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A REVISION ON COST ELEMENTS OF THE EOQ MODEL

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Abstract:

The overall objective of this paper is to investigate the fundamental cost elements of the traditional EOQ model and develop the model by expiring some of its unrealistic assumptions. Over the last few decades, there have been numerous studies developing the EOQ model, but the basic cost elements of the EOQ model have not been investigated efficiently. On the other hand, the capital cost of buying inventories seems to be important to be investigated separately as well as holding cost and ordering cost in the model. In this paper, the capital cost of the inventory and possible stepwise increases in holding and setup cost are taken into account to make a revised formula to compute the economic order quantity. The proposed model involves explicitly the capital cost of buying the inventories in the EOQ model to ensure the decision makers that their financial concerns are considered in the revised model and the new order quantity results the minimum total cost.

Key words: *EOQ; holding cost; setup cost; capital cost*

1. Introduction

The Economic order quantity (EOQ) model addresses the problem of how much to order (Piasecki, 2001; Asadabadi, 2015) where the inventory carrying and ordering cost are the only determinant factors (Piasecki, 2001). In the last few decades, the EOQ model is developed and numerous complicated models have been proposed, however so many companies are not interested in making decisions based on complicated models (Billington, 2003). Unlike most recent papers, the objective of this paper is to take the basic cost elements of the traditional EOQ model into consideration and propose a simple, but developed inventory model by removing some unrealistic assumptions.

There are some drawbacks in the previous EOQ models. First, the holding cost is assumed to be a linear function. In real world problems, there is a capacity for each warehouse facility (Singh & Singh, 2013) and if the amounts of the orders pass that specific level, a new warehouse is needed. This creates a sudden increase in the linear function of holding cost. Second, since inventories in average are more than 15 percent of the assets of organizations (Krupp, 1983) the inherent capital cost of holding them might be significant and must be considered in the EOQ general model. There is obviously a cost in acquiring the capital to buy the inventories. This cost may be considered either as the interest of the latest loans received by a company or the opportunity cost of not investing the money in somewhere else (Muhlemann & Valtis, 1980).

This paper takes both costs discussed by Muhlemann and Valtis (1980) into account, but unlike most of the previous studies that consider those as a part of holding cost, regards those costs separately in the total cost formula to make sure that they are not ignored. This consideration results in a smaller order quantity. In comparison with the previous models, this paper relaxes an unrealistic assumption of the EOQ model and develops the cost elements of the classical model by detaching the capital cost as a determinant factor of the model. The remainder of this paper is organized as follows. After this, literature review is submitted. Following that, the developed EOQ model is presented and then its applicability is examined in an example. Then the findings of this study are discussed and a conclusion ends this paper.

2. Literature Review

The EOQ model involves some unrealistic assumptions, which apparently are assumed to simplify the model (Alfares, 2014). The EOQ model consists of the ordering cost and holding cost. The ordering cost includes some cost elements such as: labor, phone calls, faxes, postage, envelopes, etc, which are to release an order (Piasecki, 2001). On the other hand, the holding cost includes the cost of the storage, insurance, spoilage, tax, etc, which are to carry the inventories (Fazel, 1997). The capital is widely ignored in the model, but some studies suggest it to be considered a part of holding cost (Berling, 2008), however it is an important managerial concern to stand alone in the formula.

A study by Piasecki (2001) deals with the concept of the capital cost in inventory management and investigates its origin. The capital cost happens as a result of the interest rate, which is paid on borrowed money to buy the inventory and is defined as a part of carrying cost. Even if the organization is debt-free, the income that the company can make with investing the money used to buy the inventory should be considered. The interest rate that can be received by the company makes decision makers consider the opportunity cost, as well as the capital cost (Asadabadi, 2015). It is computed by multiplying the interest rate by the cost of the materials (Teunter & Inderfurth, 2000).

A study by Strickland (1965) discusses the possible effects of the interest rate on the inventory level and explains how a complicated formulization might have an adverse impact on the interest of management in its applications. It states “the cost of capital assumption is essential to be made in inventory size problems" Strickland (1965). Krupp (1983) investigates the inherent capital cost of the inventory and attempts to optimize replenishment order quantities. The study by Krupp (1988) considers a model more like the economic manufacturing quantity model in which the quantities receive over time in a deterministic model with considered backorders and penalties. Billington (2003) investigates the traditional EOQ model with a reducing rate of holding cost and finds a higher optimal order quantity in comparison with the traditional quantity; "*Results show that the total cost can be reduced under specific situations. This new model is combined with previous research on setup cost reduction to show that further total cost reduction is possible*" (Billington, 2003).

Berling (2008) studies the capital cost of the inventory where the purchasing price is presented stochastically and the setup cost is assumed fixed: "*Most models of inventory control assume that the per unit purchase price is constant. The capital cost of holding inventory can then be taken into account by adding a fixed interest rate, r, times the purchase price*" (Berling, 2008). Hou and Lin (2011) focus on the setup cost and attempt to find the optimal lot sizing policies. They notice a sensible reduction in order quantity where setup cost is considered a function of capital cost with limited budget. Porteus (1985) attempts to create a framework for inventory optimal amount and focuses on reducing setups. Porteus (1985) takes the investment cost into consideration to solve the problem of the setup cost and the sales rate.

There have been several studies dealing with multi-warehousing problems. Saha et al. (2012) consider two warehouse facilities to develop the EOQ model. One of the warehouse facilities is owned by the organization and the other one is rented, then considering some other assumptions, the EOQ model is formulated. Lin and Chung (2012) also consider a case with two warehouses and question the assumptions of the EOQ model: "*Although the traditional EOQ models are still widely used in industry, practitioners frequently question validities of assumptions of these models such that their use encounters challenges and difficulties*" (Lin & Chung, 2012)

Although the study by Krupp (1983) deals with the importance of financial concerns in inventory management: "*Financial managers devote a great deal of time to working-capital management. Effective inventory management must be a critical aspect of managing working capital since inventories generally make up 16%-30% of a firm's assets*" Krupp, 1983), unfortunately, it fails to propose a sufficient and simple EOQ model, which includes holding, setup, and capital costs. Taking the capital cost of the inventory into consideration has improved numerous research insights (Billington, 2003), but further investigations are required to develop the capital cost to be regarded along with the holding and ordering cost in the EOQ formula. In the EOQ model, the capital cost is considered as a part of the holding cost (Muhlemann & Valtis, 1980). This paper, along with the study by Blocher (1992) which criticizes the idea of considering the capital cost as a part of the holding cost, detaches the capital cost from the holding cost and then considers stepwise increases for holding cost. "*It is important to realize that the investment opportunity cost of inventory should not be included as a component of holding cost*" Blocher (1992)

3. Developing EOQ Model

Nomenclature:

TC:	**Total Cost**
D:	annual demand
Q:	number of units per order
Q^*:	optimal order quantity
K	ordering cost for each order
r:	interest rate
P_{pu}:	purchase price per unit in EOQ
H_{TC}	annual holding cost
C_{TC}	annual capital cost
H_{iF}	the fixed cost of using i[th] warehouse
H_V	annual holding cost of one item

In this paper, two parts for holding cost are considered. A part of holding cost is variable, which is as a coefficient for the average of the inventory shown by H_V (like the traditional EOQ model). The other part of the holding cost is the sudden increases in the carrying cost such as the cost of renting, buying, or preparing a new space to keep the extra units of the inventory, which is shown by H_{iF}. In this paper, it is where the order quantity requires using i[th] warehouse. The new structure of holding cost can easily change to the traditional form by setting all α_is equal to zero in the equation presented below.

$$H_{TC} = H_V \frac{Q}{2} + \alpha_1 H_{1F} + \alpha_2 H_{2F} + \alpha_3 H_{3F} \ldots \quad (1)$$

$\alpha_i = 1$ if the i[th] warehouse is used, otherwise $\alpha_i = 0$.

Additionally to consider the imposed cost of order quantity on the company, the annual capital cost is considered in the new model. The bigger the order quantity, the more imposed capital cost to the system. From financial viewpoint, it is always more desirable to have smaller order quantities, which decreases the debts of an organization, and where it is a debt-free organization, it increases the organization's strength in taking advantage of investment opportunities (e.g. a bank deposit with interest) by releasing some capital. The average interest rate is shown by r.

$$C_{TC} = P_{pu} r \frac{Q}{2} \quad (2)$$

The general form of the ordering cost ($\frac{D}{Q}K$) is used and the total cost formula results as below:

$$TC = \left(H_V\frac{Q}{2} + \sum_t \alpha_t H_{tr}\right) + \frac{D}{Q}K + P_{pu}r\frac{Q}{2} + P_{pu}D \qquad (3$$

Now the order quantity that minimizes the TC should be determined. Because there are several holding costs, there are few steps to follow.

1. The value for optimal order size Q^* is calculated. The general form of optimal order size is computed as below.

$$\frac{\partial TC}{\partial Q} = \frac{H_V}{2} - \frac{D}{Q^2}(K) + \frac{P_{pu}r}{2} = 0 \qquad (4)$$

$$Q^* = \sqrt{\frac{2DK}{H_V + P_{pu}r}} \qquad (5)$$

2. The total cost for that quantity is calculated (in case of a need for renting some warehouses, obviously, the cheapest option is considered).

Note: By placing this optimal order quantity in the total cost formula, the total cost is as below:

$$TC = P_{pu}D + \sum_t \alpha_t H_{tr} + \sqrt{2KD(H + rP_{pu})} \qquad (6)$$

3. Starting from no extra warehouses, the total cost for all the orders up to the order quantity (resulted from step 1) which utilize the full capacity of warehouses are calculated (all the options resulting a quantity below the order quantity is checked, but those which go beyond that quantity are not needed to be tested).

4. The total costs resulted from step 2 and 3 are compared and the order quantity which results the lowest one is selected.

4. An Illustrative Example

Zomorrod Sofal Co. is a brick company producing light weight bricks in Iran. The new EOQ model is tested on the raw material used to make the bricks. The company is using a special soil delivered by trucks.

The details are presented below:

The annual demand of the soil is 2255 trucks and the purchasing price is 469.5$ per truck. The annual carrying cost is approximately 54$ per year for each. The ordering cost is 27$ and the annual interest rate is 3.9 percent. The company has its own warehouse with capacity of 18 trucks. There are five warehouses available to rent.

Table 1: Extra warehouse capacity and rent

	Warehouse A	Warehouse B	Warehouse C	Warehouse D	Warehouse E
Capacity	8	9	12	27	45
The annual Imposed Rent and relevant costs	200	210	250	460	550

The first step is followed (since the aim is to find the optimal order quantity, to reduce redundant calculations, $P_{pu}D$ is removed from the total costs for making the comparisons).

$$Q^* = \sqrt{\frac{2DK}{H_V + P_{pu}r}}$$

$$= \sqrt{\frac{2(2255)(27)}{54 + (469.5)(0.039)}}$$

$$= 41$$

The best warehouse to rent for the extra 23 trucks is warehouse 4, so the total cost for this quantity is (step 2):

$$TC = \sum_t \alpha_t H_{tr} + \sqrt{2DK(H + P_{pu}r)}$$

$$TC = 460 + \sqrt{2(2255)(27)\left(54 + (469.5)(0.039)\right)}$$

TC=3427.4$

Now, step 3 is performed.

No extra warehouses (Q=18):

$$TC = H_V\frac{Q}{2} + \sum_t \alpha_t H_{tr} + \frac{D}{Q}K + P_{pu}r\frac{Q}{2}$$

$$TC = (54)\frac{18}{2} + \frac{2255}{18}(27) + (469.5)(0.039)\frac{18}{2}$$

TC= 4033.3$

Full capacity of warehouse A (Q=26):

$$TC = (54)\frac{26}{2} + 200 + \frac{2255}{18}(27) + (469.5)(0.039)\frac{26}{2}$$

TC= 3481.8$

Full capacity of warehouse B (Q=27):

TC= 3441.2$

Full capacity of warehouse C (Q=30):

TC= 3364.2$

Full capacity of warehouse A and B (Q=35):

TC= 3415$

Full capacity of warehouse A and C (Q=38):

TC= 3426.1$

Full capacity of warehouse B and C (Q=39):

TC= 3431.2$

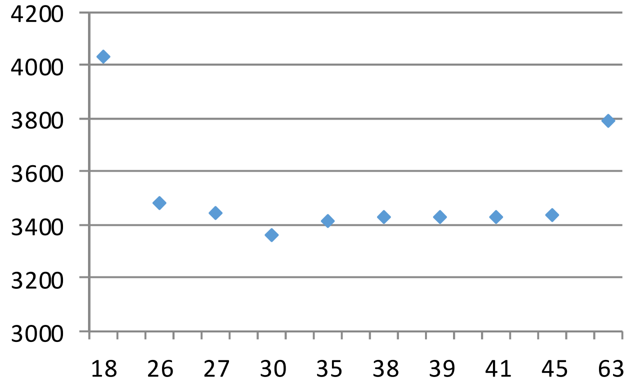

Figure 1: Total Cost/Order Quantity

The minimum of the total costs is 3364.2$ which happens when the order quantity is 30. So, in this case, the optimal order quantity is 30 trucks.

5. Discussion

The traditional EOQ model is not capable of considering sudden stepwise increases and doesn't take the capital cost into account. This paper develops the model and applies it to find the optimal order quantity for a brick company. The case which was studied in the previous section exposed the incapability of the traditional model in dealing with cases with stepwise increases in holding cost and a determinant capital cost to be seen explicitly in the formula. This makes the model much easier and applicable for those who are implementing the model in companies with one kind of inventory as well as those which keep their inventories in separate warehouses. Figure 1 represents the total costs for the critical quantities calculated in the example. As the figure represents, the minimum of the total cost happens where the order quantity is 30 trucks. The amount of economic order quantity is significantly influenced by the interest rate (Asadabadi, 2015). If the interest rate increase, the amount of order quantity (step

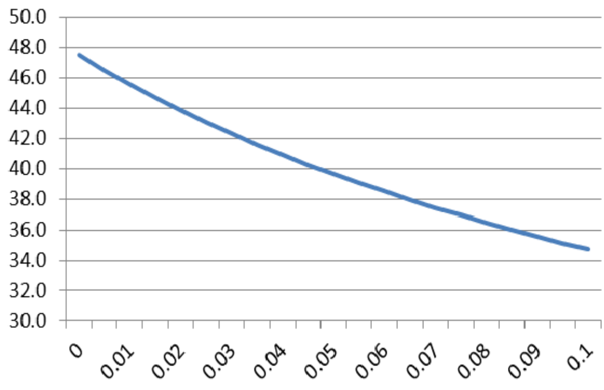

Figure 2: Order Quantity/ Interest Rate

one of the methodology) decreases. This trend is shown in figure 2. In this figure, the interest rate increases from zero to 10 percent and with regard to that the order quantity decreases from almost 48 to about 34.

The EOQ model is the most well-known inventory model widely used in industries, but some restrictive assumptions have reduced its affectivity (Alfares, 2014). This paper deals with a fundamental cost of the traditional EOQ model, holding cost, and attempts to make the model applicable where sudden increases in holding cost happen.

Besides from those cases where the goods are perishable (Chung & Li, 2014; Olsson, 2014; Azzi et al., 2014), there are few papers investigating the holding cost where it doesn't follow a linear function (Goh, 1994; Weiss, 1982). Weiss (1982) uses the classical EOQ model and makes the holding cost non-linear while all the other parameters are kept in their traditional form, and finally the optimal order quantity is presented for both deterministic and stochastic demands. In the proposed model by Goh (1994), the holding cost is allowed to differ for each of the units and then the optimal order quantity is found by considering two non-linear functions for the length of holding time of the items and the amount of on hand inventory. Investigating the holding cost has been continued until recent years. Recently, Pando et al. (2013) deal with an EOQ where the cumulative is non-linear depending on their time of storage and the quantity. Then, a procedure is developed by Pando et al. (2013) to find the optimal lot size. Despite the numerous studies investigating the holding cost, further studies are required. As a fundamental restriction, the EOQ model doesn't include the limitations in capacities of warehouses. In comparison with the studies dealing with the holding cost, in this model, the holding cost still follows a linear function, but it confronts sudden increases as the amount of order passes predetermined quantities.

Strickland (1965) analyzes the impact of the capital cost on the inventory size and highlights the obligation of decision makers to consider that cost in determining the inventory size. A study by (Muhlemann & Valtis, 1980) investigates the capital cost, but it considers it as a part of holding cost with some modifications and a revised version of the total cost is suggested at the end. Piasecki (2001) exposes the capital costs in the shape of interest rate: "*If you had to borrow money to pay for inventory, the interest rate would be part of the carrying cost. If you did not borrow for the inventory, but have loans on other capital items, the interest rate on those loans can be used since a reduction in inventory would free up money that could be used to pay the loans. If by some miracle you are debt-free, you need to determine how much you could make if the money were invested*". But, in the proposed model by Piasecki (2001), it is considered as a part of holding cost, but the elements of holding cost in EOQ model may vary over time, the capital cost of holding inventories must not be considered as a part of holding cost (Berling, 2008). Although there are some studies discussing the capital cost as a part of holding cost, but as Blocher (1992) and Berling (2008) criticize, the idea of including capital cost in holding cost is basically inappropriate. The contribution of this paper to the literature is twofold. First, it considers a stepwise holding cost to make the model applicable where the holding cost is possible to experience sudden increases for example where a new place should be rent. Second, it considers the capital cost not as a part of the holding cost, but as a separate cost. By this segregation, the effects of the interest rate and consequently the capital cost on order quantity become more observable and considerable.

There are two areas of research worthy for further studies. There are countries with inflation rate as a significant factor in managerial decisions. The inflation rate can offset the impact of the interest rate. Further studies can investigate the effects of inflation on the revised model. Furthermore, the interest rate is assumed to be constant, however, in some countries the interest rate varies continuously. Furthermore, there might be some correlations between the demand and the interest rate in the revised EOQ model worthy of further studies.

6. Conclusion

As discussed, in comparison with previous studies, the contribution of this paper is as follows. This paper aims to adjust a fundamental cost of the EOQ model, holding cost, and takes the unseen cost of capital as a determining factor of order quantity into account. It develops the classic EOQ model by investigating the model from two viewpoints. First, inventory holding cost; part of holding cost is reasonably variable and related directly to the quantity of holding items, but part of holding cost is imposed to the system as sudden increases. Therefore, there are some specific amounts (Q_i) that where the order quantity increases even for one unit (Q_i+1), the holding cost receives a sudden increase, which are considered in the revised model. Second, the capital cost; the EOQ formula doesn't explicitly consider the financial situation of the company. Even if the company is debt-free, the interest that the company could make if the money was invested should be involved in determining the order quantity. This paper considers these new critics and develops the model. Further studies can involve the inflation rate and its impact on the model.

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