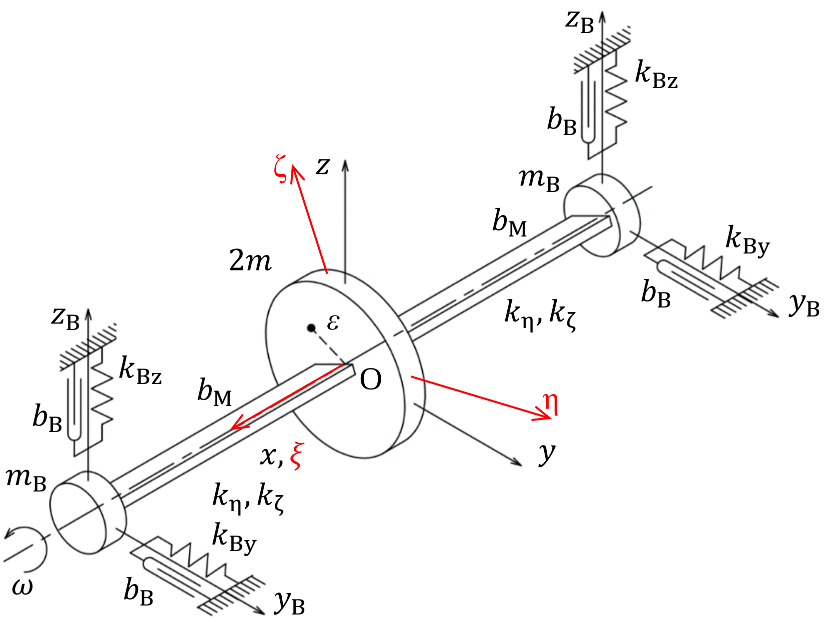


Fig. 1. Model of the Jeffcott-like rotor.

Two coordinate systems are introduced. The first one (xyz) is the stationary reference frame, its origin O coincides with the disc centre, and the x-axis lies on the undeflected bearing centreline. The second one (ξηζ) is the rotating reference frame. Its origin coincides with the origin of the stationary one, and both η-axis and ζ-axis coincide with the principal directions of the shaft stiffness.

The rotor system is symmetric with respect to the plane passing through the rotor disc and perpendicular to the axis of rotation. The following assumptions were made: (i) the stator is considered as absolutely rigid body, (ii) the disc is rigid and is fixed to the rotor shaft at the midpoint of the bearing span, (iii) the massless flexible shaft is of constant rectangular cross-section, (iv) the shaft is supported by two identical ball bearings, (v) the damping in bearing support and shaft material damping are considered as viscous, (vi) the rotor speed ω is constant, and (vii) the longitudinal and torsional vibrations are insignificant.

In Ferfecki, et al. [21] the motion equation for similar rotor system is derived by means of the Lagrange's equations of the second kind. Exploiting the symmetry of the system, the vibration of the investigated rotor system in the stationary coordinate system can be described by a set of four differential equations

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{B}\dot{\mathbf{q}} + [\mathbf{K}(t) + \omega\mathbf{K}_{\mathrm{C}}]\mathbf{q} = \mathbf{f}_{\mathrm{G}} + \mathbf{f}_{\mathrm{A}}(t), \tag{8}$$

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & m_{\mathrm{B}} & 0 \\ 0 & 0 & 0 & m_{\mathrm{B}} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} b_{\mathrm{M}} & 0 & -b_{\mathrm{M}} & 0 \\ 0 & b_{\mathrm{M}} & 0 & -b_{\mathrm{M}} \\ -b_{\mathrm{M}} & 0 & b_{\mathrm{M}} + b_{\mathrm{B}} & 0 \\ 0 & -b_{\mathrm{M}} & 0 & b_{\mathrm{M}} + b_{\mathrm{B}} \end{bmatrix},$$

$$\mathbf{K}(t) = \begin{bmatrix} k_{\eta}c^2 + k_{\xi}s^2 & sc(k_{\eta} - k_{\xi}) & -(k_{\eta}c^2 + k_{\xi}s^2) & -sc(k_{\eta} - k_{\xi}) \\ sc(k_{\eta} - k_{\xi}) & k_{\eta}s^2 + k_{\xi}c^2 & -sc(k_{\eta} - k_{\xi}) & -(k_{\eta}s^2 + k_{\xi}c^2) \\ -(k_{\eta}c^2 + k_{\xi}s^2) & -sc(k_{\eta} - k_{\xi}) & k_{\eta}c^2 + k_{\xi}s^2 + k_{\mathrm{By}} & sc(k_{\eta} - k_{\xi}) \\ -sc(k_{\eta} - k_{\xi}) & -(k_{\eta}s^2 + k_{\xi}c^2) & sc(k_{\eta} - k_{\xi}) & k_{\eta}s^2 + k_{\xi}c^2 + k_{\mathrm{Bz}} \end{bmatrix}, \tag{9}$$

$$\mathbf{K}_{\mathrm{C}} = \begin{bmatrix} 0 & b_{\mathrm{M}} & 0 & -b_{\mathrm{M}} \\ -b_{\mathrm{M}} & 0 & b_{\mathrm{M}} & 0 \\ 0 & -b_{\mathrm{M}} & 0 & b_{\mathrm{M}} \\ b_{\mathrm{M}} & 0 & -b_{\mathrm{M}} & 0 \end{bmatrix},$$

$$\begin{gathered} \mathbf{f}_{\mathrm{A}}(t) = m\omega^2\varepsilon[c\cos(\beta_0) - s\sin(\beta_0), s\cos(\beta_0) + c\sin(\beta_0), 0, 0]^{\mathrm{T}}, \\ \mathbf{f}_{\mathrm{G}} = g[0, -m, 0, -m_{\mathrm{B}}]^{\mathrm{T}}, \end{gathered} \tag{10}$$

$$s = \sin(\omega t), c = \cos(\omega t). \tag{11}$$

$\mathbf{K}_{\mathrm{C}}$ is the circulatory matrix, $\mathbf{q}, \dot{\mathbf{q}}$, and $\ddot{\mathbf{q}}$ are the vectors of displacements $[y, z, y_{\mathrm{B}}, z_{\mathrm{B}}]^{\mathrm{T}}$, velocities, and accelerations, respectively, $\mathbf{f}_{\mathrm{A}}$ is the vector of unbalance forces, $\mathbf{f}_{\mathrm{G}}$ is the vector of gravity forces, and g is the gravity acceleration.

The studied rotor is loaded by the disc unbalance and the weight of the disc and bearings. The disc and shaft are made from steel and physical parameters of the studied rotor system are listed in Table 1 and its parameters are similar to the Bently Nevada Rotor Kit test rig. It is a small rotating machine for investigation of operating regimes and a number of machinery malfunctions under realistic circumstances.

Table 1. Physical parameters of the studied rotor.

Parameter	Symbol	Value	Unit
mass lumped at the rotor midpoint	m	0.5	kg
mass lumped at the bearing stations	m_B	0.1	kg
coefficient of the shaft material damping	b_M	0 - 1 000	$kg \cdot s^{-1}$
coefficient of damping in the bearing support	b_B	0.0012 - 4 800	$kg \cdot s^{-1}$
shaft stiffness in η direction	k_η	$7.392 \cdot 10^5$	$N \cdot m^{-1}$
shaft stiffness in ζ direction	k_ζ	$2.218 \cdot 10^6$	$N \cdot m^{-1}$
bearing stiffness in y direction	k_{By}	$2 \cdot 10^7$	$N \cdot m^{-1}$
bearing stiffness in z direction	k_{Bz}	$4 \cdot 10^7$	$N \cdot m^{-1}$
eccentricity of the disc centre of mass	ε	20	μm
phase shift of the disc unbalance	β_0	0	rad

4 Results of the numerical simulations

The task is to study the effect of the shaft cross-section asymmetry, as well as damping in the bearing supports, and the shaft material damping on the rotor vibration stability. Investigation of the vibration stability of the studied rotor is judged by the stability of solution of the motion Eq. (8), which is evaluated by the moduli of eigenvalues of the transition matrix assembled over one period. Therefore, the stability was evaluated by the Floquet transition matrix method [19], and the direct integration of the motion equation was used to verify the rotor vibration stability.

Three different approaches for the approximation of the transition matrix assembled over one solution period were tested. These approaches require repeated: (i) solution of the governing equations for different initial conditions, (ii) the product of exponential matrices, and (iii) the product of matrices assembled by means of kinematic relations of the Newmark integration technique. These approaches provided the same information about the vibration stability. Nevertheless, in terms of the solution time and the solution accuracy, the approximation of the transition matrix based on kinematic relations of the Newmark integration technique proved to be the best.

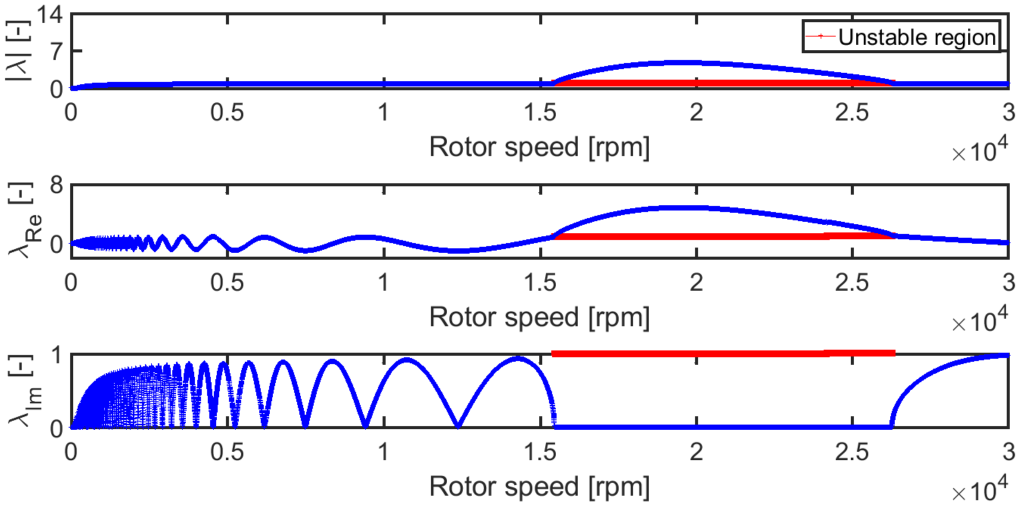


Fig. 2. Dependency of module $|\lambda|$, real λ_{Re} and imaginary λ_{Im} part of the largest eigenvalue on the rotor speed.

The vibration stability was investigated for the rotor speed ranging up to 30 000 rpm. Fig. 2 shows the dependence of the module, real, and imaginary part of the largest eigenvalue of the transition matrix on the rotor speed. Vibration in the rotor system is suppressed only by the damping in the bearing support. In the investigated range of the rotor speed, one region of unstable motion exists due to the shaft asymmetry. The unstable region of the rotor speed is denoted by the red line (Fig. 2).

For the investigated rotor system, the simulations show that the bounds of the region of unstable motion are only negligibly influenced by the amount of damping in the bearing support, see Table 2.

Table 2. Effect of damping in the bearing support on the bounds of the unstable region.

Damping in support [$kg \cdot s^{-1}$]	Damping in shaft [$kg \cdot s^{-1}$]	Lower bound [rpm]	Upper bound [rpm]
0.0012	0	15 439 - 15 441	26 053 - 26 056
0.12	0	15 439 - 15 441	26 053 - 26 056
1.2	0	15 439 - 15 441	26 053 - 26 056
12	0	15 439 - 15 441	26 053 - 26 056
1 200	0	15 441 - 15 444	26 071 - 26 073
4 800	0	15 461 - 15 464	26 277 - 26 280

Stability of the vibration evaluated by the Floquet theorem is confirmed in the vicinity of the bounds of the unstable region by direct integration of the motion Eq. (8).

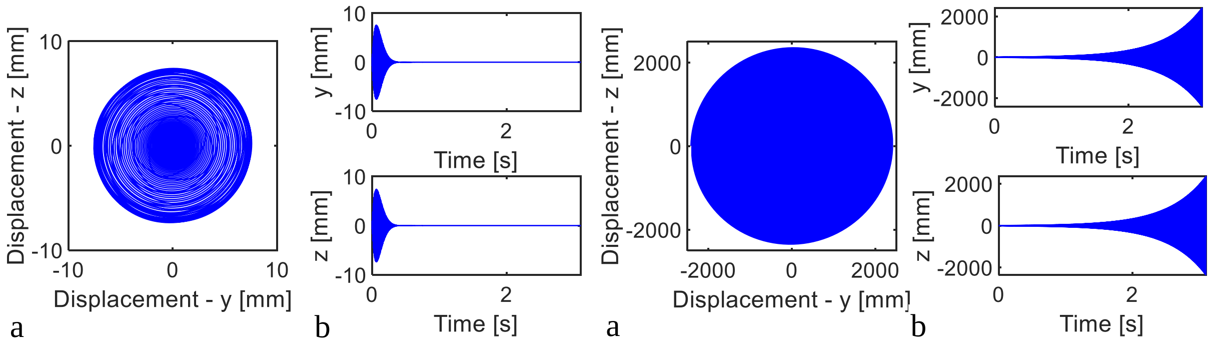


Fig. 3. Orbit (a) and time histories of the displacement components (b) of the rotor disc for the rotor speed of 15 461 rpm (left) and 15 464 rpm (right).

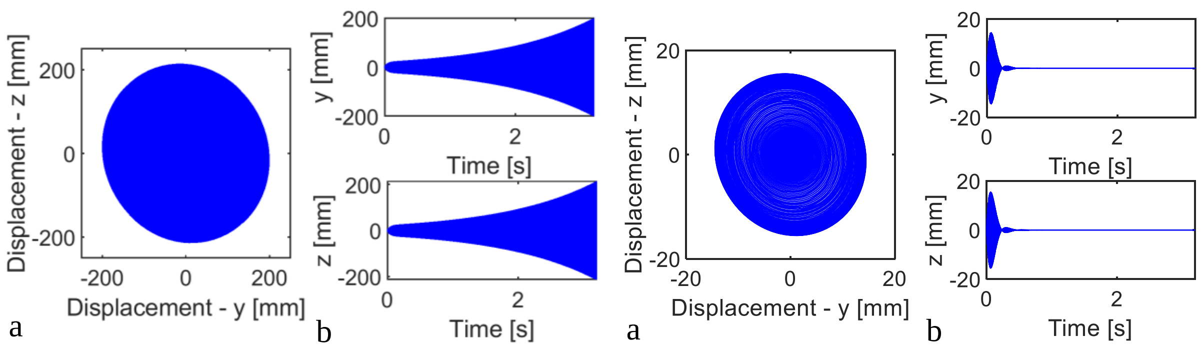


Fig. 4. Orbit (a) and time histories of the displacement components (b) of the rotor disc for the rotor speed of 26 277 rpm (left) and 26 280 rpm (right).

The orbit and the time histories of the horizontal (y) and vertical (z) components of the rotor disc displacement are drawn in Figs. 3, 4 (for the damping in support 4 800 kg·s^{-1}). As evident, the vibration amplitude outside the unstable region bounds diminishes after the transition motion. On the contrary, the vibration amplitude inside the bounds of the unstable region increases with time. The largest eigenvalue of the transition matrix crossed the unit circle through the real axis, and this type of instability is referred to as the cycle saddle-node type. In the region of unstable motion, the vibration motion amplitude increases with time.

The shaft material damping introduces another region of unstable motion. The effect of the shaft material damping on the transition matrix largest eigenvalue in dependence on the rotor speed is drawn in Fig. 5.

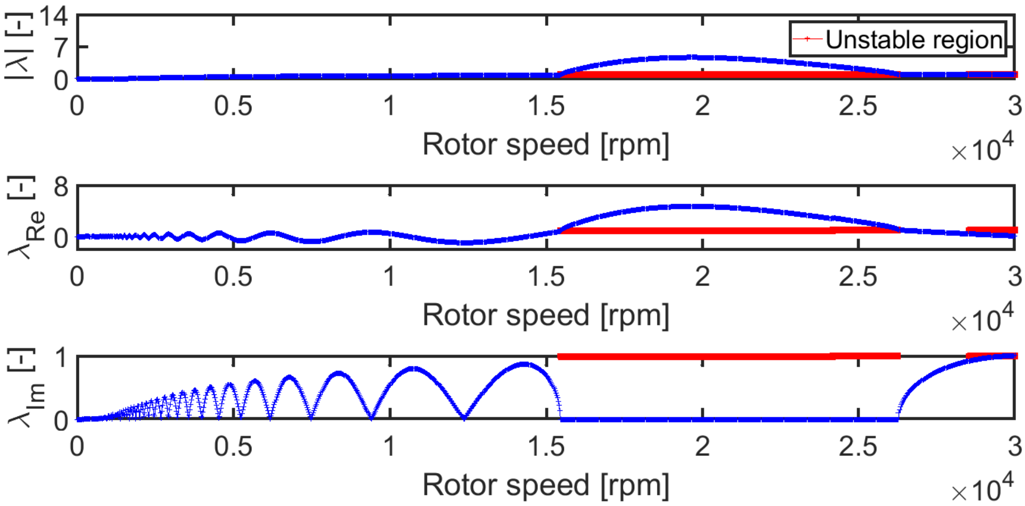


Fig. 5. Dependence of the largest eigenvalue on the rotor speed for the material damping coefficient 25 kg·s^{-1} and damping in the bearing supports 4 800 kg·s^{-1}.

For the first region of unstable motion, the dependence of the real and imaginary parts of the transition matrix largest eigenvalue on the rotor speed for the rotor system without (Fig. 2) and with (Fig. 5) the shaft material damping has the same qualitative character. Therefore, the unstable motion will be of an identical type.

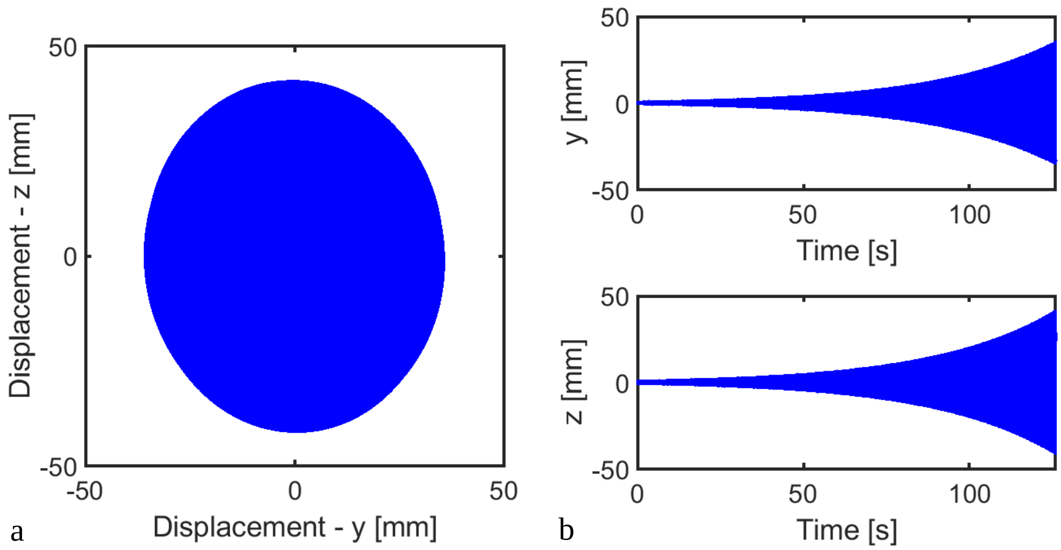
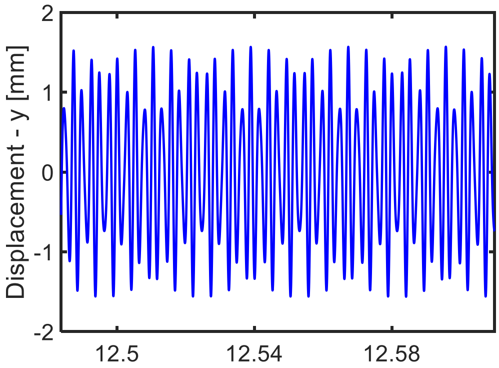


Fig. 6. Orbit (a) and time histories of the rotor disc displacement components (b) for the rotor speed 28 549 rpm at the lower bound of the 2nd region of unstable motion.

At the lower bound of the 2nd region of unstable motion, the transition matrix largest eigenvalue exceeds the unit circle as a pair of complex-conjugate multipliers ($0.396 \pm 0.919i$ for 28 549 rpm), see Fig. 5. The stable vibration becomes unstable and this bifurcation is referred to as the secondary Hopf bifurcation [19].

It has been shown that at the lower bound of the 2nd region of unstable motion, the rotor disc displacement components grow as well as the rotor disc orbit (Fig. 6). In dependence on the type of bifurcation, the response will be composed of several component frequencies. Individual frequencies are evident from the detail of the time history (Fig. 7a) and its Fourier transformation (Fig. 7b). The motion is composed of two main frequencies (Fig. 7b) and their mean value is equal to the rotor speed frequency.



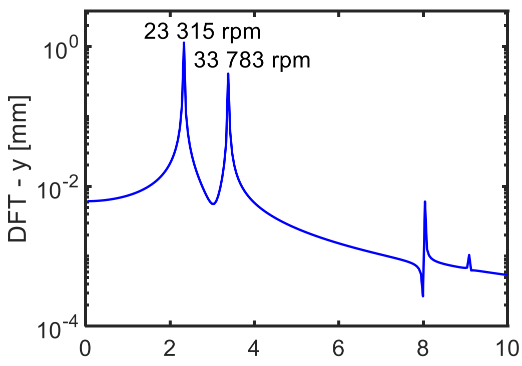


Fig. 7. Detail of the time history (a) and the discrete Fourier transform (b) of the horizontal displacement of the rotor disc for the rotor speed 28 549 rpm.

The amount of the material damping is dependent on the loss factor, the shaft stiffness, and the shaft deformation frequency [22]. The loss factor of the cast irons and steels [23] is approximately equal to $(0.2\text{-}30)\cdot 10^{-3}$, and value of the material damping can range from units to one thousand $kg\cdot s^{-1}$ (see Table 3). For the investigated rotor system, the simulations show that the lower bound of the 1st region of unstable motion is only negligibly influenced by the amount of material damping (Table 3). However, the bounds of the 2nd region of unstable motion are strongly dependent on the amount of the material damping.

Table 3. Effect of the shaft material damping on the bounds of the unstable regions for the damping in bearing supports of 4 800 $kg\cdot s^{-1}$.

Material damping [$kg\cdot s^{-1}$]	1st lower bound [rpm]	1st upper bound [rpm]	2nd lower bound [rpm]	2nd upper bound [rpm]
0	15 461 - 15 464	26 277 - 26 280	-	-
10	15 461 - 15 464	26 277 - 26 280	-	-
25	15 461 - 15 464	26 273 - 26 298	28 524 - 28 549	30 000
100	15 461 - 15 464	30 000	-	-
1 000	15 461 - 15 464	30 000	-	-

CONCLUSION

The effect of the shaft cross-section asymmetry, the damping in bearing supports, and the shaft material damping on the vibration stability using the Floquet transition matrix method was investigated. Three approaches for assembly of approximation of the transition matrix have been tested, and all of them gave the same information about the vibration stability. It was shown that the interaction between parametric excitation caused by the shaft asymmetry and self-excited vibration caused by the shaft material damping can extend the region of unstable vibrations. The developed computational procedure for investigation of the vibration stability of the Jeffcott-like rotor with asymmetric shaft was verified by the direct integration. This procedure can be easily extended for the robots, manipulators, and rotor systems with nonlinear elements such as squeeze film dampers lubricated by classical oil or magnetorheological liquid.

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REFERENCES

[1] Champneys, A. "Dynamics of Parametric Excitation", In: Meyers R. (ed.) Mathematics of Complexity and Dynamical Systems, 1st ed., Springer, New York, pp. 183 – 204, **2012**. ISBN 978-1-4614-1805-4

[2] Tondl, A., Ruijgrok, T., Verhulst, F., Nabergoj, R. "Autoparametric Resonance in Mechanical Systems", 1 st ed., Cambridge University Press, Cambridge, **2000**. ISBN 0521650798

[3] Tondl, A. "To the Problems of Self-excited Vibration Suppression", Engineering Mechanics 15 (4), pp. 297 – 307, **2008**.

[4] Tondl, A., Půst, L. "On the Phenomenon "Parametric Anti-resonance"", Strojnícky časopis – Journal of Mechanical Engineering 63(3), pp. 125 – 138, **2012**.

[5] Tondl, A. "Analysis of a Self-excited Two-mass System", Strojnícky časopis – Journal of Mechanical Engineering 59 (3), pp. 115 – 124, **2008**.

[6] Díaz-Marín, C. D., Jenkins, A. "A physical Approach to Dissipation-induced Instabilities", In: Vibration, Control and Stability of Dynamical Systems DSTA 2017, This is an extension and refinement of work presented at the DSTA 2017 conference, **2018**.

[7] Krechetnikov, R., Marsden, J. E. "Dissipation-induced Instabilities in Finite Dimensions", Reviews of Modern Physics 79 (2), pp. 519 – 553, **2007**. DOI: https://doi.org/10.1103/RevModPhys.79.519

[8] Hagedorn, P., Eckstein, M., Heffel, E., Wagner, A. "Self-excited Vibrations and Damping in Circulatory Systems", Journal of Applied Mechanics 81(10), pp. 1 – 9, **2014**. DOI: https://doi.org/10.1115/1.4028240

[9] Tůma, J., Škutová, J. "Simulation of Active Vibration Control of the Cantilever Beam", In: Proceedings of the 13th International Carpathian Control Conference (ICCC), High Tatras, Slovakia, **2012**. DOI: https://doi.org/10.1109/carpathiancc.2012.6228745

[10] Zapoměl, J., Ferfecki, P. "The Oscillation Attenuation of an Accelerating Jeffcott Rotor Damped by Magnetorheological Dampers Affected by the Delayed Yielding Phenomenon in the Lubricating Oil", Journal of Vibration and Acoustics, Transactions of the ASME 140 (1), pp. 1 – 10, **2017**. DOI: https://doi.org/10.1115/1.4037512

[11] Ferfecki, P., Zapoměl, J. "Investigation of Vibration Mitigation of Flexibly Support Rigid Rotors Equipped with Controlled Elements", Procedia Engineering 48, pp. 135 – 142, **2012**. DOI: https://doi.org/10.1016/j.proeng.2012.09.496

[12] Ferfecki, P., Zapoměl, J., Šofer, M., Pochylý, F., Fialová, S. "Numerical Computation of the Damping and Stiffness Coefficients of the Classical and Magnetorheological Squeeze Film Damper", In: MATEC Web of Conferences, 1 st ed., EDP Sciences, France, pp. 1 – 12, **2017**. DOI: https://doi.org/10.1051/matecconf/201815708001

[13] Michajłow, M., Konowrocki, R., Szolc, T. "A Semi-active Control of Lateral Vibrations of the Overhung Rotor using Dampers with the Magneto-rheological Fluid", In: Vibrations in Physical Systems Vol. XXVII, 1st ed., Poznan University of Technology and Institute of Applied Mechanics Polish Society of Theoretical and Applied Mechanics, Poznan, Poland, pp. 273-280, **2016**.

[14] Srinath, R., Sarkar, A., Sekhar, A. S. "Instability of Asymmetric Shaft System", Journal of Sound and Vibration 362 (February), pp. 276 – 291, **2016**. DOI: https://doi.org/10.1016/j.jsv.2015.10.008

[15] Hili, M. A., Bouaziz, S., Haddar M. "Stability Analysis and Dynamic Behaviour of a Flexible Asymmetric Rotor Supported by Active Magnetic Bearings", Journal of Theoretical and Applied Mechanics 55 (3), pp. 751 – 763, **2017**. DOI: https://doi.org/10.15632/jtam-pl.55.3.751

[16] Wu, G., Shen, H. "Comparison of 3-DOF Partially Decoupled Spherical Parallel Manipulators with Respect to Lateral Stabilities", In: Advances in Mechanism and Machine Science, Mechanisms and Machine Science 73, Springer Nature Switzerland AG, pp. 2099 – 2108, **2019**. DOI: https://doi.org/10.1007/978-3-030-20131-9_208

[17] Reddy, B. S., Ghosal, A. "Robustness Analysis of a Simple and Augmented Proportional Plus Derivative Controller in Trajectory Following Robots Using the Floquet Theory", Journal of Computational and Nonlinear Dynamics 13 (7), 6 pp., **2018**. DOI: https://doi.org/10.1115/1.4040022

[18] Zapoměl, J. "Computational Modelling of Transverse Vibration of Rotors Mounted on Hydrodynamic Bearings and Squeeze Film Dampers", 1st ed., VSB – Technical University of Ostrava, Ostrava, **2007**, in Czech. ISBN 978-80-248-1593-0

[19] Zhao, J. Y., Linnett, I. W., McLean, L. J. "Stability and Bifurcation of Unbalanced Response of a Squeeze Film Damped Flexible Rotor", Journal of Tribology 116 (2), pp. 361 – 368, **1994**. DOI: https://doi.org/10.1115/1.2927236

[20] Friedmann, P., Hammond, C. E., Woo, T. - H. "Efficient Numerical Treatment of Periodic System with Application to Stability Problems", International Journal for Numerical Methods in Engineering 11(7), pp. 1117 – 1136, **1977**. DOI: https://doi.org/10.1002/nme.1620110708

[21] Ferfecki, P., Zaoral, F., Zapoměl, J., Molčan, M. "Active Vibration Control of an Asymmetric Jeffcott Rotor with External and Internal Viscous Damping", In: Applied Mechanics, Ostravice, Czech Republic, pp. 48 – 53, **2019**.

[22] Bently, D. E., Muszynska, A. "Rotor Internal Friction Instability", In: NASA. Lewis Research Center Instability in Rotating Machinery, USA, pp. 337 – 348, **1985**.

[23] Zhang, J., Perez, R. J., Lavernia, E. J. "Documentation of Damping Capacity of Metallic, Ceramic and Metal-matrix Composite Materials", Journal of Materials Science 28 (9), pp. 2395 – 2404, **1993**. DOI: https://doi.org/10.1007/BF01151671