DESIGN, ANALYSIS OF 4X4 AND 8X8 MIMO SYSTEM WITH ZF, MMSE AND BF TECHNIQUES

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Abstract. This work focuses on studying signal detection using three different equalization techniques, namely: Zero Forcing (ZF), Minimum Mean Square Error (MMSE) and Beam Forming (BF), for a 4x4 MIMO-system. Results show that ZF equalization is the simplest technique for signal detection. However, Beam Forming (BF) gives better Bit Error Rate (BER) performances at high Signal to Noise Ratio (SNR) values with some complexity in design. For more antennas at the base station, it is too complex to design the weight matrix for ZF; however, it is suitable for BF with the help of good quality digital signal processors. Performance of MIMO-system, with 8 antennas at the base station using BF equalization, is analysed to get BER values at different SNR. Results show a considerable improvement in BER for 8 antennas at the base station.

Keywords: MIMO system, ZF, MMSE, BF, BER, SNR

1. INTRODUCTION

Massive – MIMO systems can send data at a higher speed than conventional MIMO systems are capable of, using spatial multiplexing techniques. At the receiver end, a large number of signals are received in addition to along with the signal from the desired transmit antenna, which act as an interference, thus making signal detection a crucial task for massive – MIMO systems [1]. There are various detection techniques which can be used for this purpose, namely: Zero Forcing (ZF), Minimum Mean Square Error (MMSE) detection and Beam Forming (BF). In a conventional communication system, there is only one antenna, with a variance of signals from the desired transmit antenna, which act as an interference, thus making signal detection a crucial task for massive – MIMO systems [1].

In a conventional communication system, with one antenna at the base station and one receive antenna, the channel gain between the ith transmit antenna and the jth receive antenna, where j=1, 2, 3,…., NR and i=1, 2, 3,…., NT. Let the multiplexed transmit data signal is denoted by X=[x₁, x₂, ..., x₉]ᵀ where x₁ denotes the transmit signal from the ith transmit antenna. The Received data denoted by y= [y₁, y₂, ..., y₉]ᵀ where yᵢ denotes the received signal at the jth receive antenna. Let nᵢ denotes the white Gaussian noise, at the jth receive antenna, with a variance σᵢ².

The NR x NT massive-MIMO system can be represented mathematically as

\[ y = \mathbf{H}x + \mathbf{n} \]  

(1)

where \( \mathbf{n} \) is the noise vector and \( \mathbf{H} \) is the channel matrix.

In matrix form MIMO system model can be written as,

\[ y = \mathbf{H}x + \mathbf{n} \]  

(2)

Where \( \mathbf{n} \) = [n₁, n₂, ..., n₉]ᵀ and \( \mathbf{H} \) denotes the ith column vector of the channel matrix \( \mathbf{H} \).

In matrix form MIMO system model can be written as,
Here $h_{ij} = \alpha + j\beta$ (complex channel).

For a sufficiently rich scattering environment with no line of sight (LOS), the channel is a Raleigh Channel with channel gain ($|h_{ij}|$) where $\alpha$ and $\beta$ are normally distributed independent random variables. Noise power for each noise component at each receiver antenna is given as

$$E(|n_i|^2) = \sigma_n^2$$

The covariance matrix for noise is given mathematically as

$$R_n = E(\bar{n}\bar{n}^H) = \sigma_n^2 I$$

The noise at each antenna is uncorrelated and white across the space, hence known as spatial noise.

### 2.2. Signal Detection

For the above mentioned model of massive – MIMO we use here two kinds of linear signal detectors, namely ZF and MMSE and an angle based signal detector named as BF. To detect the desired signal from the target transmit antenna at receiving end, all the interference signals are nullified or minimized by inverting the effect of the channel by multiplying a suitable weight matrix $\mathbf{W}$, such that

$$\mathbf{X}_D = \mathbf{W}^H \mathbf{Y}$$

where $\mathbf{X}_D$ is the detected symbol vector.

#### A. Zero Forcing Signal Detection

In Zero forcing signal detection the interferences are nullified by weight matrix $\mathbf{W}_{ZF}$, where

$$\mathbf{W}_{ZF} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \mathbf{Y}$$

It minimizes the interference by inverting the channel effect as

$$\mathbf{X}_{ZF} = \mathbf{W}_{ZF} \mathbf{Y}$$

$$\mathbf{X}_{ZF} = \mathbf{X} + \bar{\mathbf{n}}_{ZF}$$

Where, $\bar{\mathbf{n}}_{ZF} = \mathbf{W}_{ZF} \mathbf{Y} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \mathbf{n}$

Noise power can be calculated as $||\bar{\mathbf{n}}_{ZF}||_2^2$.

The expected value of noise power is given as (Figure 1)

$$E(||\bar{\mathbf{n}}_{ZF}||_2^2) = \sum_{i=1}^{N_t} \frac{\sigma_n^2}{\sigma_i^2 + \sigma_{n}^2}$$

#### 2.3. MMSE Signal Detection

In MMSE signal detection the interference is minimized by enhancing the post detection signal-to-interference plus noise ratio (SINR) by multiplying the received signal with MMSE weight matrix, given as:

$$\mathbf{W}_{MMSE} = (\mathbf{H}^H \mathbf{H} + \sigma_n^2 I)^{-1} \mathbf{H}^H \mathbf{Y}$$

It minimizes the interference by inverting the channel effect as

$$\mathbf{X}_{MMSE} = \mathbf{W}_{MMSE} \mathbf{Y}$$

$$\mathbf{X}_{MMSE} = \mathbf{X} + (\mathbf{H}^H \mathbf{H} + \sigma_n^2 I)^{-1} \mathbf{H}^H \bar{\mathbf{n}}$$

$$\mathbf{X}_{MMSE} = \mathbf{X} + \bar{\mathbf{n}}_{MMSE}$$

Where, $\bar{\mathbf{n}}_{MMSE} = (\mathbf{H}^H \mathbf{H} + \sigma_n^2 I)^{-1} \mathbf{H}^H \bar{\mathbf{n}}$.

The MMSE receiver requires the statistical information about the noise ($\sigma_{n}^2$).

Noise power can be calculated as $||\bar{\mathbf{n}}_{MMSE}||_2^2$.

The expected value of noise power is given as

$$E(||\bar{\mathbf{n}}_{MMSE}||_2^2) = \sum_{i=1}^{N_t} \frac{\sigma_n^2}{\sigma_i^2 + \sigma_{n}^2}$$

For a 4x4 MIMO system, simulation results for BER using MMSE equalization is given in Figure 2.
3. BEAM FORMING

In beam forming signal detection technique we use close-loop transmit diversity scheme where the channel is already known to the receiver. To minimize the effect of the channel, the symbol transmitted from each transmit antenna is multiplied by a complex number which is equal to the inverse of the phase of the channel, so that all the received signals at a receiver leads to constructive interference.

Mathematically, received signal (without BF) is given as:
\[ y = HX + \tilde{n} = h_1x_1 + h_2x_2 + \ldots + h_Nx_N + \tilde{n} \]  

In matrix form
\[ y = [h_1, h_2, h_3, \ldots, h_N] \cdot x + n \]  

Received signal (with BF)
\[ y = [h_1, h_2, h_3, \ldots, h_N] \cdot [e^{-j\theta_1} x_1, e^{-j\theta_2} x_2, e^{-j\theta_3} x_3, \ldots, e^{-j\theta_N} x_N] + n \]  

Where, \( h_i = |h_i|e^{j\theta_i} \)

Thus, the signal received at the receiver is
\[ y = (|h_1| + |h_2| + \ldots + |h_N|) x + n \]  

To invert the effect of the channel, the received symbols are multiply by the inverse of the channel matrix obtained using beam forming. The detected symbol can be expressed mathematically as
\[ \hat{y} = \frac{y}{(\|h_1\| + |h_2| + \ldots + |h_N|)} = x + \frac{n}{(\|h_1\| + |h_2| + \ldots + |h_N|)} \]  

For a 4x1 MIMO system, simulation results for BER using BF equalization is given below (Figure 3).

For an 8x1 MIMO system, simulation results for BER using BF equalization is given below (Figure 4).

4. RESULTS AND DISCUSSION

Simulation results for 4x4 MIMO-Systems, using three different equalization techniques (ZF, MMSE and Beam Forming), are analysed to obtain the behaviour of their performances based on BER. Zero forcing equalization is trying to nullify the interference and gives a moderate
BER value for higher SNR as it is easy to implement practical due to less complexity. The simulation results show that the MMSE is comparatively better technique for post detection equalization. Tabular values show that MMSE gives least BER for moderate SNR values. However, it is comparatively complex to design and implement practically. Another technique called Beam Forming is used here to minimize the interferences and enhance system performance. Simulation results show tremendously positive results for Beam Forming. For higher SNR, almost zero BER is observed for large amounts of transmitting bits. Simulation results for the 8x1 MIMO show that BER is considerably reduced even at lower SNR if we use beam forming.

5. CONCLUSION

In this work we have tried to implement the Massive-MIMO systems with its mathematical model and related simulation results. A 4x4 basic model of MIMO systems is designed using different equalization techniques for post detection namely, ZF, MMSE and Beam Forming. Simulation results show that although ZF equalization is simplest to implement for a 4x4 system, but it is much more complex for more antenna at base station. Based on the simulation results it is clear that beam forming (BF) is the best suitable method for massive-MIMO systems (that is, for a system with large number of antenna at the base station). A further improved system with better SNR and less BER using Beam Forming, with 8 antennas at the base station (8x1 MIMO System) is simulated. Simulation results obtained are robust and favourable with less BER. For next generation cellular communication systems, Beam Forming is one of the best suitable techniques for signal detection and pre-coding. In practice, a lot of limitations arise due to the correlation of antennas and hardware impairments.

6. REFERENCES