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PHENOMENOLOGICAL MATERIAL MODEL OF FOAM SOLIDS

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Abstract: In this contribution a new phenomenological model for pressure loaded foam materials is proposed. The presented relationship between compressive stresses and compressive strains is derived by using simple rheological models. The proposed model contains several parameters. The procedure of their identification and the influence of parameters on a curve shape are also explained. The accuracy of our model is tested and compared with other phenomenological models. These tests have been applied on aluminium and polyurethane foams.

KEYWORDS:

1 Introduction

In the beginning of the 20th century, commercial production of man-made materials has been widely expended. Their production has been focused on cellular-like materials based on polymers and metallic alloys. The structure of these materials is inspired by nature, i.e. in wood, cork, corals, etc. [1-3]. The man-made materials are most commonly used in building materials but can also be applied asthermal insulators, packaging and protective materials.

From the point of technical application the most important property of foams is their excellent ratio between pressure strength and weight. This ratio depends on the cell structure and mechanical property of material from which cellular solids are made. The experimental results show different responses in tension and in compression [1-3]. Compressive axial tests of foams with various cell structures, as well as different material compositionsexhibited strong non-linear characteristics: axial compressive loading or stress vs. relative compression or strain, with three characteristic regions(see Fig. 1). The deformation process starts with an initial linear elastic response on cell edges or cell walls. As the deformation increases the cell starts to collapse in three different ways: elastic buckling of foams (rubber-like materials), plastic yielding with plastic hinges (polymers, metals) and brittle crushing(ceramics). This region is known as the plateau region. Deformations are increasing to a value of 60 – 80 % while stresses remainroughlyunchanged. This effect results in the ability to absorb impact and vibrating loading. This collapse progresses until opposing walls meet and touch. After the opposing walls touch, the deformation stops with increasing stresses (densification or locking) i.e. cellular solids exhibit deformation until the densification is reached.

For tension there are no such plateau regionsas in compression. In this case breaking of foams occurs in the vicinity of their yielding point [3]. Therefore the application of foams is beneficial in shock or impact absorptionin the automotive, packaging industries, etc.

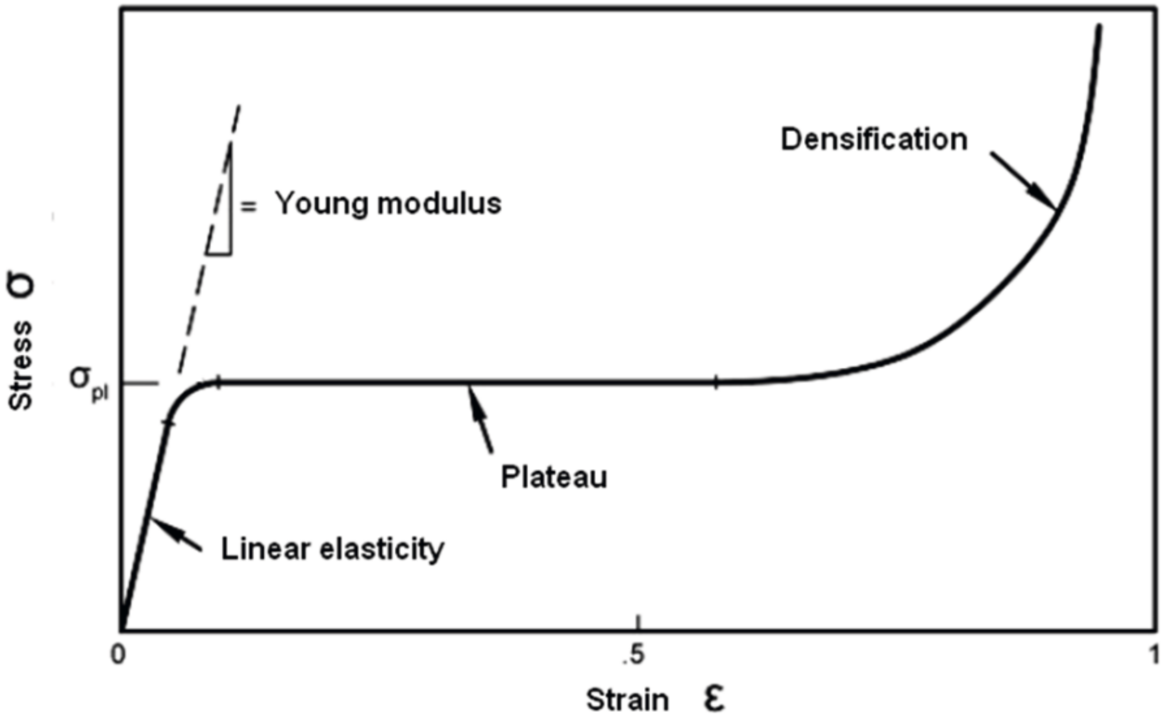


Fig. 1 Characteristic shape of compressive foam curve.

Nowadays from the point of foam applications, it is required to describe the mechanical properties of foam and predict their behaviour when loading. The functional relationships between loading and the corresponding deformations are described by modelling of the material properties.The aim of material modelling is to obtain all mechanical parameters that are necessary for any response simulations. In this way we can construct an approximation of compressive curves for foams, see Fig 1, which can be used in any practical analysis, i.e. for determining absorption properties of foams and for finite element simulations.

Modelling of foam compressive loading can be divided into two basic groups:

- Modelling at the microscopic level – this approach represents the analysis of a simple foam cell under compressive loading.

This type of modelling is represented by Patel a Finnie 1969 [4], Christensen 1986 [5], Warren and Kraynik 1988 [6], Grenestedt 1999 [7], Deshpande and Fleck 2000 [8], Evans et al. 2001 [9], Laroussiet al. 2002 [10]. The principal investigations in this area have been done by Ashby and Gibson, who describe the deformation responses of cellular solids using simple cubic cell models and predeterminedparameters for the mechanical properties as a function of relative foam densities based on experimental works [1, 2, 3].

- Modelling at the macroscopic level –the foam is assumed to be a continuum and that the foam behaviour can be described by constitutive equations.

Modelling at the macroscopic level does not investigate the internal structure of bodies and their deformation processes. However, the approach focuses on cellular solids as bodies of periodic structures with homogenised properties [22] for each cell individually;their behaviours can be described by constitutive equations. The phenomenon (shape of compressive curve) can be described by appropriate functions (principle of phenomenological approach) during the determination of constitutive equations. The most cited work in phenomenological modelling of foams has been done by Rusch [11]. Subsequently his work has been extended by Meinecke a Schwaber [12], Nagy et al. [13], Sherwoodet al. [14]. The following phenomenological models have been published by Avalleet al. [15], Liu and Subhash [16], Faruqueet al. [17], Hučko and Faria[18], Zhang et. [19]and many others.

NEW PHENOMENOLOGICAL MODEL FOR FOAM MATERIALS

In this chapter a new constitutive model describing relations between axial compressive stresses and their corresponding strains is presented. This model is applicable for all cellular solids having the same characteristic response in compression, for example honeycombs, foams, in the sense of curve quality.

From the stand point of structural applications, absorption and damping of the compressive loading establishes that the dominant loading characteristic is in comparison. In this case we were not focused on a very complex material model containing the effects of tension, bending or shear. Therefore our model contains the effects of pure compression. There is no effect of cyclic loading and the influence of temperature change. The foam material is assumed to be a continuum with homogeneous and isotropic properties.

The influence of foam density on the mechanical properties in compression is presented in Fig.2. It is clear and confirmed by many experiments [3] that the increase of density produces the increase of resistance to compression, i.e. the foam is harder. The values of the elastic modulus, elastic or plastic collapse and plateau stresses are also increasing.The strain locking region shifts to lower values of strains due to prior contact of opposite cell sides.

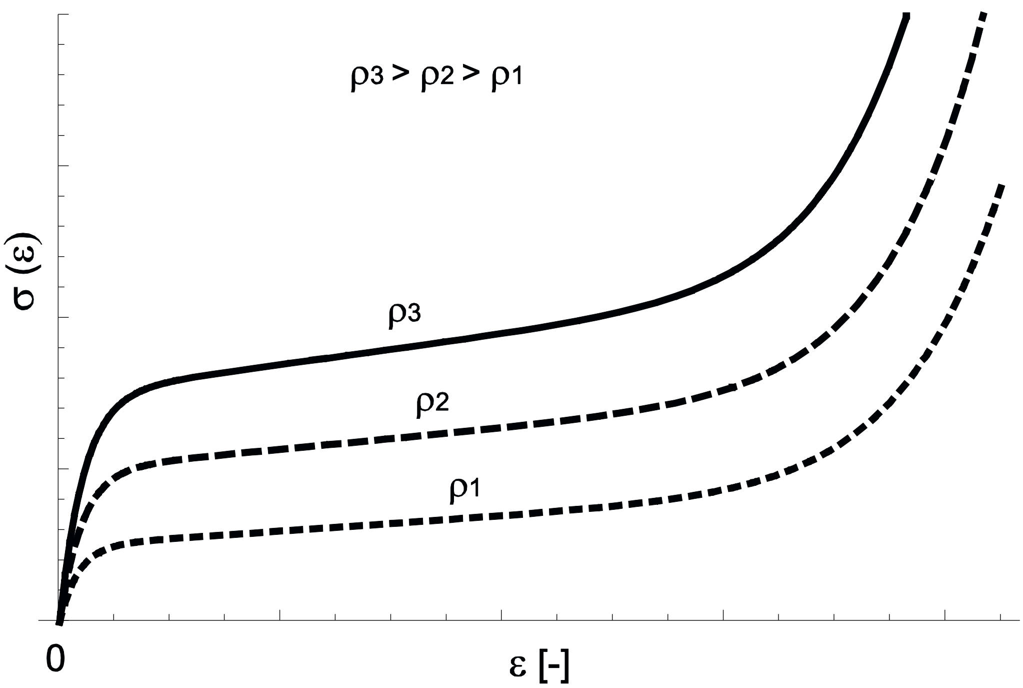


Fig. 2 The influence of density on the mechanical properties of foams in compression.

The plateau region is relatively constant, but the stiffening significantly increases as the foam density increases. For some elastomeric foams, softening can also be observed in this region.Therefore our proposed model could be able to reflect both the softening and stiffening processes in the plateau region see Fig. 3

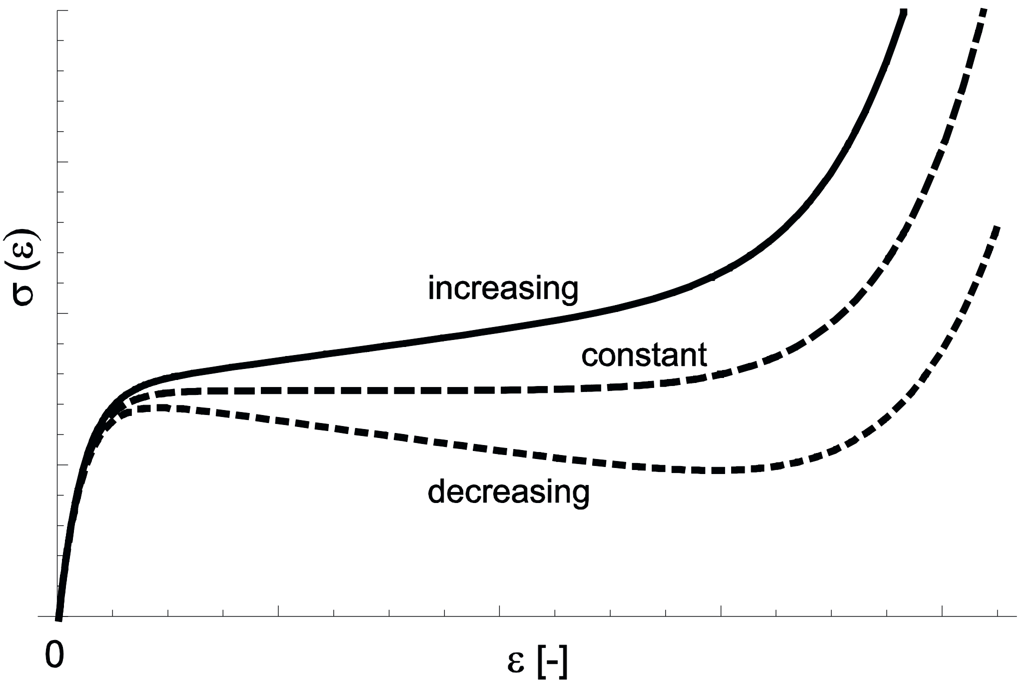


Fig. 3 Stress curves in plateau region.

Each region is modelled separately by the corresponding rheological model determining the meaning and influences of all parameters. Parameters are numbered by lower indexes which correspond to each region (1-elastic, 2-plateau, 3-locking). Putting together these individual models establishes a more complex rheological foam model.

2 Modelling of the linear elastic region

In the linear elastic region there is a linear dependence of stressesuntil the collapse loading is reached, and then the stress level remains relatively constant. This curve shape can be compared with the relaxation curve of metals. Therefore the Maxwell model is suitable for describing this region.

The serial Maxwell model (see Fig. 4) consists of a linear spring and a viscous damper. To distinguish between variables belonging to a spring or a damper we can add the lower subscript “*P*“ for a spring and about “*T*“ for a damper.

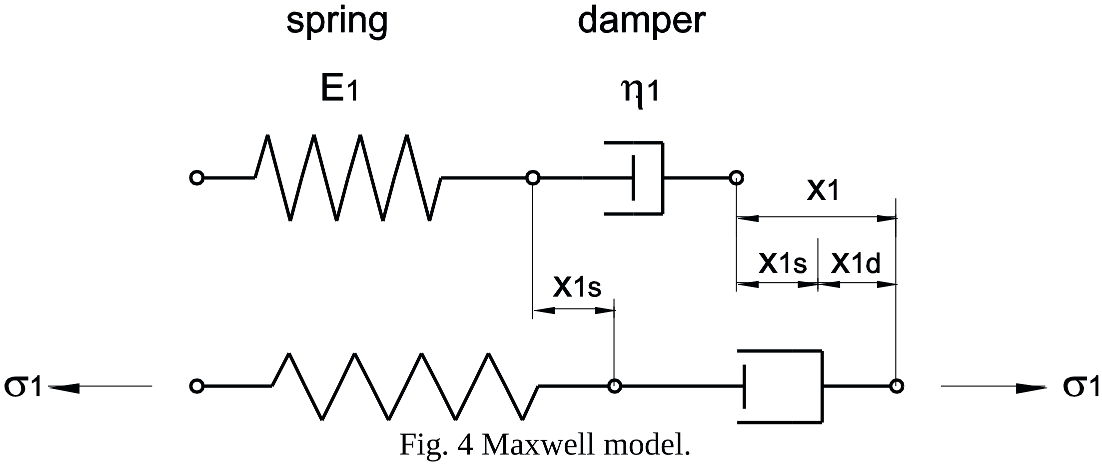


Fig. 4 Maxwell model.

The force acting in a spring is linearly dependent on itsdeformation. The ratio of applied force and its corresponding deformation determine the equivalent spring stiffnessE_1. Then the force in a spring $F_{1P}$is defined as:

$$F_{1P} = E_1 x_{1P}, \tag{1}$$

where x_{1P} is the axial deformation of the spring.

The force of the viscous damper F_{1T} is proportional to the deformation rate. The ratio of damper force and deformation rate is equal to the damping coefficientη_1. Thus for the damper force F_{1T} we can write:

$$F_{1T} = \eta_1 x_{1T}, \tag{2}$$

where $\dot{x}_{1T}$ is the deformation rate of the damper.

Then the total deformation for the serial Maxwell model equals the sum of both deformations:

$$x_1 = x_{1P} + x_{1T}. \tag{3}$$

The spring force is changing with increasing deformation and this force corresponds with the force of the damper. If the spring force reaches the collapse value, then it will remain constant and the damper force in the Maxwell model will have the same value. This is valid under the assumption that there is a constant deformation rate. Then the total force F_1 in our Maxwell model can be expressed as:

$$F_1 = F_{1P} = F_{1T}. \tag{4}$$

Subsequently the force-deformation ratio can be rewritten as:

$$x_1 = \frac{F_{1P}}{E_1} + \int \frac{F_{1T}}{\eta_1} dt. \tag{5}$$

The ratio between the deformation rate, the spring force rate and the damper force can be derived by the time derivation of eq.(5) as follows:

$$x_1 = \frac{F_{1P}}{E_1} + \frac{F_{1T}}{\eta_1}. \tag{6}$$

Substituting eq.(4) into eq. (6) we get:

$$x_1 = \frac{F_1}{E_1} + \frac{F_1}{\eta_1}. \tag{7}$$

The eq.(7) represents a differential equation of the first order under the assumption of constant deformation rate $\dot{x}_{1T}$. The eq.(7) contains the time as an independent variable only, therefore we can get the solution assuming the initial condition *F(t = 0) = 0*, as follows:

$$F(t)_1 = e^{-\frac{E_1 t}{\eta_1}} \left(-1 + e^{\frac{E_1 t}{\eta_1}} \right) \eta_1 \dot{x}_1. \tag{8}$$

Thus we can transform eq.(8) from the force vs. deformation space into the stress vs. strain space. Then we can get:

$$\sigma(t)_1 = e^{-\frac{E_1 t}{\eta_1}} \left(-1 + e^{\frac{E_1 t}{\eta_1}} \right) \eta_{1T} \dot{\varepsilon}_1. \tag{9}$$

The stress in eq.(9) is a function of time and strain rate. But the constitutive equation requires that the stress is a function of strain and strain only. Therefore we recommend the following substitution:

$$t = \frac{\varepsilon_1}{\varepsilon_1}. \tag{10}$$

The eq.(9) then becomes:

$$\sigma(\varepsilon, \varepsilon)_1 = e^{-\frac{E_{1P}}{\eta_1}\frac{\varepsilon_1}{\varepsilon_1}} \left(-1 + e^{\frac{E_{1P}}{\eta_1}\frac{\varepsilon_1}{\varepsilon_1}} \right) \eta_1 \varepsilon_1. \tag{11}$$

We are assuming that the strain rate is constant (this assumption is valid for static loading) and has the value $\varepsilon_1 = 1$ s^{-1}. Thus eq. (11) is simplifiedinto a final form:

$$\sigma(\varepsilon)_1 = e^{-\frac{E_1}{\eta_1}\varepsilon_1} \left(-1 + e^{\frac{E_1}{\eta_1}\varepsilon_1} \right) \eta_1. \tag{12}$$

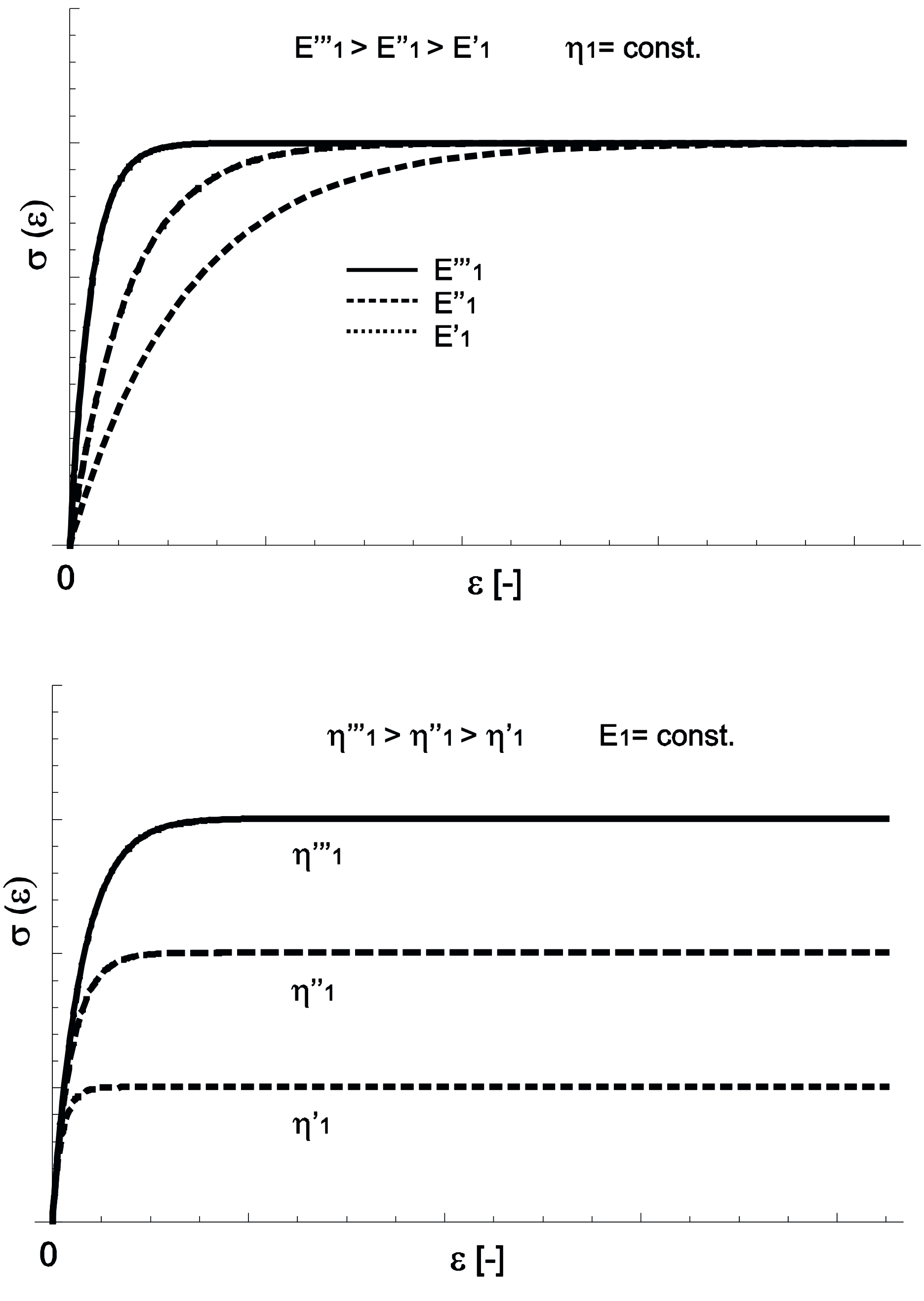


Fig. 5 Influences of selected parameters in the Maxwell model.

The diagrams in Fig.5 represent eq. (12). The increasing stiffness of the spring (the increase of elastic modulus) causes the increase of stress steepness at the initial state of loading. The viscosity (damping coefficient) determines the stress level that is equal in the spring and the damper and remains constant after collapse deformations.The damping coefficient is equivalent to the elastic or plastic collapse stress in the presented diagrams. These stresses are close to the so-called plateau stresses; therefore we can make the damping coefficient identical with the plateau stress.

3 Modelling of plateau region

For the plateau region we can add a second spring into the Maxwell model in a parallel configuration, see Fig. 6. The spring stiffness $E_2$is coincidently also linear. Ifthis stiffness is zero then the stress in plateau region will be constant and be equalled to the damper viscosity $\eta_1$in the Maxwell model. If the stiffness $E_2$is positive, the stress willhigher, if it is negative, the stress will be lower.

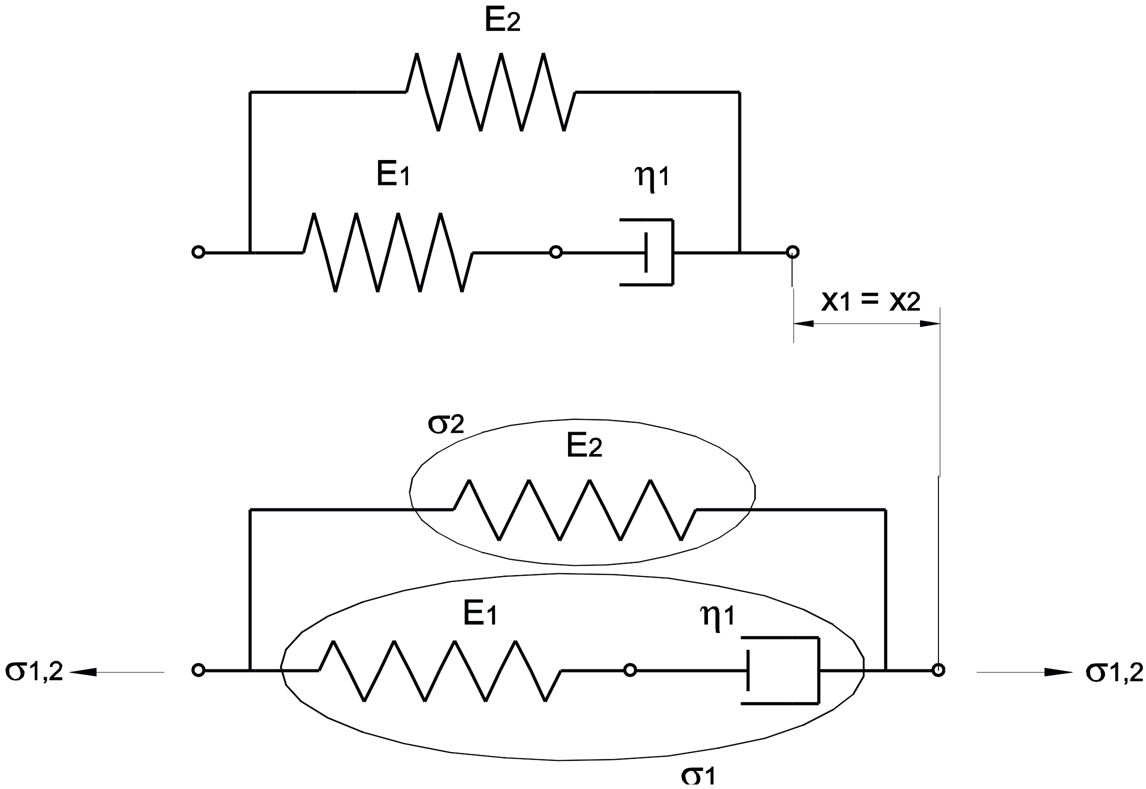


Fig. 6 Model containing linear and plateau regions

The deformation for a parallel spring $x_2$is equal to the total deformationx_1 from the Maxwell model:

$$x_2 = x_1 \approx \varepsilon \ . \tag{13}$$

Then the stress in parallel the spring is:

$$\sigma_2 = E_2 x_2 = E_2 x_1 = E_2 \varepsilon \ . \tag{14}$$

The total stress is determined by a sum of the individual stresses in each part:

$$\sigma_{1,2} = \sum_{i=1}^{2} \sigma_i = \sigma_1 + \sigma_2 \ . \tag{15}$$

Substituting eqs.(12) and (14) into eq.(15) we get:

$$\sigma(\varepsilon)_{1,2} = e^{-\frac{E_1}{\eta_1}\varepsilon} \left(-1 + e^{\frac{E_1}{\eta_1}\varepsilon} \right) \eta_1 + E_2 \varepsilon \ . \tag{16}$$

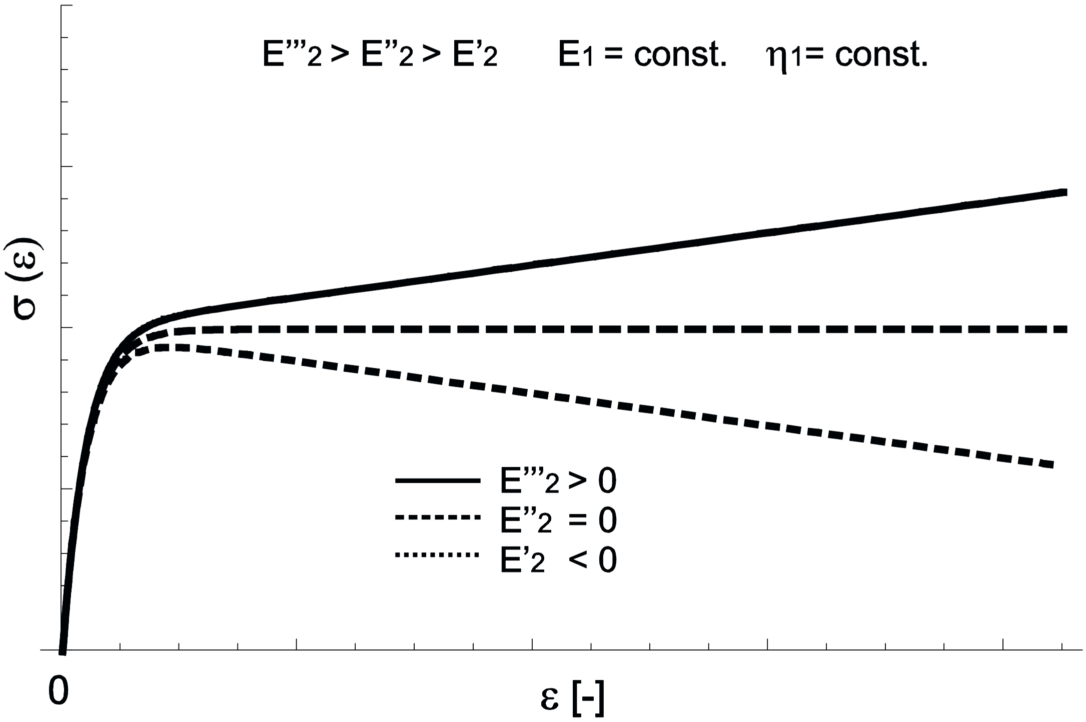


Fig. 7 The influence of parameterE_2 on a curve shape.

The Fig.7 shows that the stiffness E_2 has no influence on the steepness inthe linear part. Therefore this influence can be omitted.

4 Modelling of strain locking region

The strain locking region has a strong non-linear character that can be described by the curve of an exponential function. This results in the addition of components which ensure the increase in exponential stress. This can be done by inserting the non-linear spring stiffness E_3 into the previous model. The non-linear stiffnessE_3can be defined as:

$$E_3 = \gamma\left(1 - e^{x_3}\right)^h, \qquad (17)$$

where$x_3$is the deformation of springE_3 andγ , $h$are parameters of the proposed model which are identified experimentally.
The parallel structure leads to the following equation:

$$x_3 = x_2 = x_1 \approx \varepsilon\ . \qquad (18)$$

Finally we get the complete rheological model for cellular solids, see Fig.8:

The stress acting in a non-linear spring is determined as:

$$\sigma_3 = E_3 x_3 = E_3 \varepsilon\ . \qquad (19)$$

The influence of parameters $\gamma$and $h$on non-linear stress curves are displayed in Fig.9.

The total stress for a complete model $\sigma$is found through the summation of stresses:

$$\sigma = \sigma_{1,2,3} = \sum_{i=1}^{3} \sigma_i = \sigma_1 + \sigma_2 + \sigma_3\ . \qquad (20)$$

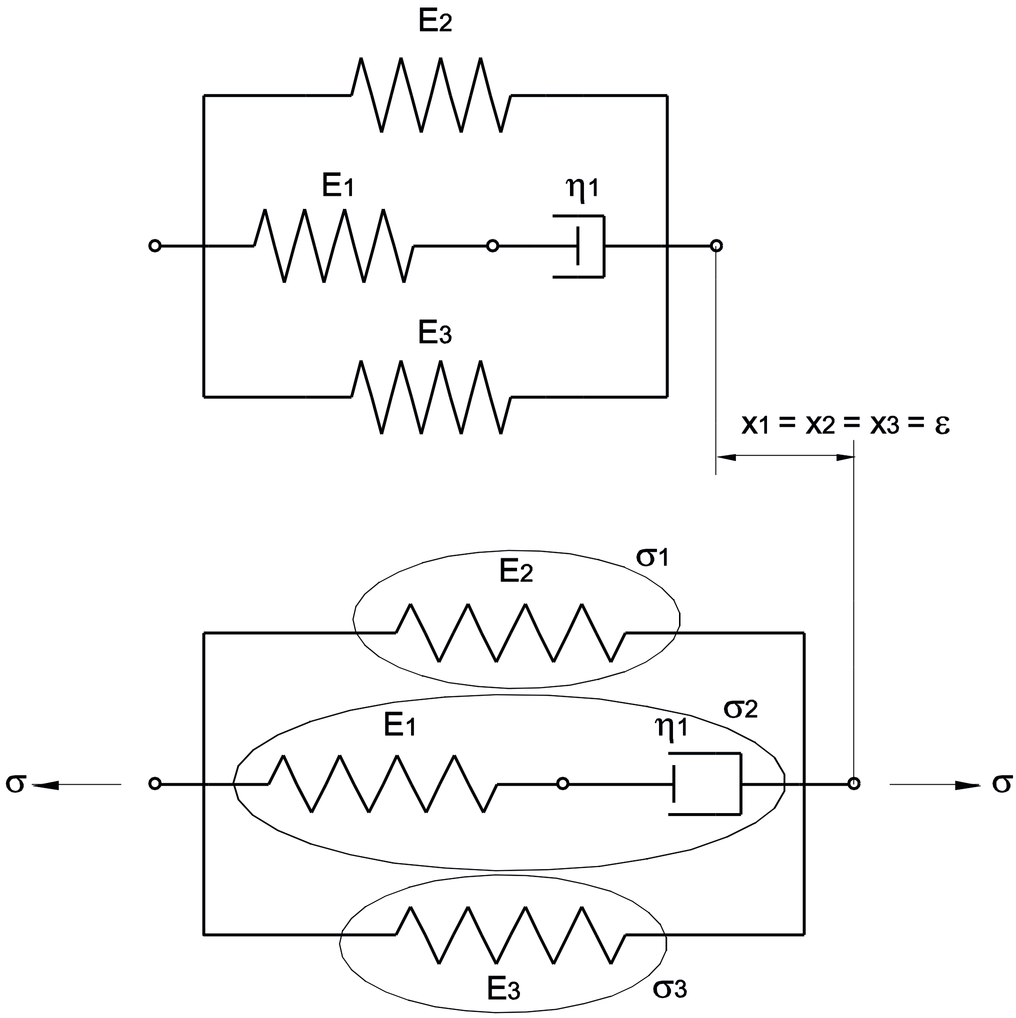


Fig. 8 The complete rheological model for cellular solids

Then the final relationship between stresses and strains is:

$$\sigma(\varepsilon) = e^{-\frac{E_1}{\eta_1}\varepsilon}\left(-1 + e^{\frac{E_1}{\eta_1}\varepsilon}\right)\eta_1 + E_2\varepsilon + E_3\varepsilon \,. \tag{21}$$

Substituting eq.(17) into eq.(21) we get:

$$\sigma(\varepsilon) = e^{-\frac{E_1}{\eta_1}\varepsilon}\left(-1 + e^{\frac{E_1}{\eta_1}\varepsilon}\right)\eta_1 + E_2\varepsilon + \gamma\left(1 - e^{\varepsilon}\right)^h \varepsilon \,, \tag{22}$$

and after some simple manipulation:

$$\sigma(\varepsilon) = e^{-\frac{E_1}{\eta_1}\varepsilon}\left(-1 + e^{\frac{E_1}{\eta_1}\varepsilon}\right)\eta_1 + \left[E_2 + \gamma\left(1 - e^{\varepsilon}\right)^h\right]\varepsilon \,. \tag{23}$$

The eq.(23) represents the constitutive model of foam materials for axial compressive loading. The model contains five parameters with the following physical significance: parametersE_1, E_2, η_1 a γ represent stresses and with the exception ofE_2 all of them are positive; the parameter h is dimensionless and is always positive (usually 4 or 6 [20]). As usual, all model parameters in the proposed constitutive model must be determined precisely by experiments. Our advantage is that we already known the influences of all parameters on the compressive stress-strain curves.

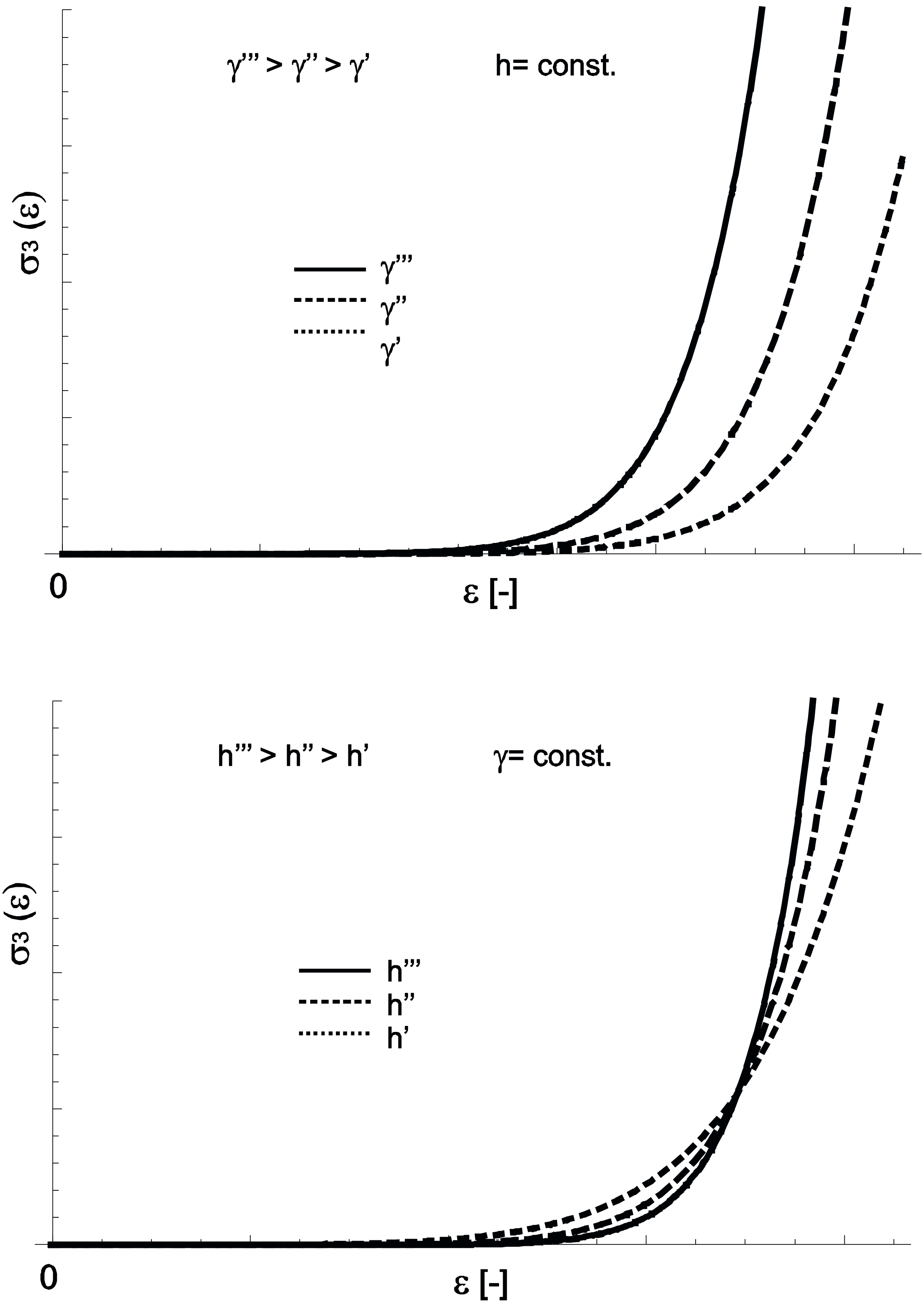


Fig. 9 The influence of parameters γ and h.

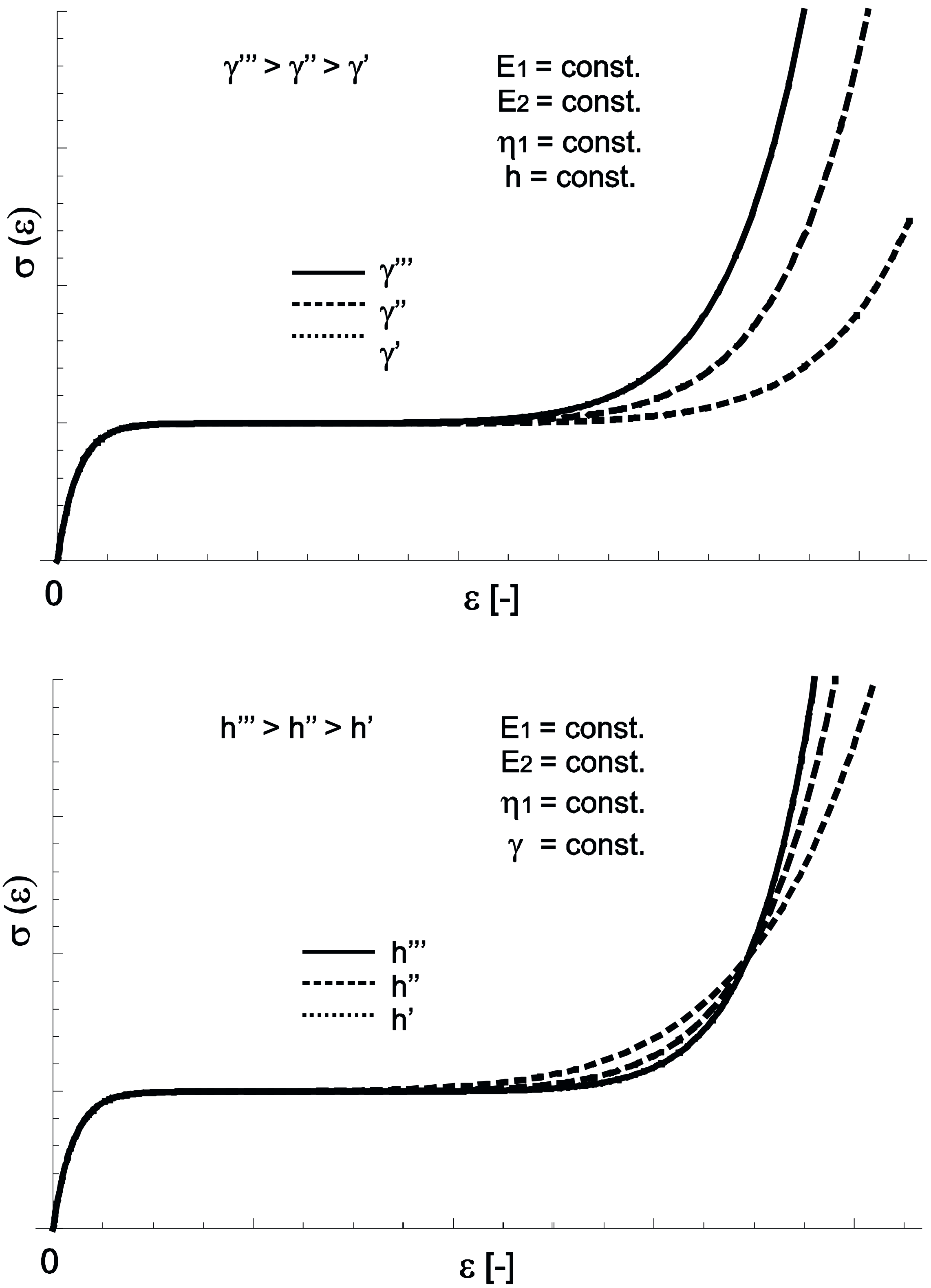


Fig. 10 The influence of parameters γ and $h$on the final stress curves.

This constitutive model can be simplified for foams having a horizontal plateau (with constant collapse stress) by omitting the parameter E_2 in eq. (23), thus we obtain:

$$\sigma(\varepsilon) = e^{-\frac{E_1}{\eta_1}\varepsilon}\left(-1 + e^{\frac{E_1}{\eta_1}\varepsilon}\right)\eta_1 + \gamma\left(1 - e^{\varepsilon}\right)^h \varepsilon . \quad (24)$$

CURVE FITTING OF TESTED FOAMS

The accuracy of the proposed model has been compared with two other phenomenological models [15, 16]. In this study all three models have been used for the fitmentof experimental compressive curves for polyurethane and aluminium foams with various densities. The program MATLAB and its application Curve Fitting Toolbox have been used for making this study. The Sum of Squares Due to Error (SSE) and R-Squaremeasures have been compared. The SSE measures the total deviation between the fitting curve and the experimentalone.Itis also knownas the accumulated square of residuals. The value closer to 0 indicates a better fit. The R-Square measures how successful the fit is in explaining the variation in the data. The R-Square is the square of the correlation between the response values and the predicted response values. It can take on any value between 0 and 1, with a value closer to 1 indicating a better fit. [21]

The Avalle model [15] is presented by the following equation:

$$\sigma(\varepsilon) = A\left(1 - e^{(-E/A)\varepsilon(1-\varepsilon)^m}\right) + B\left(\frac{\varepsilon}{1-\varepsilon}\right)^n . \quad (25)$$

and the Liumodel [16] is presented by the following equation:

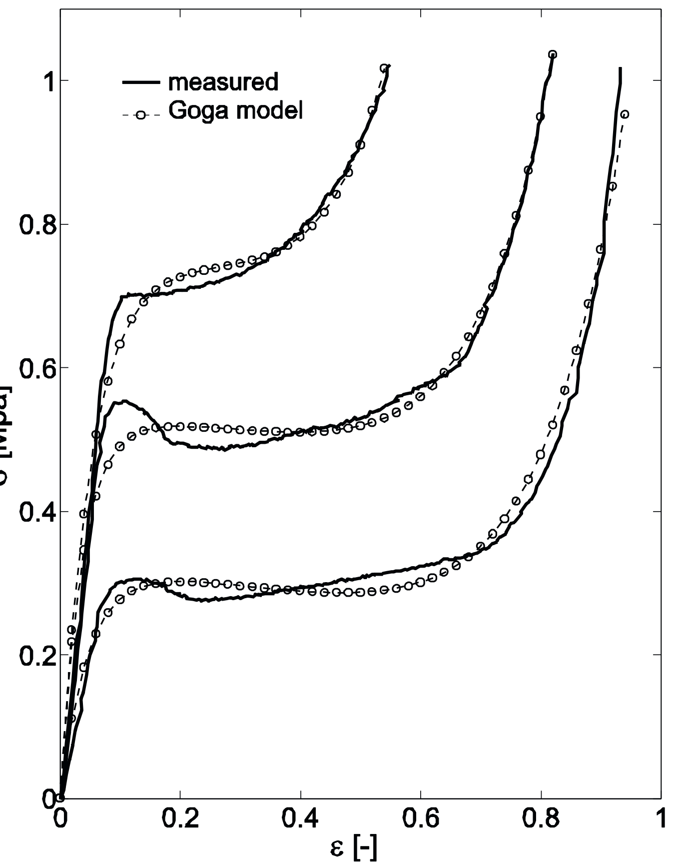
$$\sigma(\varepsilon) = a\left(\frac{e^{\alpha\varepsilon} - 1}{b + e^{\beta\varepsilon}}\right) + e^c\left(e^{\chi\varepsilon} - 1\right), \quad (26)$$

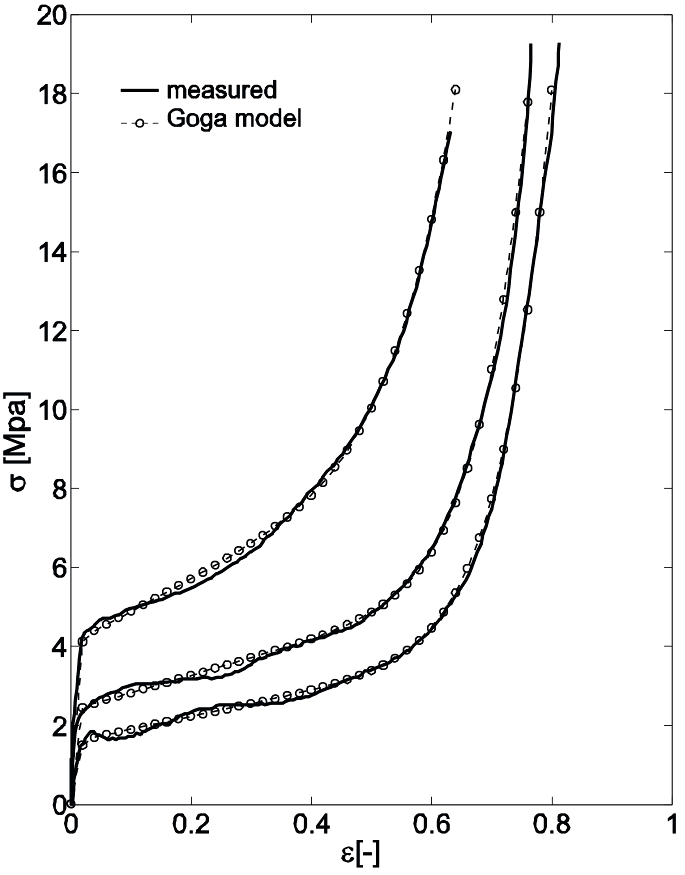
where A, B, E, m, $n$are parameters of the Avalle model and a,b, c, α, β, $\chi$are parameters of the Liu model.

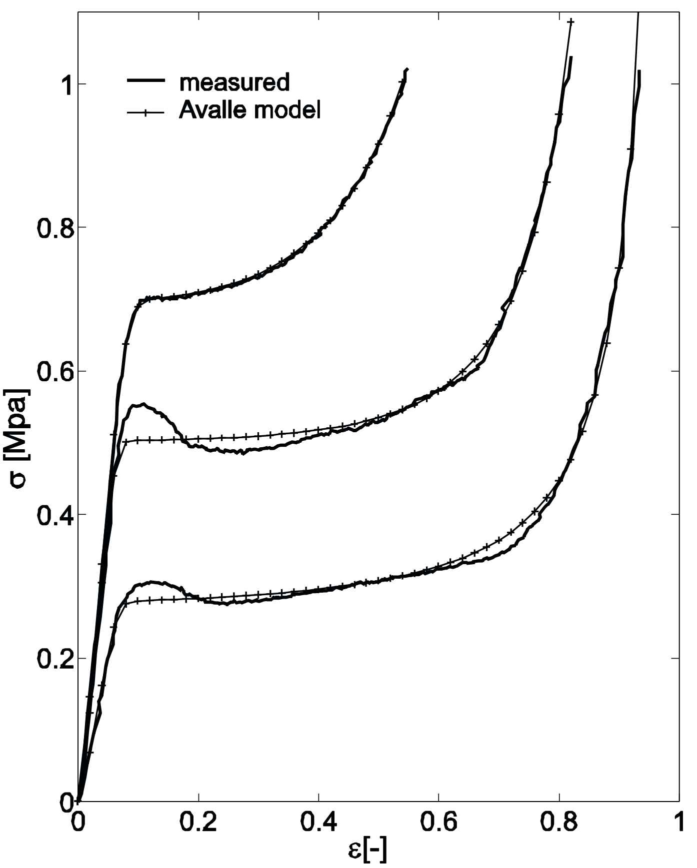
All experiments have been done on polyurethane (PUR) and aluminium (AL) foams. Three samples of each foam with different density have been tested under quasi-static compressive loading using auniversal testing machine with a constant strain rate of0,02 s^{-1} at constant temperature. The samples are50x50x30mm bricks. The experimental and fitting curves are presented on Fig. 11. Table 1 represents a comparison of three models in the sense of curve fitting.

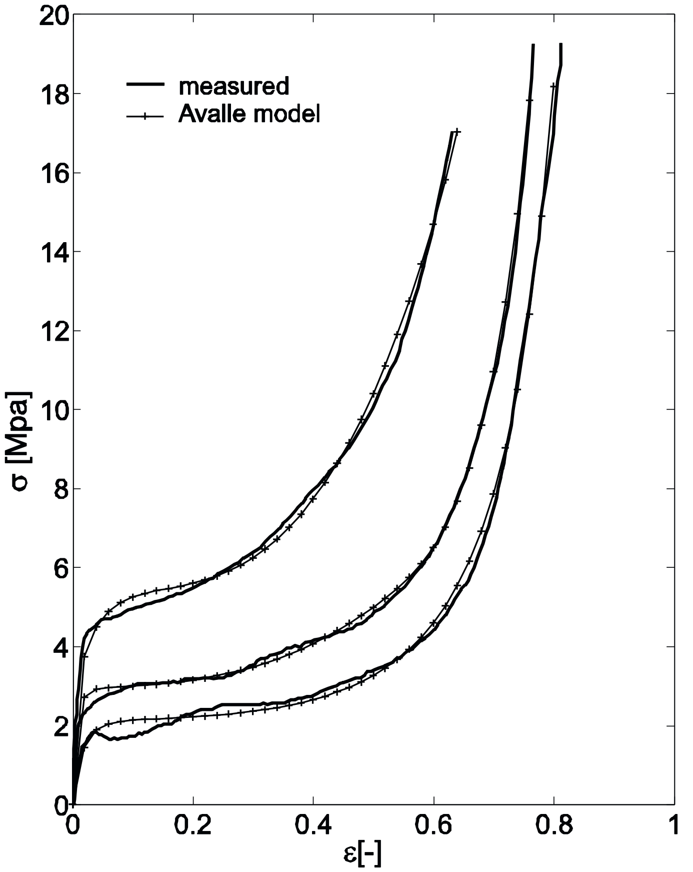
Tab. 1 The values of SSE a E-square fitting for different models.

foam	model	SSE	R-square		foam	model	SSE	R-square
PUR 37	Avalle	0,0404	0,9922		AL 30	Avalle	9,0872	0,9958
	Liu	0,0492	0,9905			Liu	5,9688	0,9982
	Goga	0,1267	0,9755			Goga	3,3847	0,9984
PUR 57	Avalle	0,0648	0,9866		AL 35	Avalle	4,2336	0,9984
	Liu	0,1513	0,9687			Liu	3,0369	0,9989
	Goga	0,1604	0,9668			Goga	4,3854	0,9984
PUR 95	Avalle	0,0035	0,9993		AL 55	Avalle	7,4062	0,9957
	Liu	0,0793	0,9840			Liu	4,0513	0,9977
	Goga	0,1202	0,9758			Goga	3,3132	0,9979


measured
Goga model
σ [Mpa]
ε [-]
0
0.2
0.4
0.6
0.8
1


measured
Goga model
σ [Mpa]
ε[-]
0
2
4
6
8
10
12
14
16
18
20
0.2
0.4
0.6
0.8
1


measured
Avalle model
1
0.8
0.6
0.4
0.2
0
σ [Mpa]
0
0.2
0.4
0.6
0.8
1
ε[-]


measured
Avalle model
20
18
16
14
12
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σ [Mpa]
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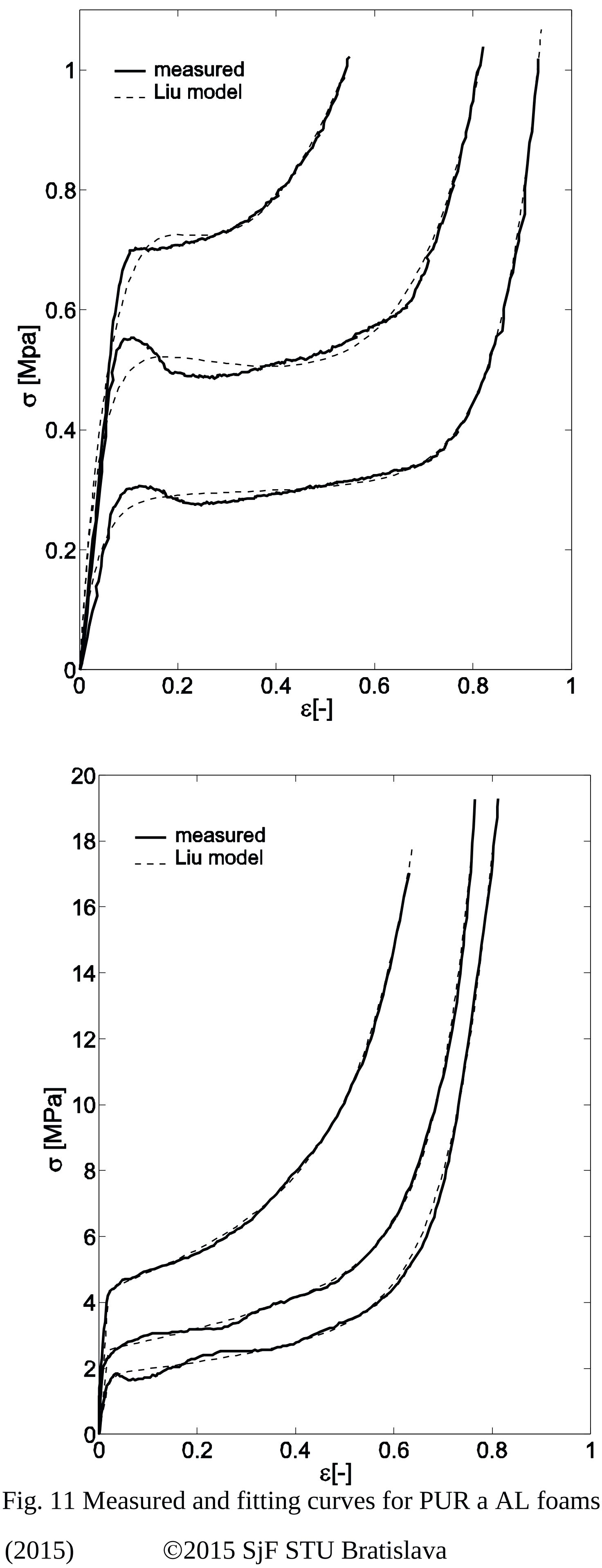

Fig. 11 Measured and fitting curves for PUR a AL foams.

Discussion and conclusions

This paper presents the new phenomenological material model for foam materials subject to axial compressive loading. The proposed model is a simple model that consists ofa small number of parameters with the clear physical interpretation. The influence of each parameter on curve fitting has been precisely determinedand to confirm the physical meaning of all parameters. The proposed model has been tested and verified on polyurethane and aluminium foams with different densities. The results presented in Fig. 11 and Tab. 1 confirm the correctness of our new model. Our model can be applied for various types of foams as can be presented above.

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