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EQUITY RISK: MEASURING RETURN VOLATILITY USING HISTORICAL HIGH-FREQUENCY DATA

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Abstract:

Market Volatility has been investigated at great lengths, but the measure of historical volatility, referred to as the relative volatility, is inconsistent. Using historical return data to calculate the volatility of a stock return provides a measure of the realized volatility. Realized volatility is often measured using some method of calculating a deviation from the mean of the returns for the stock price, the summation of squared returns, or the summation of absolute returns. We look to the stocks that make up the DJIA, using tick-by-tick data from June 2015 - May 2016. This research helps to address the question of what is the better measure of realized volatility? Several measures of volatility are used as proxies and are compared at four estimation time intervals. We review these measures to determine a closer/better fit estimator to the true realized volatility, using MSE, MAD, Diebold-Mariano test, and Pitman Closeness. We find that when using a standard deviation based on transaction level returns, shorter increments of time, while containing some levels of noise, are better estimates of volatility than longer increments.

Key words: *Risk, Volatility, Equity, Return, High-Frequency*

1. Introduction

The volatility of the return of an equity is an important measure for investors interested in maximizing profits over some period of time. Volatility from the standpoint of the investor, is the fundamental risk and return tradeoff of the investment. An investment that is purchased at the "right price" is a good investment, provided that the investor can meet his/her investment objectives relative to return. The price of an investment is determined by several factors, including the risk associated with the investment, and volatility is one way we measure risk.

Many applications of volatility exist in trying to determine pricing for both stocks and their associated derivatives. Several well-known models, like Black-Scholes, have been used for decades, and incorporate volatility into the pricing model. In these models, the volatility is based on existing prices and other variables, thus resulting in an implied

volatility. The challenge in these models is that the measure of volatility is not one that can be calculated independently of the model. The other parameters of the model are identified, along with the current price, and the volatility is found as the unknown.

A problem in financial literature is that there are many proposed methods utilized to measure the volatility of a stock's return. As noted by Anderson and Bollerslev (1998),

"The computation of daily return variances from high-frequency intraday returns parallels the use of daily returns calculating monthly ex-post volatility, as exemplified by Schwert (1989, 1990a) and Schwert and Sequin (1990). This idea has previously been applied by others, Hsieh (1991) and Schwert (1990b) in measuring daily equity market volatility from the sample standard deviations for bonds and currencies."

Statistically, return volatility is simply a measure of dispersion of returns. With this in mind, the standard deviation of the returns should provide a simple and measureable indication of the historical variation of the returns.

This is not a new concept, though the utilization of high-frequency data, particularly the transaction level trade data (tick level data), has been ignored as it introduces various levels of noise. The noise, while difficult to estimate, is none the less part of the inherent variation of the returns, so why shouldn't we include this in our measure of "true" volatility? Anderson, Bollerslev, Diebold, and Ebens (2001) suggest that the best estimation approach is found when using a time interval of five minutes to mitigate the effects of "market microstructure frictions, including price discreteness, infrequent trading, and bid-ask bounce effects". Liu, Patton and Sheppard (2015) support the use of the five minute interval when measuring relative volatility based on a broad study across several asset classes, although their sample within each asset class was relatively small. In contrast, this paper attempts to provide evidence to support the use of the standard deviation of transaction level change in return as the correct measure of intra-day volatility for a stock price.

Ñíguez (2016) used proxies that include the increments of quadratic variation discussed by Ghysels et al. (2006), and compared these proxies to four models of monthly volatility measures to estimate the EuroStoxx 50 index. In that analysis, Ñíguez used daily and monthly returns over a 19 year period, providing 5176 daily and 239 monthly observations. In our study, we utilized transaction level data for all 30 Dow Component Stocks over a single year, providing approximately 75 million unique trade observations. This large amount of data translates to an average of 10,000 transactions per stock, and 250 trading days.

2. Methodology

The approach taken in this study is to use transaction level data of the individual stock, not of the index, to measure the standard deviation of the return. We take the transaction data for each trade of all Dow Jones Industrial Average (DJIA) stocks over a 12 month period. The time line provides 250 trading days observed, with the number of daily trades ranging between a few thousand trades per day up to over fifty thousand trades per day. The data provides approximately 75 million observations across the 30 DJIA components for this time period.

To establish the measure of volatility we take the sequential trades for each equity through the course of each trading day. Hansen and Lunde (2005) utilized a log based method for calculating return, thus we calculate the return from one transaction to the next as

$$R_t = \log \frac{P_t}{P_{t-1}} \quad (1)$$

There could be a need to adjust for variation in volume when dealing with daily or monthly time periods to account for the highly skewed distribution of the trade volumes and to offset for extreme values of the trading volumes (Barron, 1995). When using high frequency data at the transaction level, we see that a vast majority of trades are round lots of 100 shares. Since the adjustment would essentially create a constant, we chose not to adjust for the minimal volume variation. The unadjusted return is used to calculate the daily standard deviation of the return as

$$SD = \sqrt{\frac{\sum_{t=1}^n (R_t - \bar{R})^2}{n-1}} \quad (2)$$

where $\bar{R}$ is the mean of the return for all transactions in the day and n is the number of transactions in the day. The SD is the proposed measure of volatility of the daily returns for each stock evaluated.

We additionally compare two alternative proxies for return volatility, using the summed squared return and the summed absolute return as suggested by Koopman et al. (2005). For the summed squared return are calculate SS proxy as

$$SS = \sum_{t=1}^n (R_t)^2 \quad (3)$$

For the summed absolute returns we calculate the ABS proxy as

$$ABS = \sum_{t=1}^n |R_t| \quad (4)$$

Liu, Patton, and Sheppard (2015) suggest that the increment of time used to estimate the volatility is best when a five minute interval is used to estimate the realized volatility. Their investigation used data from 31 assets across 5 asset classes, using data over an 11 year period, though for some assets there were limitations on data availability. In their study, they used a proxy for the quadratic variation using the squared open to close returns from transaction prices in their main analysis. In order to test this against our proposed measure of volatility, we take time sequenced intervals to assess their claim. Using intervals of one, five, 15, and 30 minutes, we estimate the realized volatility by measuring the volatility over each of the time intervals.

We calculate the return from one 1-minute interval to the next as

$$R_{t1} = \log \frac{P_t}{P_{t-1}} \quad (5)$$

The return for the one minute interval is used to calculate the one minute interval estimate of the daily standard deviation of the return as

$$SD1 = \sqrt{\frac{\sum_{t=1}^n (R_{t1} - \bar{R1})^2}{n-1}} \quad (6)$$

where $\overline{R1}$ is the mean return for all one minute intervals in the day and n is the number of one minute intervals in the day.

Following the suggestion of Morgan (1996), we then scale the measured daily standard deviation based on the square of the number of intervals in each trading day (450 minutes per trading day, giving a scaling of $\sqrt{450}$ for the 1 minute interval, $\sqrt{90}$ for the 5 minute interval, etc.) to give the scaled standard deviation for each of the four intervals. These become the proxy comparisons for each time interval against the daily standard deviation estimates.

This method of scaling for the number of time intervals was referred to as $\sqrt{h}$ (Christoffson et.al. (1998)), where h is the number of time intervals being scaled. Christofferson et.al. (1998) suggest that scaling works when log prices are independently and identically distributed, and also for percentiles “if log changes are not only iid, but also normally distributed. As this applies in our case, we extend for this investigation that the $\sqrt{h}$ also holds when converting for intraday trading. Investigation into this assumption that the $\sqrt{h}$ will hold when applied to scaling for intraday trading is another area we would consider for future studies. Of consideration when making that investigation is the length of the time interval that the scaling holds (Danielsson & Zigrand, 2006).

$$ScSD1ScSD1 = SD\sqrt{450} \quad (7)$$

We proceed to replicate this approach for the five minute, the 15 minute and the 30 minute intervals using the same methods. We then compare each estimate based on the time interval back to the proxy. For SS and ABS proxies we do not make the time interval scaling adjustment.

Liu, Patton, and Sheppard (2015) measured the accuracy of the estimators using the QLIKE loss function, as well as pair-wise comparisons using a “robust t-statistic on the average difference in loss.” In our analysis we chose to use several methods to assess the performance of the interval estimators of the true realized volatility. The Mean Squared Error (MSE) and Mean Absolute Deviation (MAD) are common measures of performance of estimators. A third, less common method is the Pitman Closeness measure. As an additional pair-wise comparison technique we also employed the Diebold and Mariano (1995) style pairwise tests between our volatility estimators.

The methodology for calculating the MSE for the one minute intervals was to take the squared deviation between the estimate SD1 and the ScSD1 for each trading day and to calculate the mean of this squared deviation as

$$MSE1 = \frac{\sum_{d=1}^N (SD1_d - ScSD1_d)^2}{N} \quad (7)$$

where d is the trading day and N is the total number of trading days.

The methodology for calculating the MAD for the one minute intervals was to take the absolute deviation of between the SD1 and the ScSD1 for each trading day and to calculate the mean of this absolute deviation as

$$MAD1 = \frac{\sum_{d=1}^N |SD1_d - ScSD1_d|}{N} \quad (8)$$

where d is the trading day and N is the total number of trading days.

While less utilized in the applied literature, Pitman Closeness has been a topic of discussion as a method of assessing the performance of an estimator of a parameter for decades. To address a discussion on methods to assess how well an estimator performed, Pitman [1937] proposed a method of comparing the closeness of two estimators of a parameter to determine which of the two is “closer” to the true value of the parameter being estimated. Using Pitman Closeness, if $\hat{\theta}_1$ and $\hat{\theta}_2$ are both estimators of θ , then $\hat{\theta}_1$ would be considered Pitman Closer to θ than $\hat{\theta}_2$ would be to θ if

$$\Pr(|\hat{\theta}_1 - \theta| < |\hat{\theta}_2 - \theta|) > \frac{1}{2}. \quad (9)$$

For empirically estimating the Pitman Closeness (PC) of the one minute interval based estimator of ScSD1 and the five minute interval estimator of ScSD5, we find the absolute deviations of each estimator from their respective proxies and see which is closer as follows

For each trading day evaluated from $d=1$ to N ,

$$\text{if } |SD1 - ScSD1| < |SD5 - ScSD5| \quad (10)$$

then we increment a counter value PC1 by 1, if not then we do not increment the counter value. After assessing comparisons across all N days, we compare the performance of the one minute interval estimator versus the five minute interval estimator as

$$\text{if } \frac{PC1}{N} > 0.50 \quad (11)$$

then the one minute interval estimator is Pitman Closer to its proxy than the five minute interval estimator. Otherwise,

$$\text{if } \frac{PC1}{N} < 0.50 \quad (12)$$

then the five minute interval estimator is Pitman Closer to its proxy than the one minute interval estimator. In the unlikely event that the ratio is equal to 0.50, the two estimators would be equally likely, and neither would be Pitman Closer than the other.

3. Results

In our evaluation of the DOW components over the 250 trading days, we calculated the SD and its time interval scaling, for each equity. In comparing the time based interval estimators for each equity, we found that the smaller time interval was always a better performing estimator than all intervals tested of longer duration. Summary level data is shown in Table 1. Panel A provides the mean, standard deviation, minimum, and maximum MAD, MSE, and Pitman Closeness for each of the time based estimators of calculated standard deviation proxy across the 30 equities. Panel B provides the mean, standard deviation, minimum, and maximum MAD, MSE, and Pitman Closeness for each of the time based estimators of the summed squared proxy across all 30 equities. Panel C provides the mean, standard deviation, minimum, and maximum MAD, MSE, and Pitman Closeness for each of the time based estimators of the summed absolute value proxy across all 30 equities.

Table 1

This table contains summary statistics for MSE, MAD, and Pitman Closeness for each estimator compared with its respective proxy. Panel A reports the summary statistics for the Scaled Standard Deviation for each of the time interval estimators. Panel B reports the summary statistics for the Sum of Squared Returns for each of the time interval estimators. Panel C reports the summary statistics for the Sum of Absolute Returns for each of the time interval estimators.

Panel A: Scaled Standard Deviation						
	MSE-ScSD1	MSE-ScSD5	MSE-ScSD15	MSE-ScSD30		
mean	1.9643E-05	2.0073E-06	2.0398E-06	5.5493E-06		
st dev	1.9845E-05	3.4134E-06	1.0946E-06	2.0188E-06		
min	3.4093E-06	1.0314E-07	4.7309E-07	2.4355E-06		
max	1.0239E-04	1.6892E-05	5.3801E-06	1.0778E-05		
	MAD-ScSD1	MAD-ScSD5	MAD-ScSD15	MAD-ScSD30		
mean	0.00299	0.00050	0.00106	0.00197		
st dev	0.00070	0.00022	0.00029	0.00039		
min	0.00176	0.00023	0.00052	0.00133		
max	0.00486	0.00108	0.00163	0.00284		
	PC1v5	PC1v15	PC1v30	PC5v15	PC5v30	PC15v30
mean	0.9996	0.9607	0.2104	0.7422	0.9206	0.9530
st dev	0.0012	0.0718	0.1884	0.2096	0.0853	0.0141
min	0.9960	0.6680	0.0000	0.2360	0.6680	0.9277
max	1.0000	1.0000	0.7480	1.0000	1.0000	0.9760
Panel B: Sum of Squared Returns						
	MSE-SS1	MSE-SS5	MSE-SS15	MSE-SS30		
mean	0.000152	0.000157	0.000158	0.000158		
st dev	0.000542	0.000548	0.000549	0.000551		
min	0.000000	0.000000	0.000000	0.000000		
max	0.002940	0.002963	0.002968	0.002974		
	MAD-SS1	MAD-SS5	MAD-SS15	MAD-SS30		
mean	0.000478	0.000504	0.000525	0.000541		
st dev	0.000727	0.000736	0.000737	0.000737		
min	0.000027	0.000040	0.000050	0.000058		
max	0.003694	0.003725	0.003745	0.003762		
	PC1v5	PC1v15	PC1v30	PC5v15	PC5v30	PC15v30
mean	0.76806	0.86351	0.89022	0.75081	0.81450	0.67869
st dev	0.03456	0.02407	0.01851	0.03089	0.02069	0.03482
min	0.70000	0.82400	0.83600	0.69200	0.76800	0.59200
max	0.84000	0.93200	0.92800	0.83200	0.84800	0.74000
Panel C: Sum of Absolute Returns						
	MSE-ABS1	MSE-ABS5	MSE-ABS15	MSE-ABS30		
mean	3.84973	4.03155	4.10254	4.13216		
st dev	9.48859	9.61113	9.65697	9.67490		
min	0.05790	0.09298	0.10728	0.11384		
max	50.33480	51.12248	51.42028	51.53666		
	MAD-ABS1	MAD-ABS5	MAD-ABS15	MAD-ABS30		
mean	0.6939526	0.7813460	0.8127927	0.8256064		
st dev	0.5105169	0.5184427	0.5212314	0.5224091		
min	0.1799537	0.2444480	0.2676599	0.2773632		
max	2.8804114	2.9849891	3.0223052	3.0371899		
	PC1v5	PC1v15	PC1v30	PC5v15	PC5v30	PC15v30
mean			1.0000	1.0000		

	1.0000	1.0000			1.0000	1.0000
st dev	-	-	-	-	-	-
min	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
max	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

The equity level comparisons using the MSE, MAD, and Pitman Closeness are provided in the appendix.

For robustness, we examine the performance of our volatility proxies using the process found in Diebold and Mariano (1995). Diebold and Mariano (1995) compare the accuracy of two different forecasts of a time series. Relying on the notation used by Diebold and Mariano (1995), let the two forecasts be $\{\hat{y}_{it}\}_{t=1}^T$ and $\{\hat{y}_{jt}\}_{t=1}^T$, and let the time series be $\{y_{it}\}_{t=1}^T$. The errors in forecasting the time series are denoted as $\{e_{it}\}_{t=1}^T$ and $\{e_{jt}\}_{t=1}^T$. The test statistic, S_1 , for testing a null hypothesis of whether one forecast outperforms the other is

$$S_1 = \frac{\bar{d}}{\sqrt{\frac{2\pi f_d(0)}{T}}}$$

where $\bar{d}$ is the sample mean loss differential and $\hat{f}_d(0)$ is a consistent estimate of the loss differential at frequency 0.

Table 2 contains the results for Diebold and Mariano (1995) style pairwise tests between our volatility estimators. Pairwise tests are performed for each security of the thirty securities that we examine. The reported values reflect the number of securities where the row measure outperformed the column measure. For example, in Panel A the intersection between the row labeled “15 Minute” and the column labeled “5 Minute” contains a value of 2. This means that the 15 minute proxy outperformed the 5 minute proxy for two of our thirty securities. All results reported in this table are significant at least at the 5% level.

Panel A reports the results of the scaled standard deviation measure. For this metric, the Diebold and Mariano tests show that the 5 minute estimator consistently outperforms the other three estimators. Panel B reports results for the sum of squared returns measure. For this metric, the results show that the shorter the time interval of the estimators, then the better the results. Overall, the 1 minute estimators outperforms all others. Panel C reports the results for the sum of absolute returns measure. The results for this metric are similar to those in Panel B; the shorter the time interval the better, with the 1 minute estimators performing the best. These results tend to support our other findings.

Table 2

This table contains results for Diebold and Mariano (1995) style pairwise tests between our volatility estimators. Pairwise tests are performed for each security. Panel A reports the results of the scaled standard deviation measure. Panel B reports results for the sum of squared returns measure. Panel C reports the results for the sum of absolute returns measure. The reported values reflect the number securities where the row measure outperformed the column measure. All results reported in this table are significant at least at the 5% level.

Panel A: Scaled Standard Deviation				
	1 Minute	5 Minute	15 Minute	30 Minute
1 Minute	-	-	-	-
5 Minute	30	-	-	-
15 Minute	8	2	-	-
30 Minute	1	1	1	-
Panel B: Sum of Squared Returns				
	1 Minute	5 Minute	15 Minute	30 Minute
1 Minute	-	-	-	-
5 Minute	7	-	-	-
15 Minute	4	4	-	-
30 Minute	3	4	5	-
Panel C: Sum of Absolute Returns				
	1 Minute	5 Minute	15 Minute	30 Minute
1 Minute	-	-	-	-
5 Minute	1	-	-	-
15 Minute	1	1	-	-
30 Minute	1	1	1	-

4. Conclusions

While the results of this evaluation are not surprising, they do provide a basis for contradicting the suggestions of Anderson et al (2001) and Liu, Patton, & Sheppard (2015) that the use of the five minute interval for estimating the realized volatility is superior. A second concern of the Liu, Patton & Sheppard (2015) research is that the measure of return and relative volatility come from the open to close measure across the interval, rather than the incremental changes in price. Our results support that a shorter time interval, as economically available, should be used when estimating the realized volatility of stock price returns as the traditional standard deviation.

Further investigation into this area of realized volatility study should include extended models of estimation including GARCH models, ranged based estimation models, and other models.

Declaration of Interest

The authors report no conflicts of interest. The authors alone are responsible for the content and writing of this paper.

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Appendix

Composite comparisons of MSE, MAD, and PC ScSD1, ScSD5, ScSD15, ScSD30 against proxies

Symbol	MSE-Ad SD1	MSE-Ad SD5	MSE-Ad SD15	MSE-Ad SD30	MAD-Ad SD1	MAD-Ad SD5	MAD-Ad SD15	MAD-Ad SD30	PCtV5	PCtV15	PCtV30	PC5v15	PC5v30	PC15v30
AAPL	3.409E-06	2.011E-07	3.207E-06	8.080E-06	0.001761	0.000316	0.001548	0.002470	1.0000	0.6680	0.7480	1.0000	1.0000	0.9280
AXP	9.924E-06	3.632E-07	1.314E-06	4.552E-06	0.002821	0.000420	0.000935	0.001792	1.0000	0.9960	0.1320	0.7440	0.9280	0.9320
BA	2.457E-05	2.116E-06	1.722E-06	7.048E-06	0.003864	0.000788	0.000959	0.002056	1.0000	0.9960	0.1000	0.5160	0.8120	0.9680
CAT	1.451E-05	3.271E-07	2.743E-06	9.038E-06	0.003734	0.000493	0.001385	0.002622	1.0000	0.9920	0.1600	0.8080	0.9480	0.9680
CSGO	9.281E-06	4.168E-07	1.878E-06	5.719E-06	0.002399	0.000243	0.001250	0.002162	1.0000	0.9480	0.3840	0.9760	0.9920	0.9640
CYX	9.058E-06	1.445E-07	3.610E-06	1.078E-05	0.002923	0.000298	0.001633	0.002843	1.0000	0.9400	0.3680	0.9480	1.0000	0.9680
DD	1.513E-05	6.263E-07	1.951E-06	6.451E-06	0.003665	0.000619	0.001127	0.002213	1.0000	0.9920	0.1000	0.6840	0.9320	0.9560
DIS	7.331E-06	1.875E-07	1.925E-06	6.104E-06	0.002420	0.000262	0.001180	0.002107	1.0000	0.9640	0.2840	0.9560	0.9920	0.9520
GE	1.024E-04	1.689E-05	5.380E-06	5.959E-06	0.003515	0.000727	0.001132	0.001926	1.0000	0.9840	0.1760	0.7440	0.9360	0.9520
GS	2.520E-05	1.324E-06	1.198E-06	5.358E-06	0.004856	0.001082	0.000837	0.001962	1.0000	1.0000	0.0120	0.3120	0.7560	0.9480
HD	3.641E-05	5.085E-06	2.164E-06	4.777E-06	0.003282	0.000592	0.001002	0.001917	1.0000	0.9960	0.0840	0.7480	0.9440	0.9560
IBM	1.236E-05	5.180E-07	8.691E-07	3.910E-06	0.003408	0.000659	0.000739	0.001727	1.0000	1.0000	0.0240	0.4840	0.8840	0.9760
INTC	5.363E-06	1.031E-07	2.545E-06	7.158E-06	0.002271	0.000227	0.001401	0.002360	1.0000	0.9120	0.4680	0.9720	0.9880	0.9440
JNI	2.143E-05	2.352E-06	1.303E-06	3.172E-06	0.002389	0.000380	0.000850	0.001538	1.0000	0.9880	0.2040	0.8040	0.9480	0.9600
JPM	3.478E-05	5.390E-06	3.120E-06	6.209E-06	0.002486	0.000346	0.001323	0.002190	1.0000	0.9480	0.4120	0.9600	0.9920	0.9600
KO	6.143E-06	1.863E-07	6.548E-07	2.436E-06	0.002389	0.000377	0.000673	0.001362	1.0000	1.0000	0.0800	0.6760	0.9200	0.9600
MCD	7.555E-06	2.411E-07	1.022E-06	3.399E-06	0.002620	0.000413	0.000802	0.001571	1.0000	0.9840	0.1080	0.6960	0.9320	0.9440
MMM	1.205E-05	6.947E-07	5.287E-07	2.601E-06	0.003344	0.000787	0.000537	0.001340	1.0000	1.0000	0.0160	0.2840	0.6680	0.9760
MRK	4.605E-05	5.873E-06	3.189E-06	5.780E-06	0.003102	0.000495	0.001174	0.002007	1.0000	0.9680	0.1800	0.8360	0.9480	0.9320
MSFT	4.471E-06	2.855E-07	3.100E-06	7.727E-06	0.001948	0.000328	0.001555	0.002489	0.9960	0.7590	0.7149	0.9799	0.9880	0.9317
NKE	1.367E-05	5.161E-07	1.984E-06	6.411E-06	0.003356	0.000534	0.001143	0.002194	0.9960	0.9600	0.1520	0.7400	0.9200	0.9600
PFE	5.095E-05	6.741E-06	3.528E-06	6.265E-06	0.003167	0.000493	0.001235	0.002112	1.0000	0.9759	0.1807	0.8675	0.9478	0.9277
PG	4.900E-06	1.098E-07	8.428E-07	2.708E-06	0.002149	0.000281	0.000775	0.001448	1.0000	0.9840	0.1840	0.8040	0.9720	0.9400
TRV	1.463E-05	9.313E-07	4.731E-07	2.469E-06	0.003646	0.000902	0.000519	0.001327	1.0000	1.0000	0.0000	0.2360	0.6880	0.9600
UNH	1.898E-05	7.350E-07	1.623E-06	6.297E-06	0.004217	0.000795	0.001015	0.002138	1.0000	0.9880	0.0560	0.5600	0.8800	0.9680
UTX	1.647E-05	1.649E-06	1.831E-06	5.038E-06	0.003125	0.000590	0.000937	0.001790	0.9960	0.9880	0.1200	0.6560	0.8880	0.9560
V	1.583E-05	8.546E-07	1.584E-06	5.353E-06	0.003075	0.000458	0.001061	0.002027	1.0000	0.9799	0.1526	0.7791	0.9478	0.9679
VZ	2.981E-05	4.369E-06	1.684E-06	2.959E-06	0.002759	0.000512	0.000857	0.001519	1.0000	0.9959	0.1057	0.7561	0.9309	0.9472
WMT	1.100E-05	8.728E-07	1.806E-06	6.040E-06	0.002584	0.000381	0.000985	0.001784	1.0000	0.9797	0.2154	0.7764	0.9431	0.9431
XOM	5.622E-06	1.133E-07	2.414E-06	6.682E-06	0.002299	0.000241	0.001326	0.002231	1.0000	0.9357	0.3896	0.9639	0.9920	0.9438

Composite comparisons of MSE, MAD, and PC SS1, SS5, SS15, SS30 against proxies

Symbol	MSE-SS1	MSE-SS5	MSE-SS15	MSE-SS30	MAD-SS1	MAD-SS5	MAD-SS15	MAD-SS30	PC1v5	PC1v15	PC1v30	PC5v15	PC5v30	PC15v30
AAPL	5.23E-06	5.23E-06	5.49E-06	5.67E-06	0.00067114	0.000676785	0.00070642	0.00072708	0.7520	0.8800	0.8760	0.7720	0.8200	0.6840
AXP	6.48E-07	5.95E-07	5.96E-07	5.95E-07	0.000136382	0.000146692	0.000165745	0.000178301	0.7560	0.8360	0.8840	0.7520	0.8240	0.6640
BA	1.23E-05	1.26E-05	1.26E-05	1.25E-05	0.000300901	0.000312906	0.000340405	0.000342265	0.7680	0.8440	0.8880	0.7360	0.7720	0.6840
CAT	4.37E-08	6.4E-08	7.77E-08	1.1E-07	9.20292E-05	0.000122403	0.000154794	0.000184453	0.7440	0.8600	0.8960	0.7280	0.8200	0.6880
CSCO	2.98E-06	3.25E-06	3.35E-06	3.58E-06	0.000322903	0.0003453	0.000367231	0.000388195	0.7440	0.8640	0.8720	0.7360	0.8320	0.7160
CVX	5.72E-08	8.76E-08	1.13E-07	1.33E-07	0.000144385	0.00017636	0.000209944	0.000230961	0.7520	0.8480	0.8680	0.7240	0.7920	0.6680
DD	3.51E-07	4.25E-07	4.21E-07	4.85E-07	0.00015459	0.000180516	0.000196929	0.000215523	0.7480	0.8240	0.8360	0.6920	0.7680	0.6040
DIS	3.55E-06	3.61E-06	3.65E-06	3.7E-06	0.000213118	0.000227007	0.000247369	0.000264306	0.7280	0.8320	0.8960	0.7400	0.8160	0.7040
GE	0.00294	0.002963	0.002968	0.002974	0.003693503	0.003725495	0.003745478	0.003761706	0.7720	0.8600	0.8840	0.7720	0.8480	0.7240
GS	4.1E-08	5.47E-08	6.52E-08	8.51E-08	5.81903E-05	8.66766E-05	0.000109476	0.000135598	0.7600	0.8640	0.9160	0.7400	0.8320	0.6960
HD	0.000168	0.000199	0.0002	0.000201	0.000903948	0.000993945	0.001011163	0.001026248	0.7680	0.8480	0.8760	0.6960	0.7880	0.6520
IBM	2.38E-08	2.98E-08	3.75E-08	5.18E-08	5.91812E-05	7.35949E-05	9.30347E-05	0.000104529	0.7720	0.9120	0.9080	0.8000	0.8080	0.5960
INTC	3.15E-07	3.37E-07	3.61E-07	4.02E-07	0.000264114	0.000286317	0.000311427	0.000329469	0.7920	0.8880	0.8800	0.7440	0.8200	0.6840
JNI	0.000134	0.000136	0.000139	0.000139	0.000791466	0.000807552	0.000827118	0.00083722	0.8280	0.8520	0.9000	0.7680	0.8400	0.6960
JPM	0.000516	0.000606	0.000611	0.000612	0.001491954	0.001632446	0.001658077	0.001672846	0.7840	0.8640	0.9200	0.7560	0.8400	0.7000
KO	3.58E-07	4.23E-07	4.44E-07	4.86E-07	6.63269E-05	8.22996E-05	9.5375E-05	0.000104657	0.8400	0.9320	0.9280	0.8320	0.8320	0.6920
MCD	1.07E-06	1.07E-06	1.08E-06	1.11E-06	0.000135097	0.00014302	0.000159171	0.000172353	0.7360	0.8960	0.8960	0.7880	0.8280	0.6560
MMM	3.84E-08	4.31E-08	4.73E-08	6.35E-08	4.63549E-05	6.07716E-05	7.087E-05	8.23079E-05	0.8160	0.8640	0.9200	0.7120	0.8120	0.6680
MRK	0.000221	0.000218	0.000223	0.000224	0.001001853	0.001012083	0.001036735	0.001058412	0.7840	0.8520	0.9040	0.7120	0.8400	0.7400
MSFT	1.22E-06	1.28E-06	1.36E-06	1.49E-06	0.000460426	0.000479788	0.000505483	0.000528588	0.7711	0.8434	0.8755	0.7510	0.7711	0.6908
NKE	1.39E-06	1.34E-06	1.53E-06	1.65E-06	0.00018869	0.000204619	0.000235179	0.000256111	0.7000	0.8600	0.8880	0.7920	0.8160	0.5920
PFE	0.000445	0.000441	0.000447	0.000448	0.001462111	0.001472207	0.001499856	0.001521574	0.7791	0.8594	0.9036	0.7470	0.8072	0.7068
PG	4.4E-08	1E-07	1.19E-07	1.45E-07	5.87532E-05	7.74334E-05	9.03985E-05	0.000100855	0.7880	0.8680	0.8840	0.7480	0.8080	0.6880
TRV	1.22E-08	2E-08	2.07E-08	2.66E-08	2.68118E-05	3.95242E-05	4.96604E-05	5.76237E-05	0.7800	0.8720	0.8840	0.7600	0.8120	0.6720
UNH	3.52E-08	9.51E-08	1.43E-07	1.78E-07	7.41415E-05	0.00010923	0.000147371	0.000168752	0.7240	0.8880	0.8920	0.7840	0.8240	0.7000
UTX	9.56E-07	9.61E-07	9.52E-07	9.58E-07	0.000131974	0.000147211	0.000155619	0.000165432	0.7960	0.8640	0.8880	0.7160	0.8000	0.6720
V	1.36E-05	1.4E-05	1.52E-05	1.55E-05	0.000318817	0.000338129	0.000365283	0.000381463	0.7189	0.8594	0.8755	0.7430	0.8153	0.6466
VZ	8.34E-05	9.13E-05	9.17E-05	9.16E-05	0.000636759	0.000676201	0.000689531	0.000699036	0.8333	0.8943	0.9024	0.7642	0.8333	0.6667
WMT	3.16E-06	3.01E-06	3.09E-06	3.01E-06	0.000256694	0.000263792	0.000278631	0.000282623	0.7886	0.8415	0.8780	0.7520	0.8008	0.6951
XOM	7.77E-07	8.07E-07	8.21E-07	8.46E-07	0.000197554	0.000219426	0.00024109	0.000260466	0.7189	0.8353	0.8876	0.7671	0.8153	0.7149
MSE-SS1														
MSE-SS5														
MSE-SS15														
MSE-SS30														
MAD-SS1														
MAD-SS5														
MAD-SS15														
MAD-SS30														
PC1v5														
PC1v15														
PC1v30														
PC5v15														
PC5v30														
PC15v30														

Composite comparisons of MSE, MAD, and PC ABS1, ABS5, ABS15, ABS30 against proxies

Symbol	MSE-ABS1	MSE-ABS5	MSE-ABS15	MSE-ABS30	MAD-ABS1	MAD-ABS5	MAD-ABS15	MAD-ABS30	PC1v5	PC1v15	PC1v30	PC5v15	PC5v30	PC15v30
AAPL	50.334795	51.122477	51.420277	51.536662	2.8804114	2.9849891	3.0223052	3.0371899	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
AXP	0.5106884	0.6203695	0.6628814	0.6818021	0.4650323	0.542975	0.5721495	0.5841353	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
BA	0.5718917	0.6981671	0.7491157	0.7663811	0.4551079	0.5436409	0.5758152	0.5883925	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
CAT	0.6076318	0.7979046	0.82729444	0.90691	0.6476874	0.7598199	0.800295	0.8171405	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
CS CO	0.9020016	1.0650023	1.1328246	1.159868	0.6798971	0.7659199	0.7985408	0.8118668	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
CXV	1.8098535	2.1594905	2.2896072	2.3448614	1.0594624	1.1805208	1.2228426	1.2406603	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
DD	0.7572217	0.9249887	0.9862339	1.0151541	0.5781076	0.6749675	0.7092867	0.7236298	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
DJS	1.329492	1.5241634	1.6034423	1.6359802	0.7443993	0.8319808	0.8636457	0.8766355	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
GE	4.2256727	4.4164871	4.4925753	4.5242229	0.6430538	0.727462	0.7579703	0.7705216	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
GS	0.2646999	0.3593962	0.3992373	0.416027	0.3342661	0.4320394	0.4675321	0.481793	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HD	2.9380111	3.0900706	3.1477874	3.1711761	0.5861429	0.6720074	0.7027356	0.7150114	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
IBM	0.2249023	0.3076611	0.3417879	0.3552702	0.362735	0.4425848	0.471709	0.4824304	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
INTC	1.7067711	1.960283	2.0609364	2.1005657	0.9582435	1.056669	1.0925514	1.1067761	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
JNJ	1.0722059	1.1867337	1.2350299	1.2541313	0.5523842	0.6217923	0.6467528	0.6564173	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
JPM	1.9012308	2.132419	2.2151493	2.2472283	0.7286697	0.8240719	0.8574284	0.8709679	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
KO	0.2848966	0.3513617	0.3763191	0.3869083	0.3076796	0.3736689	0.3974718	0.4065854	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
MCD	11.741897	11.866885	11.918687	11.941915	0.6701839	0.7418513	0.7686126	0.7792241	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
MMM	0.488307	0.5440561	0.5661925	0.5759352	0.2881978	0.3560343	0.3795586	0.3891901	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
MRK	0.8117388	0.9429238	0.9987344	1.0242057	0.542913	0.6316281	0.6634677	0.6770872	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
MSFT	5.4506886	5.9335964	6.1155494	6.191858	1.6603847	1.7647833	1.8018713	1.81715	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
NKE	5.6090419	5.8113891	5.8921774	5.9253985	0.7597417	0.8564985	0.8921372	0.9064115	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
PE E	1.596443	1.7580672	1.8301899	1.8614506	0.6569842	0.7491689	0.7821692	0.7963476	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
PG	0.3971366	0.4889358	0.5227405	0.5373811	0.4521007	0.5212545	0.5454623	0.5553145	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
TRV	0.0579024	0.0929781	0.1072796	0.1138366	0.1799537	0.244448	0.2676599	0.2773632	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
UNH	0.6842853	0.8150988	0.8672552	0.8897836	0.4759233	0.5783303	0.6149498	0.6302826	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
UTX	0.3160432	0.4053398	0.4401995	0.455644	0.4227986	0.5019317	0.5300507	0.5414132	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
V	0.6535176	0.8013697	0.8631901	0.8884337	0.5609771	0.6503364	0.6833876	0.6964834	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
VZ	0.4757433	0.5507612	0.5805658	0.5927522	0.3501187	0.4198749	0.4449263	0.4552099	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
WMT	1.1853403	1.324464	1.3797441	1.403332	0.5727867	0.6496847	0.6770558	0.6884438	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
XOM	16.581788	16.893613	17.007495	17.057788	1.2422348	1.3394288	1.3734385	1.3879178	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000