INTRODUCTION

Milk is an emulsion or colloid of butterfat globules within a water-based fluid that contains dissolved carbohydrates and protein aggregates with minerals (Rolf, 2002). Because it is produced as a food source for a neonate, all of its components are beneficial to the growing young. Milk is a white liquid produced by the mammary glands of mammals. It is the primary source of nutrition for young mammals before they are able to digest other types of food. Early-lactation milk contains colostrum, which carries the mother’s antibodies to the baby and can reduce the risk of many diseases in the baby (Pehrsson et al., 2000; Wright et al., 2013). It also contains many other nutrients. The principal requirements of the neonate are energy (lipids, lactose, and protein), biosynthesis of non-essential amino acids supplied by proteins (essential amino acids and amino groups), essential fatty acids, vitamins and inorganic elements, and water (Fox, 1995; Boutelle et al., 2013). Milk is often transported by tube, in cans, in tanks etc. Therefore it is important to understand its physical behaviour, especially flow behaviour, covering among others the density, dynamic viscosity, and kinematic viscosity. The knowledge of flow behaviour may help optimize milk transport and also reduce technical and economic losses (Novakovic et al., 2000; Xiang et al., 2011; Brodziak, 2012; Kumber, Dostal, 2014). Ultra-high temperature processing, less often ultra-heat treatment (both abbreviated UHT), or ultra-pasteurization is the sterilization of food by heating it for an extremely short period (around 1–2 s) to a temperature exceeding 135 °C, which is the

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temperature required for killing spores in milk. The most common UHT-treated product is milk, but the process is used also in fruit juices, cream, soy milk, yogurt, wine, soups, honey, and stews. UHT milk was invented in the 1960s, and became generally available for consumption in the 1970s. The high heat during the UHT process can cause Maillard browning and change the taste and smell of dairy products. UHT milk has a typical shelf life of 6–9 months, until opened. It can be contrasted with HTST pasteurization (high temperature/short time), during which milk is heated to 72 °C for at least 15 s (Clare et al., 2005; Aguir et al., 2012).

The aim of this study was to describe viscosity, density, and analytical differences of four milk samples from Czech cows – three samples of UHT milk (0.5%, 1.5%, and 3.5% fat) and one sample of raw milk from a Czech bio-farm. Mathematical models using Power law and Gaussian fitting were constructed.

MATERIAL AND METHODS

Samples of four different cow milks were analyzed. Three milks were prepared using the UHT technology. The UHT milks had different commercial signature – ‘skimmed UHT milk’ with 0.5% fat content, ‘semi-fat UHT milk’ with 1.5% fat content, and ‘full-fat UHT milk’ with 3.5% fat content. The fourth milk sample was raw milk (untreated) purchased at a bio-farm. All the cow milks were obtained and processed in the Czech Republic.

Analytical properties

The individual analytical properties were determined according to the following procedures and relevant standards:

• Titratable acidity was determined using Soxhlet-Henkel’s method (ČSN EN 570105)
• Milk fat content was assessed by Gerber’s method (i.e. the proportion of fat which is separated in butyrometers after dilution of phosphorus-lipid envelope of fat bubbles by treatment with sulfuric acid)
• Dry matter content was determined according to ČSN EN 1 ISO 6731 standard (milk samples were drying at the temperature of 102 ± 2 °C to constant mass)
• Nitrogen content and the following conversion to protein content was determined using Kjeldahl method (ČSN EN ISO 8968-1)

Density

Cow milk density was measured using a digital densitometer Densito 30PX (Mettler Toledo, Schwerzenbach, Switzerland). Firstly, all cow milk samples were cooled down to the temperature of 0 °C and after that they were slowly heated to 60 °C (Muramatsu, 1996; Dinkov et al., 2008). Milk density was measured at 10 °C intervals.

Dynamic viscosity

Dynamic viscosity of the cow milk samples was measured using a rotary viscometer DV-3P (Anton Paar, Graz, Austria) with a temperature sensor Pt100 (Fig. 1). A standard spindle LCP, the best for measuring low-viscosity fluids (water, milk, petrol, etc.), was used. The samples were first cooled to the temperature of 0 °C and after that slowly heated to 100 °C (Alcántara, 2012; Atasav et al., 2012).

Kinematic viscosity is more illustrative for this kind of fluid. It is a ratio of dynamic viscosity and density, as shown in Eq. (1):

\[ \nu = \frac{\eta}{\rho} \]  

where:
\( \eta \) = dynamic viscosity (Pa.s)
\( \rho \) = density (kg.m\(^{-3}\))

RESULTS

Analytical properties

Analytical properties of the four studied cow milks are shown in Table 1 and Fig. 2. The titratable acidity (TA) and dry matter content (DMC) in cow milk decreased with the increasing fat content (FC). The measured values of fat content approximately corresponded with data given by the milk manufacturers, in raw milk the fat content was 3.60%, depending on
Density of the cow milks investigated was measured at temperatures ranging 0–60 °C. The results are shown in Fig. 2.

Temperature dependence of density

The dependence of density on temperature was measured. The temperature range was set at about 0–100 °C. The 0.5% UHT milk showed the highest values of dynamic viscosity. Dynamic viscosity was decreasing with the increasing milk fat content. The temperature dependence of dynamic viscosity was manifested in all the cow milk samples. At about 100 °C the dynamic viscosity of milk was rising. This could be due to precipitation of proteins in the milk samples. Results of the studied temperature dependence of dynamic viscosity in the individual milk samples are shown in Fig. 3.

Temperature dependence of dynamic viscosity

Based on experimental values of density and dynamic viscosity of the four cow milk samples, kinematic viscosity can be calculated using Eq. (1). Results of

![Fig. 2. Temperature dependence density of cow milk](image)

Table 1. Analytical properties of cow milk

<table>
<thead>
<tr>
<th>Cow milk</th>
<th>TA (SH)</th>
<th>FC (%)</th>
<th>DMC (%)</th>
<th>PC (g 100 g⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5% UHT</td>
<td>7.43</td>
<td>0.38</td>
<td>10.07</td>
<td>3.57</td>
</tr>
<tr>
<td>1.5% UHT</td>
<td>7.24</td>
<td>1.58</td>
<td>11.09</td>
<td>3.51</td>
</tr>
<tr>
<td>3.5% UHT</td>
<td>6.74</td>
<td>3.69</td>
<td>12.89</td>
<td>3.53</td>
</tr>
<tr>
<td>Raw milk</td>
<td>6.54</td>
<td>3.60</td>
<td>12.07</td>
<td>3.54</td>
</tr>
</tbody>
</table>

TA = titratable acidity, FC = fat content, DMC = dry matter content, PC = protein content; *P* < 0.05

Table 2. Parameters of fitting temperature dependence of density for Eq. (2)

<table>
<thead>
<tr>
<th>Cow milk</th>
<th><em>a</em>₀</th>
<th><em>a</em>₁</th>
<th><em>a</em>₂</th>
<th>SSE</th>
<th>RMSE</th>
<th><em>R</em>²</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5% UHT</td>
<td>1031</td>
<td>-2.977E-5</td>
<td>3.183</td>
<td>17.34</td>
<td>2.0820</td>
<td>0.8997</td>
</tr>
<tr>
<td>1.5% UHT</td>
<td>1032</td>
<td>-0.00237</td>
<td>2.129</td>
<td>17.05</td>
<td>2.0640</td>
<td>0.9120</td>
</tr>
<tr>
<td>3.5% UHT</td>
<td>1029</td>
<td>-0.01804</td>
<td>1.666</td>
<td>32.46</td>
<td>2.8490</td>
<td>0.8756</td>
</tr>
<tr>
<td>Raw milk</td>
<td>1025</td>
<td>-0.01165</td>
<td>1.819</td>
<td>0.859</td>
<td>0.4634</td>
<td>0.9974</td>
</tr>
</tbody>
</table>

SSE = sum of squared error, RMSE = root-mean-square error, *R*² = coefficient of determination
The temperature dependence of kinematic viscosity of the milk samples are shown in Fig. 4.

Temperature dependence of kinematic viscosity of the milks was calculated using Power law model – see Eq. (2). Parameter \( a_0 \) represents kinematic viscosity at \( 0^\circ C \). All parameters and statistical values are given in Table 4.

### DISCUSSION

The titratable acidity and dry matter content in the cow milk were decreasing with increasing content of milk fat, which was expected and in accordance with other studies (Chen et al., 2004; Montanholi et al., 2013). The measured milk fat content approximately
corresponded to data reported by the milk manufacturers, the fat content found in the raw milk sample was 3.60%. This depends on the cow breed (for details see e.g. Miciński et al., 2012; Poulsen et al., 2012; Sobotka et al., 2014). The protein content of the cow milks studied ranged 3.51–3.57 g per 100 g cow milk, depending on nutrition and lactation level of cows (Crovetto et al., 2009; Tena-Martinez et al., 2009).

It was proved that with increasing temperature, the density of the cow milks was decreasing. These decreases were non-linear (Tagawa et al., 1997). The 0.5% UHT milk exhibited the highest values of dynamic viscosity. Dynamic viscosity of milk was decreasing with the increasing milk fat content (Oguntunde, Akintoye, 1991). The temperature dependence of dynamic viscosity was demonstrated in all cow milk samples (Božíková, Hlaváč, 2013). At the temperature around 100 °C the dynamic viscosity of milk increased, which could be due to precipitation of proteins in the samples, similarly as it is e.g. in the milk of camel (Aludatt et al., 2010) and goat (Fonseca et al., 2011). In Fig. 4 kinematic viscosity was used because it is more illustrative for this kind of fluid. Kinematic viscosity works in line with density and dynamic viscosity (Kumar, Dostał, 2014).

Finally, mathematical models using Power law and Gaussian fitting were constructed in accordance with Rohm et al., 1996. Coefficient of determination $R^2$ achieved very high values (0.843–0.997).

**CONCLUSION**

To put it briefly, the titratable acidity and dry matter content in the studied cow milk samples were decreasing with increasing milk fat content. The protein content in cow milk, which depends on nutrition and lactation of cows, was in the interval 3.51–3.57 g per 100 g milk. It was proved that with increasing temperature, the density and dynamic viscosity of cow milks lowered. All cow milk samples demonstrated the temperature dependence of dynamic (and/or kinematic) viscosity. To describe the milk flow behaviour better, kinematic viscosity was used because it is more illustrative for this kind of fluid. Kinematic viscosity works in line with density and dynamic viscosity. Mathematical models using Power law and Gaussian fitting were constructed. These very accurate models can be used for the prediction of flow behaviour of cow milk and maybe of other milks, e.g. of goat, sheep, camel or horse.

**REFERENCES**


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Table 4. Parameters of fitting temperature dependence of kinematic viscosity for Eq. (2)

<table>
<thead>
<tr>
<th>Cow milk</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>SSE</th>
<th>RMSE</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5% UHT</td>
<td>3.267</td>
<td>-0.5115</td>
<td>0.36680</td>
<td>0.661501</td>
<td>0.40670</td>
<td>0.8432</td>
</tr>
<tr>
<td>1.5% UHT</td>
<td>4.408</td>
<td>-2.5850</td>
<td>0.07225</td>
<td>0.007235</td>
<td>0.04253</td>
<td>0.9973</td>
</tr>
<tr>
<td>3.5% UHT</td>
<td>2.211</td>
<td>-0.3232</td>
<td>0.30620</td>
<td>0.035270</td>
<td>0.09390</td>
<td>0.9596</td>
</tr>
<tr>
<td>Raw milk</td>
<td>2.184</td>
<td>-0.1396</td>
<td>0.51201</td>
<td>0.005577</td>
<td>0.03734</td>
<td>0.9939</td>
</tr>
</tbody>
</table>

SSE = sum of squared error, RMSE = root-mean-square error, $R^2$ = coefficient of determination

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