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IDENTIFICATION OF FATIGUE CONSTANTS BY MEANS OF 3D METHOD

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Abstract: The conventional method for evaluation of the fatigue constants uses one set of experimental data from strain-controlled uniaxial fatigue tests. However, these constants do not ensure the compatibility conditions. The new 3D method retains the mathematical and physical relationships between curves considered. This paper presents a way of implementation of the identification procedure and shows results obtained for three types of materials.

KEYWORDS: Fatigue constant, low-cycle fatigue, 3D method.

1 Introduction

Nowadays, fatigue behaviour of materials is described mostly by the stress–strain curve in the form of Ramberg–Osgood equation and by the Manson–Coffin–Basquin equation in the form of the strain–life curve. It is well known that the assumption of equality of the plastic and elastic components in both equations leads to the so called compatibility condition. The conventional evaluation method of fatigue constants uses one set of experimental data from strain-controlled uniaxial fatigue tests. However, these constants do not ensure the compatibility conditions. A new method proposed by A. Nieslony [1] for determining the stress–strain and strain–life curves retains the mathematical and physical relationships between the curves considered. This paper presents a way of implementation of the identification procedure and shows some interesting results obtained for three types of materials: aluminum alloy 2124T851, stainless steel 316L and structural steel ST52.

2 Identification of fatigue constant

For evaluation of low cycle fatigue tension-compression tests, the Manson-Coffin-Basquin equation [2] is applied usually. It illustrates the relationship between the amplitude of total strain ε_{at} and number of cycles to crack initiation N_f

$$\varepsilon_{at} = \varepsilon_{ae} + \varepsilon_{ap} = \frac{\sigma'_f}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c, \tag{1}$$

where ε_{ae} - amplitude of elastic strain, ε_{ap} - amplitude of plastic strain, ε'_f - fatigue ductility coefficient, c - fatigue ductility exponent, σ'_f - fatigue strength coefficient, b - fatigue strength exponent.

Analogical relation is used usually for evaluation of torsional low-cycle fatigue tests

$$\gamma_{at} = \gamma_{ae} + \gamma_{ap} = \frac{\tau_f'}{G}\left(2N_f\right)^{b_\gamma} + \gamma_f'\left(2N_f\right)^{c_\gamma}, \tag{2}$$

where γ_{at} - amplitude of total shear strain, γ_{ae} - amplitude of elastic shear strain, γ_{ap} - amplitude of plastic shear strain, γ_f' - fatigue shear ductility coefficient, c_γ - fatigue ductility exponent, τ_f' - fatigue shear strength coefficient, b_γ - fatigue shear strength exponent.

The amplitude of the total strain can be expressed also by the Ramberg-Osgood equation [3] as a function of stress amplitude in the case of tension-compression

$$\varepsilon_{at} = \varepsilon_{ae} + \varepsilon_{ap} = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{K'}\right)^{n'}, \tag{3}$$

and in the case of torsion

$$\gamma_{at} = \gamma_{ae} + \gamma_{ap} = \frac{\tau_a}{G} + \left(\frac{\tau_a}{K_\gamma'}\right)^{n_\gamma'} \tag{4}$$

where *K'* - cyclic strength coefficient, *n'* - cyclic strain hardening exponent, K_γ' - cyclic shear strength coefficient, n_γ' - cyclic shear strain hardening exponent. Six fatigue constants have been usually determined by the conventional method. A simple regression through experimental results is used in the form

$$Y = B + AX \tag{5}$$

Then, the least square method is applied. Estimators A and B define material fatigue parameters searched in an explicit way or as a simple function. At first, experimental data are subjected to linearization by calculating the logarithms of their values. Next, linearized forms of suitable equations are derived using simple mathematical operations. Three equations obtained in this way are shown in Table 1. Each equation represents a line with two coefficients – slope of the line and y-intercept of the line. Thus, three independent linear regressions must be performed in order to determine six parameters characterizing the fatigue properties of the material. This procedure does not preserve compatibility of the strain-life and stress–strain curves.

Table 1: Equations used for the determination of the material constants with the conventional method.

The primary form	The linearized form		
	Y=B+AX	X	Y
$\varepsilon_{ae} = \frac{\sigma_f'}{E}\left(2N_f\right)^b$	$Y = log(\frac{\sigma_f'}{E}) + bX$	$log(2N_f)$	$log(\varepsilon_{ae})$
$\varepsilon_{ap} = \varepsilon_f'\left(2N_f\right)^c$	$Y = log(\varepsilon_f') + cX$	$log(2N_f)$	$log(\varepsilon_{ap})$
$\sigma_a = K'\left(\varepsilon_{ap}\right)^{n'}$	$Y = log(K') + n'X$	$log(\varepsilon_{ap})$	$log(\sigma_a)$

Niesłony [1] has proposed a new method for identifying fatigue constants - so-called 3D method. The main advantage of this method is that it ensures the compatibility of six fatigue constants. The method is based on the approximation by a straight line in xyz space where

$x = log(\varepsilon_{ap})$, $y = log(\sigma_a)$, $z = log(2N_f)$. The regression line is determined by the direction $\mathbf{r}$ (r_1, r_2, r_3) and a point P (x_P, y_P, z_P), see Fig1.

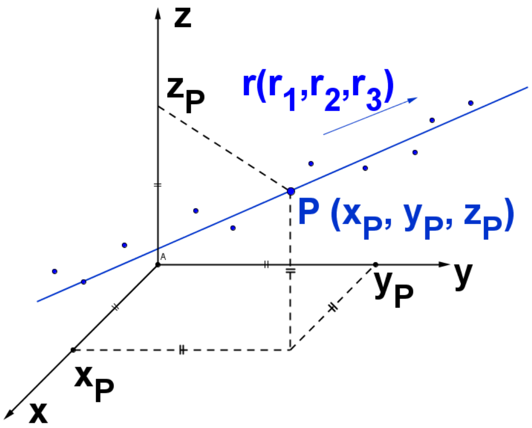

Fig. 1 Points in the three-dimensional space with regression line determined by point P and directional vector r

Based on the coordinates of the direction vector and coordinates of point P the six fatigue coefficients can be calculated as follows

$$n' = \frac{r_2}{r_1}, \tag{6}$$

$$K' = 10^{y_P - x_P n'}, \tag{7}$$

$$c = \frac{r_1}{r_3}, \tag{8}$$

$$\varepsilon_f' = 10^{x_P - z_P c}, \tag{9}$$

$$b = \frac{r_2}{r_3}, \tag{10}$$

$$\sigma_f' = 10^{y_P - z_P b}. \tag{11}$$

Same relations can be used for torsional fatigue constants as well. When the problem is solved, four regressions are made. The first regression determines the equation of a plane in space and the other three are used to calculate regression lines in the planes $x - y$, $y - z$ and $z - x$, where $x = log(\varepsilon_{ap})$, $y = log(\sigma_a)$, $z = log(2N_f)$. Subsequently, two regressions with the highest coefficient of determination are selected. It is obvious that at least one of selected regressions is a line in plane, i.e. it is necessary to establish a plane which is given by the straight line and which is, at the same time, perpendicular to the plane in which it lies. The final regression line is determined as the intersection of selected planes. In order to make methods user friendly, the algorithm in MATLAB was developed.

3 Results of fatigue parameters estimation - 2124T851 aluminum alloy

Low-cycle fatigue tests were performed on smooth specimens made of 2124T851 aluminum alloy at the Department of Applied Mechanics at the VŠB – Technical University of Ostrava using the LabControl 100kN/1000Nm testing machine. In the case of uniaxial loading, the solid specimen with diameter of 5 millimeters was used for strain amplitudes lower than 1.3%. Otherwise, the tubular specimen with diameters ϕ10/ϕ12.5 mm was used to avoid buckling. The EPSILON 3442 extensometer with a gauge length of 10 mm was used to measure/control the axial strain. The EPSILON 3550 extensometer with a gauge length of

25.4 mm was used to measure/control the shear strain under the torsion. All fatigue tests were realized under strain control with zero mean strain during the cycle. The strain rate was equal to 0.01 s^{-1}. The number of cycles to crack initiation N_f was determined in all tests based on the decrease of force/torque amplitude (axial stress σ_a / shear stress τ_a) by twenty percent in comparison to saturated values. Results of uniaxial and torsional fatigue tests with constant axial (ε_a) and shear strain (γ_a) amplitude respectively are summarized in Table 2 and Table 3.

Table 2: Results of uniaxial fatigue tests of AA2124T851

id. test	ε_a	σ_a [MPa]	N_f
1	0.025	455.4	36
2	0.02	444.4	67
3	0.015	424	209
4	0.0125	406	520
5	0.01	389	642
6	0.008	370	1206
7	0.0065	358	3194
8	0.0055	328	7062

Table 3: Results of torsional fatigue tests of AA2124T851

id. test	γ_a	τ_a [MPa]	N_f	Note
1	0.02635	252.9	41	sp.12.5/10
2	0.02184	239.8	85	sp.12.5/10
3	0.01800	236.0	125	sp.12.5/10
4	0.01350	229.6	152	sp.12.5/10
5	0.01037	214.6	322	sp.12.5/10
6	0.00730	183.3	2257	sp.12.5/10
7	0.00630	164.4	6800	sp.12.5/10
8	0.00490	128	27890	sp.12.5/10

It should be mentioned that subscript “a” means amplitude of variable in the half of fatigue life in the Table 2 and Table 3.

Both methods (conventional and 3D method) were applied to obtain fatigue parameters of AA2124T851 material. The regression dependencies obtained for 2124T851 aluminum alloy based on uniaxial and torsional data are shown in Fig. 2 and Fig. 3.

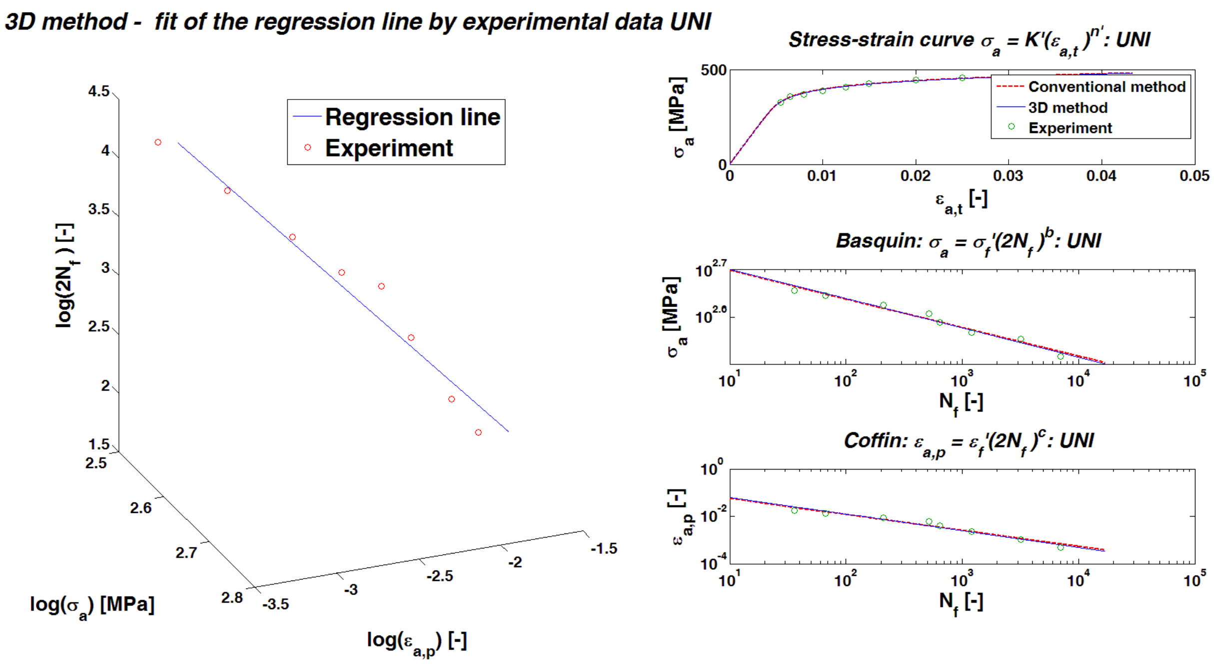

Fig. 2 Graphical representation of 3D method application and its comparison with conventional method for AA2124T851 – tension-compression

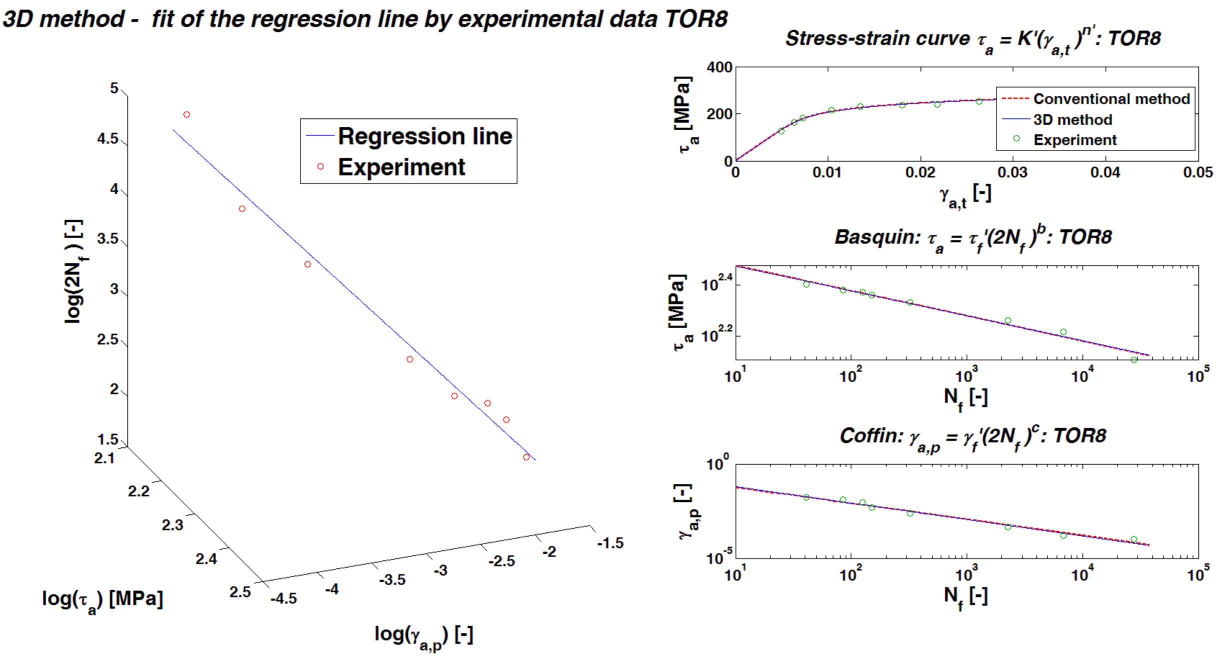

Fig. 3 Graphical representation of 3D method application and its comparison with conventional method for AA 2124T851 – torsion

Values of fatigue constants obtained after application of the 3D and conventional methods are presented in Table 4 and Table 5. Fatigue parameters AA 2124T851 are comparable excluding fatigue ductility coefficient and fatigue shear ductility coefficient. There's approximately 20 percent difference between both methods.

Table 4: Results of 3D method and conventional method application (AA 2124T851 - uniaxial)

Parameter	Conventional method	3D method	Difference [%]
E [MPa]	65540	65540	
ε'_f	0.529	0.424	24.764
c	-0.706	-0.674	4.748
σ'_f [MPa]	611	603	1.327
b	-0.063	-0.061	3.279
K' [MPa]	646	646	0.000
n'	0.0892	0.089	0.225

Table 5: Results of 3D method and conventional method application (AA 2124T851 - torsion)

Parameter	Conventional method	3D method	Difference [%]
E [MPa]	26700	26700	
ε'_f	0.875	0.757	15.588
c	-0.874	-0.854	2.342
σ'_f [MPa]	400	405	1.235
b	-0.0978	-0.0995	1.709
K' [MPa]	406	406	0.000
n'	0.111	0.111	0.000

It is obvious from indicators presented in Table 6 that the 3D method provides comparable statistical values as the conventional method in both types of loading.

Table 6: Statistical values obtained by both approaches - AA 2124T851

Loading		$R^2(\varepsilon)$	$R^2(p)$	R^2(total)	$R^2(\sigma\sigma)$	SLOG(σ)	SLOG(ε)	SLOG(p)
Uniaxial	C	0.971768	0.954617	0.988816	0.983503	0.00254	0.003324	0.046905
	3D	0.999999	0.999999	0.999999	0.983503	0.00254	0.003419	0.047284
Torsion	C	0.976169	0.976383	0.98378	0.919281	0.011118	0.005898	0.057694
	3D	0.941517	0.999999	0.999999	0.919281	0.011118	0.005843	0.058305

where C means Conventional method and 3D means new 3D method. Further,
$R^2(\varepsilon)$ is coefficient of determination in plane $\log(\varepsilon_{ae})$ - $\log(2N_f)$ or $\log(\gamma_{ae})$ - $\log(2N_f)$,
$R^2(p)$ is coefficient of determination in plane $\log(\varepsilon_{ap})$ -$\log(2N_f)$ or $\log(\gamma_{ap})$ -$\log(2N_f)$,
$R^2(\sigma\sigma)$ is coefficient of determination in plane $\log(\varepsilon_{ap})$ - $\log(\sigma_a)$ or $\log(\gamma_{ap})$ - $\log(\sigma_a)$,
$R^2(t)$ is total coefficient of determination in space
SLOG(ε) is coefficient of variation in plane $\log(\varepsilon_{ae})$ - $\log(2N_f)$ or $\log(\gamma_{ae})$ - $\log(2N_f)$,
SLOG(σ) is coefficient of variation in plane $\log(\varepsilon_{ap})$ - $\log(2N_f)$ or $\log(\gamma_{ap})$ -$\log(2N_f)$,
SLOG(p) is coefficient of variation in plane $\log(\varepsilon_{ap})$ - $\log(\sigma_a)$ or $\log(\gamma_{ap})$ - $\log(\sigma_a)$.

4 Results of fatigue parameters estimation - the stainless steel 316L

The electro-servo-hydraulic testing machine LabControl 100kN/1000Nm located at the Faculty of Mechanical Engineering of the VŠB - Technical university of Ostrava was used to realize fatigue tests in low-cycle domain on specimens made of 316L stainless steel. Uniaxial fatigue tests with strain amplitudes lower than 1.2% were realized on solid round-type specimen with diameter of 5 mm. Otherwise, the tubular specimen with diameters ϕ10/ϕ12.5 mm was used to avoid buckling. All tests were realized on smooth specimens under strain control with zero mean strain during the cycle. The EPSILON 3442 extensometer with a gauge length of 10 mm was used to measure/control the axial strain. The strain rate of about 6.10^{-3} s^{-1} was considered. The number of cycles to crack initiation was determined in all tests based on the decrease of force amplitude by twenty percent in comparison to the saturated values.

Results of uniaxial fatigue tests with constant axial (ε_a) amplitude are summarized in Table 7.

Table 7: Results of uniaxial fatigue tests of SS316L

id. test	ε_a	σ_a [MPa]	N_f
1	0.02433	627	24
2	0.01752	564	56
3	0.01349	501	133
4	0.00989	466	594
5	0.00699	424	897
6	0.00488	380	5700
7	0.00395	362	8630
8	0.00293	336	124450

Both methods (conventional and 3D method) were applied to obtain fatigue parameters of SS316L material. Resulting regression dependencies for the stainless steel 316L based on uniaxial data are shown as output from the MATLAB program in Fig. 4. Values of fatigue constants obtained after application of the 3D and the conventional method are presented in Table 8. Fatigue parameters of SS316L are comparable because differences in values are within six percent.

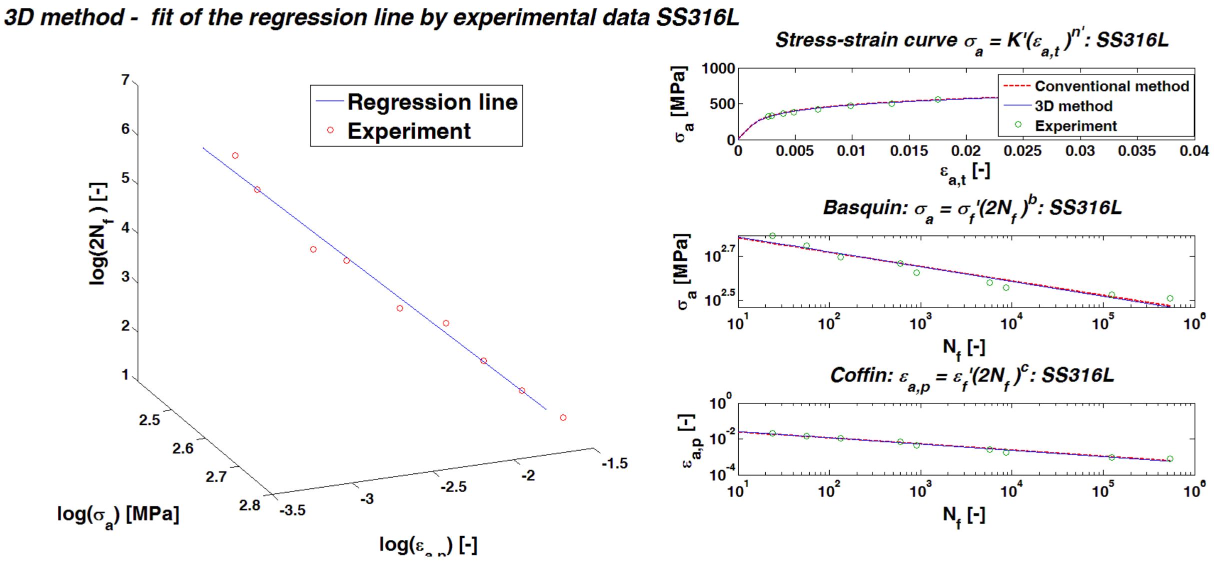

Fig. 4: Graphical representation of 3D method application and its comparison with conventional method for the stainless steel 316L – tension-compression

Table 8: Results of 3D method and conventional method application – SS316L

Parameter	Conventional method	3D method	Difference [%]
E [MPa]	169785	169785	
ε'_f	0.070	0.074	5.405
c	-0.343	-0.350	2.000
σ'_f [MPa]	742	755	1.722
b	-0.066	-0.068	2.941
K' [MPa]	1251	1251	0.000
n'	0.194	0.194	0.000

It is obvious from the indicators presented in Table 9 that the 3D method provides comparable statistical values as the conventional method in both types of loading.

Table 9: Statistical values obtained by both approaches

Loading		$R^2(\varepsilon)$	$R^2(p)$	R^2(total)	$R^2(\sigma\sigma)$	SLOG(σ)	SLOG(ε)	SLOG(p)
Uniaxial	C	0.931864	0.980842	0.989221	0.971871	0.007631	0.010848	0.026896
	3D	0.990854	0.999999	0.999999	0.971871	0.007631	0.01104	0.025079

5 Results of fatigue parameters estimation - the structural steel ST52

The last experimental set contains results of low-cycle fatigue tests carried out at the VUHZ Dobra on specimens made of ST52 structural steel. Main results in the form of axial strain amplitude (ε_a), axial stress amplitude (σ_a) and number of cycles to fracture N_f are summarized in Table 10.

Table 10: Results of uniaxial fatigue tests of ST52 steel

id. test	ε_a	σ_a [MPa]	N_f
1	0.002	290	58066
2	0.0028	330	23534
3	0.003	335	15990
4	0.004	382	6562
5	0.005	396	3338
6	0.0075	426	1294
7	0.01	471	884
8	0.015	509	236
9	0.02	517	282
10	0.025	527	150

Both methods (conventional and 3D method) were applied to obtain fatigue parameters of ST52 material. The resulting regression dependencies for the ST52 structural steel based on uniaxial data are shown as output from the MATLAB program in Fig. 5. Values of fatigue constants obtained after application of 3D and conventional methods are presented in Table 11. Fatigue parameters of ST52 are comparable because differences in the values are within six percent.

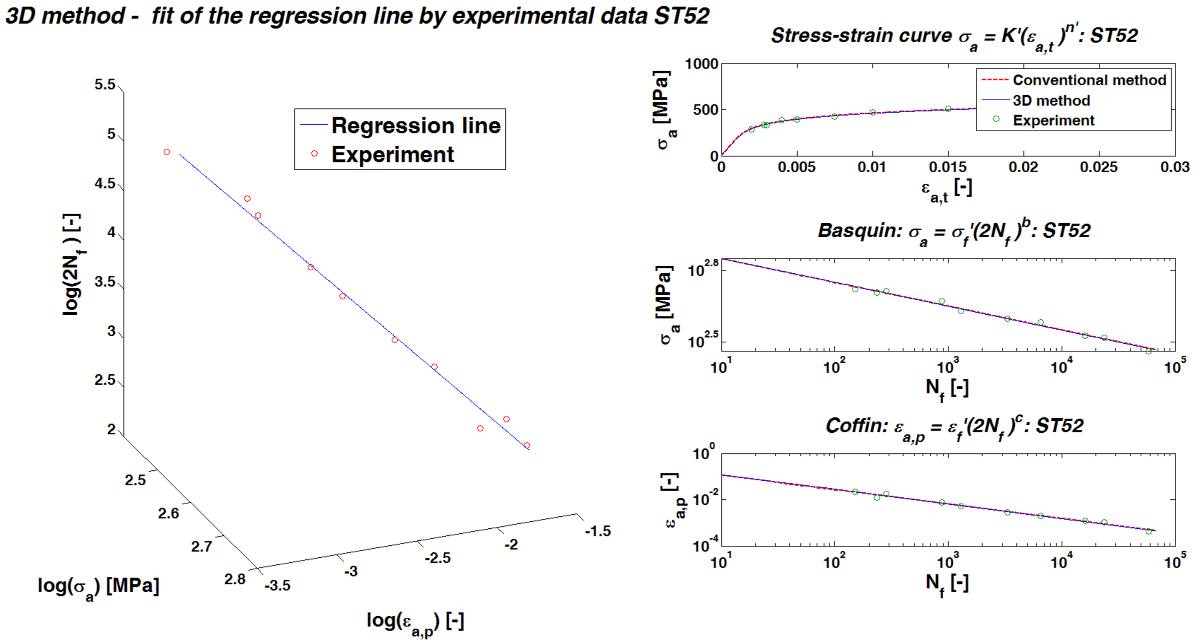

Fig. 5: Graphical representation of 3D method application and its comparison with conventional method for the ST52 structural steel – tension-compression

Table 11: Results of 3D method and conventional method application

Parameter	Conventional method	3D method	Difference [%]
E [MPa]	185000	185000	
ε'_f	0.757	0.803	5.729
c	-0.627	-0.634	1.104
σ'_f [MPa]	958	963	0.519
b	-0.100	-0.101	0.990
K' [MPa]	997	997	0.000
n'	0.159	0.159	0.000

It is obvious from indicators presented in Table 12 that the 3D method provides comparable statistical values as the conventional method in both types of loading.

Table 12 Statistical values obtained by both approaches - ST52 structural steel

Loading		$R^2(\varepsilon)$	$R^2(p)$	R^2(total)	$R^2(\sigma\sigma)$	SLOG(σ)	SLOG(ε)	SLOG(p)
Uniaxial	C	0,986861219	0,988943	0,992421	0,987976	0,004221	0,004408	0,0231754
	3D	0,999022111	0,999999	0,999999	0,987976	0,004221	0,004414	0,0226700

CONCLUSION

The conventional method has a problem to ensure the compatibility conditions. However, the new 3D method retains mathematical and physical relationships between curves considered. In order to make both methods user friendly, the algorithm in MATLAB was developed. This paper presents the way of implementation of the identification procedure and shows results obtained for three materials - 2124T851 aluminum alloy, stainless steel 316L and structural steel ST52. It is obvious from indicators presented in Tables 4, 6 and 7 that the 3D method provides comparable statistical values as the conventional method in both types of loading. In some cases, the quality of approximation measured by statistical quantities is even better for 3D method. The biggest difference between fatigue parameters was found out in the case of the aluminum alloy for ε_f' and σ_f'.

The further research will be focused on materials commonly used for production of railway-wheels [4], [5] and on the comparison of life-time prediction for variable amplitude loading based on fatigue constants identified by conventional and 3D methods.

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