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Evaluation of Predictive Capabilities of Regression Models and Artificial Neural Networks for Density and Viscosity Measurements of Different Biodiesel-Diesel-Vegetable Oil Ternary Blends

Mert GULUM[1]*, Funda KUTLU ONAY[2], Atilla BILGIN[3]

[1,3]*Mechanical Engineering Department, Karadeniz Technical University, Trabzon, 61080, Turkey*
[2]*Computer Engineering Department, Amasya University, Amasya, 05000, Turkey*

***Abstract* – Nowadays, biodiesel and vegetable oils have received increasing attention as renewable clean alternative fuels to fossil diesel fuel because of decreasing petroleum reserves and increasing environmental concerns. However, the straight use of biodiesel and vegetable oils in pure form results in several operational and durability problems in diesel engines because of their higher viscosity than fossil diesel fuel. One of the most used methods for solving the high viscosity problem is to blend them with fossil diesel fuel or alcohol. The reliable viscosity and density data of various biodiesel-diesel-alcohol ternary blends or biodiesel-diesel binary blends are plentifully available in existing literature, however, there is still the scarcity of dependable measurement values on different biodiesel-diesel-vegetable oil ternary blends at various temperatures. Therefore, in this study, waste cooking oil biodiesel (ethyl ester) was produced, and it was blended with fossil diesel fuel and waste cooking oil at different volume ratios to prepare ternary blends. Viscosities and densities of the ternary blends were determined at different temperatures according to DIN 53015 and ISO 4787 standards, respectively. The variation in viscosity with respect to temperature and oil fraction and the change of density vs. temperature were evaluated, rational and exponential models were proposed for these variations, and these models were tested against the density and viscosity data measured by the authors, Nogueira et al. and Baroutian et al. by comparing them to Gupta et al. model, linear model, Cragoe model and ANN (artificial neural networks) previously recommended in existing literature.**

***Keywords* – Artificial neural networks; biodiesel; density; ethyl ester; exponential model; prediction; rational model; renewable energy; viscosity.**

1. INTRODUCTION

Recently, renewable fuels have become more necessary throughout the world due to the depletion of world petroleum reserves, increases in crude oil prices and environmental concerns [1], [2]. Among renewable fuels, biodiesel and vegetable oils are at the forefront of the substitutes to petroleum-based diesel fuel in the transportation sector [3].

Vegetable oils can be used directly in direct injection or in-direct injection diesel engines for the short-term operation without major modifications [4], [5]. In fact, the idea of using vegetable oils as fuel is not new [6] since Rudolf Diesel first used peanut oil in his engine at the World Exhibition in Paris in 1900 [7]. Since then, significant efforts have been made by lots of scientists

* Corresponding author.
E-mail address: gulum@ktu.edu.tr

to use different vegetable oils as fuel in diesel engines especially for emergency situations [8], [9]. However, the use of vegetable oils can result in operational and durability problems (such as clogged filters, deposits in the combustion chamber and injector tip, injector cocking, injector pump failure, etc. [6], [10]) for long-term operation because of their higher viscosity, cloud point and pour point temperatures, and relatively more reactive to oxygen [5], [6]. These problems can be eliminated by chemically converting vegetable oils to biodiesel [10].

Biodiesel, consisting of a mixture of methyl or ethyl esters of long-chain fatty acids [11], has many advantages over fossil diesel fuel [12] such as: (i) it is non-toxic, non-aromatic and biodegradable fuel [13], (ii) it can be directly used in diesel engines without major modification [14], (iii) it composes of 10 % to 12 % oxygen by mass in the molecular structure, thus significantly reducing HC, CO and PM emissions [15], and finally (iv) it has higher density (contributing to deliver greater mass of fuel to the engine [16]), flash point (making safer handling and storage [17]) and cetane number (giving higher resistance to diesel knock by shortening ignition delay [15] and allowing it to be utilized in higher compression ratio [18]). However, the important disadvantages of biodiesel are (i) reactivity of unsaturated hydrocarbon chains in the molecular structure, (ii) cold start problems, (iii) lower energy content and volatility, and (iv) higher viscosity, copper strip corrosion, price and generally nitrogen oxide emission [19]–[21].

To overcome the high viscosity problem of vegetable oils and biodiesel leading to poor atomization and incomplete combustion, many researchers have carried out blending them with fossil diesel fuel or alcohol. In the existing literature, many studies have been performed on measurements of important fuel properties of binary blends including vegetable oil or biodiesel with diesel fuel. However, there are still no comprehensive studies about (1) determination of density and viscosity of biodiesel-diesel-vegetable oil ternary blends at different temperatures or about (2) development of mathematical correlations between density or viscosity with temperature and oil fraction in blend to predict them. Therefore, in this study, (1) the lowest viscosity waste cooking oil ethyl ester (biodiesel) was synthesized via basic transesterification according to the specific reaction parameters given in [22], and it was blended with fossil diesel fuel at 20 % volume ratio. The biodiesel-diesel binary blend was referred as base fuel (BF). (2) Waste cooking oil was added to the binary blend (BF) on the volume ratios of 2 %, 4 %, 6 %, 8 %, 10 %, 15 % and 20 %. The resulting ternary blends were named to reflect their composition. For example, the name WCO2 indicates a blend consisting of 2 % waste cooking oil-98 % base fuel. Similar naming (WCO4, WCO6, WCO8, WCO10, WCO15 and WCO20) and fractions are also valid for the other ternary blends. (3) Density and viscosity of the resulting ternary blends were measured at different temperatures (278.15–368.15 K) according to the related international standards. Finally (4), the exponential and rational models, previously suggested by the authors [15], [23], [24], were used to predict densities and viscosities of biodiesel-diesel-vegetable oil ternary blends measured by the authors, Nogueira et al. [25] and Baroutian et al. [26], respectively, by comparing to Gupta et al. model, linear model, Cragoe model and ANN (artificial neural networks) previously suggested by the other authors in the literature.

2. EXPERIMENTAL STUDY

2.1. Biodiesel Production

In this study, waste cooking oil biodiesel was produced by means of basic transesterification using ethanol, sodium hydroxide and anhydrous sodium sulphate which were of analytical grades. After the detailed parametric investigation conducted by the authors in [22], transesterification

reaction parameters giving the lowest viscosity were determined as: catalyst concentration of 1.25 %, reaction temperature of 70 °C, reaction time of 120 minutes and ethyl alcohol to oil molar ratio of 12:1. More details about biodiesel production process can be also found in [15], [17], [22]–[24].

2.2. Density and Viscosity Measurements

The densities and viscosities of ternary blends (WCO2, WCO4, WCO8, WCO10, WCO15 and WCO20) were measured at different temperatures (278.15–368.15 K) according to ISO 4787 and DIN 53015 test standards from the following equations, respectively:

$$\rho_{\text{ternary blends}} = \frac{m_{\text{total}} - m_{\text{pycnometer}}}{m_{\text{water}}} \rho_{\text{water}}, \tag{1}$$

$$\mu_{\text{ternary blends}} = K_{\text{ball}} (\rho_{\text{ball}} - \rho_{\text{biodiesel}}) t, \tag{2}$$

where ρ, m, μ, K_{ball} and t are density, mass, dynamic viscosity, coefficient of the viscometer ball and falling time of the ball moving between two horizontal lines marked on the viscometer tube at limit velocity, respectively. Density measurements were conducted by means of ISO pycnometer and a top loading balance with an accuracy of ±0.01 g while viscosity measurements were conducted using universal Haake Falling Ball Viscometer, Haake Water Bath, a stopwatch (±0.01 s) and a thermometer (±0.5 °C). The density and viscosity measurements were carried out three times for each sample and the results were averaged. Moreover, as well known, kinematic viscosities of the ternary blends were computed by dividing dynamic viscosity with density at the same temperature.

Some important fuel properties (viscosity, density, flash point, cold filter plug point (CFPP), higher heating value (HHV), saponification number (SN), iodine value (IV) and cetane number (CN)), average molecular masses and typical formulas of diesel fuel (DF) and produced ethyl ester (biodiesel) are given in Table 1. Table 2 lists also the ethyl ester composition of produced biodiesel. The major and minor fatty acids are oleic and behenic, respectively.

TABLE 1. SOME IMPORTANT FUEL PROPERTIES OF DIESEL FUEL AND ETHYL ESTER, MEASUREMENT STANDARDS AND INTERNATIONAL BIODIESEL STANDARDS

Property	Unit	Measurement standards	DF	Ethyl ester	EN14214	ASTM-D6751
Viscosity at 40 °C*	mm^2/s	DIN 53015	2.700	4.387	3.50–5.00	1.90–6.00
Density at 15 °C*	kg/m^3	ISO 4787	832.62	876.50	860–900	***
Flash Point**	°C	EN ISO 3679	63	–	101≤	130≤
CFPP**	°C	EN 116	−6.0	–	< +5 (summer) −15 < (winter)	***
Average molecular mass	g/mol	–	169.883****	305.654*****	***	***
Typical formula	–	–	C12.31H21.975******	C19.68H36.99O2****	***	***

Property	Unit	Measurement standards	DF	Ethyl ester	EN14214	ASTM-D6751
HHV**	kJ/kg	DIN 51900-2	45950	–	***	***
SN	mg KOH/g	–	–	183.391	***	<500
IV	g I_2/100 g	–	–	95.930	120 max	***
CN	–	–	–	54.477	>51	≥47

* Measured in Internal Combustion Engines Lab. at Karadeniz Technical University; ** Measured in Prof. Dr. Saadettin GÜNER Fuel Research and Application Center at Karadeniz Technical University; *** Not specified; **** Calculated from typical formula; ***** Calculated from fatty acid distribution; ****** Calculated from HHV and Mendeleev's formula.

TABLE 2. FATTY ACID ETHYL ESTER COMPOSITION OF PRODUCED BIODIESEL BASED ON PERCENT MASS

Fatty acid*	BD
Palmitic (C16:0)	17.647
Oleic (C18:1)	46.491
Linoleic (C18:2)	32.581
α-Linolenic acid (C18:3)	1.505
Arachidic (C20:0)	0.785
Gadoleic acid (C20:1)	0.679
Behenic (C22:0)	0.311

*Measured at Mustafa Kemal University with gas chromatography device.

2.3. Uncertainty Analysis

Measured physical quantities are used to compute targeted results in experimental studies. Uncertainties of measuring devices naturally result in uncertainties in the computed quantities, too. Uncertainty analysis provides information to determine uncertainties in the targeted results to be aware of the reliability of them. A method of Kline and McClintock [27] was generally used in scientific studies to compute uncertainties of targeted results (such as density and viscosity in this study). The highest uncertainty of 0.8538 % was calculated for all the targeted results, making them extremely reliable. An example calculation for uncertainty analysis was found in the authors' previous study [24].

2.4. Artificial Neural Networks

Artificial neural networks (ANN) have been developed by inspiring human biological nervous systems [28]. In a similar way to the human brain, the artificial nerve cells, named neurons, process external information and generate the appropriate output [28]. This requires three basic layers such as input, hidden and output layers [28]. The information is served to the network from the input layer [28]. They are processed in the hidden layer(s) and then sent to the output layer. The processing of the information means to convert information coming from the network into output with the help of the network weight values [28]. The most widely used model is artificial neural networks (ANN). These networks are especially capable of producing solutions for various linear and nonlinear problems [29]. The problem decides how many neurons must be in the input and output layers. However, there is no definite method for determining the number of hidden layers and the number of neurons in each hidden layer. This is determined by trial-and-error or cross-validation techniques [30]. To measure the training performance of ANN,

samples of the network that do not participate in training are given as input to the network after the training is finished, and the network decision about them is checked. If the network is producing correct responses to these samples, it can be said that the network performance is good and the network has learned the event since a successful network must have the ability to generalize to new samples [31].

In this study, viscosity and density values were estimated by means of ANN. For this, the oil fraction and temperature values were given as input to ANN and the viscosity or density values determined experimentally were defined as the output layer, as shown in Fig. 1. The training set consists of 60 % of the existing data and the test set is 40 %. The test set was used to measure the performance of the network. The network training was carried out with 10 iterations, and the training and test data were extracted randomly at each iteration. The network accuracy was calculated as a result of each iteration and average network performance was obtained when all the iterations were completed. The properties of the network consisting of one input layer with two neurons, two hidden layers and one output as follows: the created network type was feed-forward back-propagation neural network and Levenberg-Marquardt (LM) was used as the learning algorithm [32]. The transfer function was tangent sigmoid (tansig) for layers, and the optimal number of neurons in the hidden layers was specified as 10 by scanning between 10 and 40. The designs of the network and the training and testing of the data were carried out entirely using *MATLAB* nntool software.

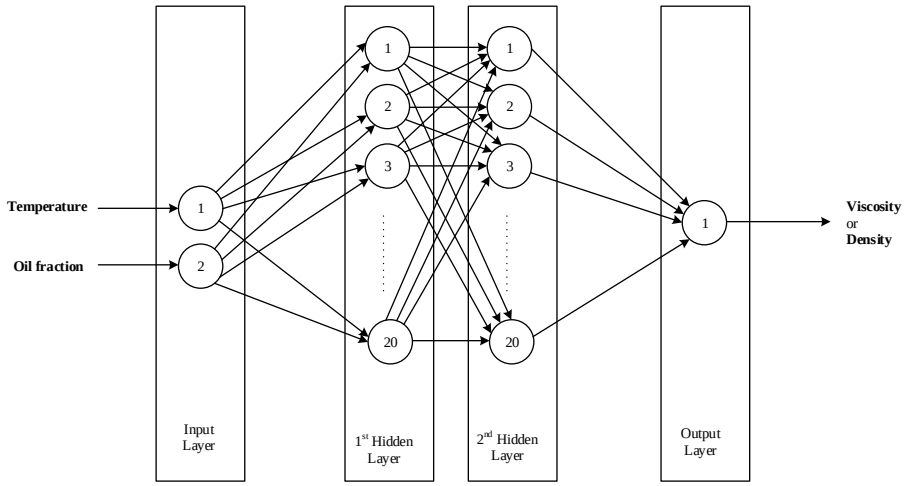

Fig. 1. ANN structure used in this study.

3. RESULTS AND DISCUSSION

3.1. Effects of Temperature on Viscosity and Density

3.1.1. Viscosity Variation

The dependence of kinematic viscosity on temperature for WCO2 ternary blend is illustrated in Fig. 2 in which points and lines represent measured data by the authors and computed values from rational model (Eq. (3)) previously suggested by the authors [23], [24], Gupta et al. model (Eq. (4)) [33], [34] and ANN such as:

$$\nu_{\text{blend}} = (a + T)/(b + c \cdot T), \tag{3}$$

$$\nu_{\text{blend}} = \exp(a + b \cdot \text{In}T), \tag{4}$$

where T is the temperature of ternary blends in K or °C, and a, b and c are regression constants.

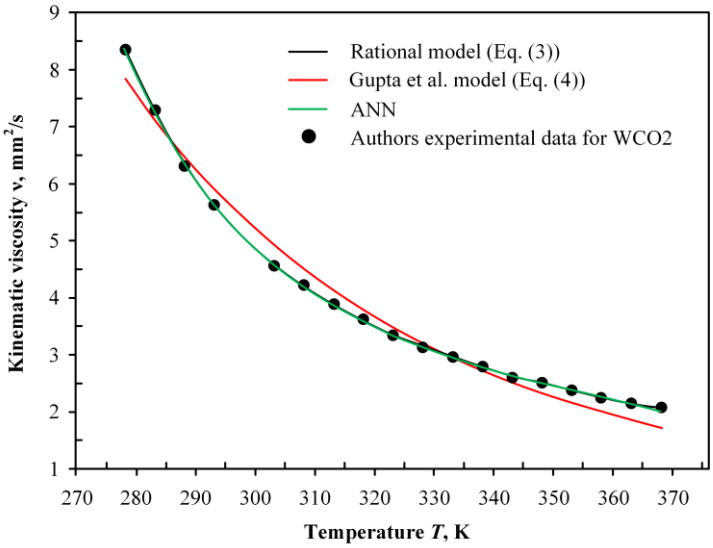

Fig. 2. Variation in kinematic viscosity of WCO2 vs. temperature.

The variations in kinematic viscosity with respect to temperature for BF and the other ternary blends (WCO4, WCO6, WCO8, WCO10, WCO15 and WCO20) were given in [35, Fig. 1–7]. As shown in these figures, BF and ternary blends exhibit the same trends in the viscosity variation corresponding to temperature. In other words, the behavior of variation in viscosity with respect to temperature is exponential, as expected, that is, viscosity values exponentially decrease as temperature of fuel (T) increases for all cases. A quite good qualitative agreement between estimated values and measured data is captured by the rational model and ANN (except for the measured value of WCO6 at 278.15 K, as seen in [35, Fig. 3]) while Gupta et al. model somewhat under predicts the measured data between approximately 278.15–283.15 K and 338.15–368.15 K, and over predicts them between about 288.15–328.15 K. Table 3 reports the kinematic viscosity data of base fuel and vegetable oil biodiesel (ethyl ester)-diesel fuel-vegetable oil ternary blends (WCO2, WCO4, WCO6, WCO8, WCO10, WCO15 and WCO20) measured by the authors at different temperatures (278.15–368.15 K), percent relative errors between measured data and calculated values from Eq. (3), Eq. (4) and ANN at the measurement points, and correlation parameters (i.e. regression constants and R values representing the goodness of fit) of Eq. (3) and Eq. (4). The maximum relative errors were determined as 2.2418 %, 18.5151 % and 4.8482 % for Eq. (3), Eq. (4) and ANN while the minimum R values were computed as 0.9998 and 0.9723 for Eq. (3) and Eq. (4). According to Fig. 2, and [35, Fig. 1–7 and Table 3], the rational model qualitatively and quantitatively provides an excellent fit for representation of the viscosity data of ternary blends measured by the authors.

TABLE 3. VISCOSITY DATA OF BLENDS MEASURED BY THE AUTHORS, RELATIVE ERRORS, AND REGRESSION PARAMETERS

Temperature *T*, K	Measured *v*, mm^2/s							
	Oil volume fraction *X*, %							
	0	2	4	6	8	10	15	20
278.15	7.721	8.350	9.170	9.597	10.484	10.964	11.921	12.960
283.15	6.458	7.288	8.113	8.568	9.193	9.728	10.794	11.735
288.15	5.503	6.308	7.159	7.606	8.282	8.812	9.812	10.924
293.15	4.748	5.628	6.378	6.846	7.477	7.980	9.168	10.097
303.15	3.767	4.565	5.335	5.857	6.432	6.938	8.066	8.985
308.15	3.363	4.223	4.923	5.517	6.084	6.583	7.676	8.565
313.15	3.094	3.884	4.546	5.186	5.799	6.242	7.350	8.254
318.15	2.754	3.623	4.294	4.935	5.487	5.965	7.072	7.905
323.15	2.457	3.346	4.046	4.688	5.263	5.708	6.766	7.636
328.15	2.308	3.129	3.821	4.503	5.047	5.502	6.622	7.406
333.15	2.130	2.957	3.644	4.270	4.877	5.297	6.488	7.211
338.15	1.960	2.788	3.451	4.169	4.706	5.132	6.268	7.012
343.15	1.801	2.596	3.288	4.043	4.589	5.011	6.119	6.898
348.15	1.681	2.504	3.160	3.897	4.438	4.852	5.987	6.695
353.15	1.573	2.371	3.093	3.791	4.309	4.708	5.873	6.497
358.15	1.426	2.244	2.956	3.678	4.218	4.620	5.760	6.435
363.15	1.327	2.142	2.871	3.581	4.121	4.517	5.658	6.322
368.15	1.267	2.077	2.795	3.493	4.034	4.420	5.567	6.218

TABLE 3 (CONTINUED)

Oil volume fraction *X*, %	Eq.	Regression Constants			*R*
		a	b	c	
0	(3)	$-5.554e^2$	$3.131e^2$	–1.255	0.9999
2		$-1.910e^3$	$1.525e^3$	–6.182	0.9999
4		$2.588e^2$	$-4.688e^2$	1.894	0.9998
6		$-8.857e^1$	$-1.675e^2$	$6.726e^{-1}$	0.9998
8		$-1.400e^2$	$-1.208e^2$	$4.818e^{-1}$	0.9999
10		$-1.395e^2$	$-1.082e^2$	$4.342e^{-1}$	0.9999
15		$-1.732e^2$	$-7.248e^1$	$2.921e^{-1}$	0.9998
20		$-1.591e^2$	$-6.658e^1$	$2.724e^{-1}$	0.9999
0	(4)	41.09	–6.949	–	0.9945
2		32.44	–5.398	–	0.9909
4		28.36	–4.657	–	0.9858
6		23.90	–3.859	–	0.9789
8		22.50	–3.598	–	0.9746
10		21.48	–3.407	–	0.9759
15		18.26	–2.819	–	0.9723
20		17.73	–2.708	–	0.9767

TABLE 3 (CONTINUED)

Eq.	Relative errors, %*							
	Oil volume fraction *X*, %							
	0	2	4	6	8	10	15	20
(3)	0.1937	0.4667	0.9290	0.8701	0.2683	0.5822	0.4126	0.0229
	0.2278	0.9804	1.0164	1.0311	0.3203	0.1555	0.4097	0.1979
	0.0745	0.2991	0.7222	0.2656	0.7902	0.2701	0.2278	0.8279
	0.7856	0.0117	0.1314	0.7092	0.2111	0.8837	0.4986	0.0163
	0.5793	0.8365	0.0317	0.6534	0.4264	0.6819	0.2534	0.2139
	0.1461	0.1757	0.2851	0.0979	0.1034	0.0793	0.2861	0.2428
	2.0106	0.0589	1.2130	0.4182	0.7218	0.1796	0.2617	0.3118
	0.0358	0.5500	0.4379	0.0712	0.0524	0.0300	0.2186	0.1798
	2.2418	0.3257	0.4100	0.3765	0.2698	0.1944	1.1417	0.1758
	0.2698	0.3816	0.5864	0.0172	0.0619	0.0140	0.1141	0.0790
	0.6291	0.2382	0.1597	1.2408	0.2705	0.2872	0.7245	0.1379
	0.3922	0.2922	0.7706	0.1235	0.0353	0.2159	0.0859	0.0107
	0.2535	1.2090	1.0759	0.5286	0.5856	0.3803	0.0722	0.7898
	0.4349	0.5614	0.7748	0.0832	0.0786	0.0841	0.0241	0.0657
	1.1739	0.2357	1.1077	0.3385	0.2403	0.5372	0.1138	0.8426
	1.4267	0.3587	0.3937	0.1314	0.0731	0.0383	0.0817	0.1538
	1.5579	0.2998	1.0841	0.1013	0.0299	0.0671	0.0720	0.2037
	0.7643	1.1398	1.8229	0.0969	0.0316	0.1550	0.1077	0.2490
(4)	6.2025	6.1124	6.4132	7.7365	9.5137	8.5850	8.0648	7.0331
	0.9170	2.2945	2.6426	3.5223	3.2139	3.0380	3.4394	2.1643
	2.9606	2.7070	1.6940	1.5812	0.8748	0.8441	1.1102	0.2333
	5.8868	4.9078	5.3584	5.6091	5.0289	5.0189	3.0899	3.5069
	5.7125	7.9157	7.7402	8.4542	8.2121	7.7467	6.6022	6.2171
	5.6878	6.7957	8.1923	8.0941	7.8625	7.4011	6.9699	6.5972
	2.7203	6.4541	8.7034	8.0680	6.7961	7.2243	6.7588	5.8959
	3.3730	4.7701	6.8987	6.8303	6.6157	6.3085	6.1098	5.9283
	3.9694	4.2854	5.5044	5.8912	5.0888	5.3468	6.1390	5.1257
	0.5197	2.6475	4.0076	3.8993	3.6965	3.7209	3.8532	3.9761
	2.9587	0.1041	1.6435	3.3578	1.6284	2.3250	1.5745	2.5031
	4.9126	2.0312	0.1346	0.0522	0.1752	0.3883	0.8159	1.2444
	6.5524	2.8001	1.8454	2.6129	2.8962	2.2027	0.9151	1.0929
	9.4564	6.7983	4.5231	4.4499	4.6846	3.8550	2.7771	2.0088
	12.3679	8.8624	8.7221	7.0371	6.7407	5.6130	4.7949	2.8473
	12.3315	10.7419	10.5446	9.2410	9.4281	8.3138	6.6991	5.5755
	14.4438	13.2341	13.6549	11.6388	11.8071	10.5496	8.6577	7.4293
	18.5151	16.8859	16.7791	14.0690	14.2305	12.7477	10.6752	9.3025

ANN	0.6187	0.2961	0.2184	2.8368	0.0932	0.4001	0.0899	0.6624
	0.0852	0.5791	0.7081	0.1185	0.0664	0.0661	0.3419	0.1941
	0.3950	0.4157	0.1101	0.3242	0.3983	0.2007	0.3476	1.0049
	0.0874	0.4746	0.1578	1.0102	0.3211	0.2181	0.5432	0.2209
	0.2533	0.4525	0.2113	0.5327	0.2388	0.4496	0.6877	0.2074
	0.4418	0.4360	0.7410	0.1742	0.1267	0.5959	0.1767	0.1140
	2.2275	0.2851	1.5848	0.3870	0.5991	0.1787	0.0056	0.0000
	0.7239	1.1872	0.1280	0.1166	0.0825	0.1155	0.2153	0.3240
	1.6159	0.4521	0.4635	0.1462	0.6702	0.2323	0.5667	0.0182
	0.3096	0.0043	0.5585	0.2411	0.7211	0.2901	0.2941	0.0565
	0.0553	0.1217	1.0733	1.2633	0.9153	0.7053	0.7889	0.0555
	0.4649	0.2050	0.2148	0.2696	0.5088	0.5811	0.1987	0.4301
	0.6460	1.9633	0.6173	0.0874	0.7846	0.2543	0.0102	0.3981
	0.9308	0.2028	0.9332	0.5127	0.0447	0.0576	0.3129	0.1470
	2.3801	0.2982	0.4156	0.1353	0.5734	0.3426	0.4454	0.8836
	0.5272	0.4320	0.8132	0.2649	0.2357	0.0540	0.1827	0.1469
	2.8881	0.2315	0.8304	0.1390	0.1771	0.0417	0.1605	0.0328
	4.8482	2.2497	1.1555	0.3113	0.3189	1.4400	1.1359	0.2833

*Relative error $= \left|\nu_{\text{est.}} - \nu_{\text{exp.}}\right| / \nu_{\text{exp.}}$

The rational model (Eq. (3)) [23], [24], Gupta et al. model (Eq. (4)) [33] and ANN were also tested against the dynamic viscosity data of different biodiesel-diesel fuel-vegetable oil ternary blends measured by Nogueira et al. [25] and Baroutian et al. [26] at different temperatures (293.15–373.15 K and 20–90 °C, respectively), as shown in Fig. 3, Fig. 4 and [35, Fig. 8–25]. The ternary blends (S0.804, S0.810, S0.838, S0.844, S0.847, S0.878, S0.880, S0.884, S0.912, S0.913, S0.919, S0.943, S0.945 and S0.946) given in [25] were prepared on mass fraction basis while the ternary blends (P1.5, P2.5, P3, P4.5, P6 and P9) given in [26] were prepared on volume fraction basis. The ternary blends were named to reflect oil content in blend. For example, the name of S0.804 shows a ternary blend consisting of 0.804 % soybean oil in blend on mass fraction basis while P1.5 shows a ternary blend consisting of 1.5 % palm oil in blend on volume fraction basis. The other ternary blends in [25], [26] also were similarly named. The best qualitative correlation with the variation in viscosity data through the studied temperature ranges was obtained using the rational model for all ternary blends (S0.804, S0.810, S0.838, S0.844, S0.847, S0.878, S0.880, S0.884, S0.912, S0.913, S0.919, S0.943, S0.945, S0.946, P1.5, P2.5, P3, P4.5, P6 and P9). Gupta et al. model does not result in the satisfactory agreement of viscosity estimation for all ternary blends. In other words, the predicted values from Gupta et al. model are higher between about 295.15–338.15 K and lower between 345.15–373.15 K than the measured data of soybean oil biodiesel (methyl ester)-diesel fuel-soybean oil ternary blends given by Nogueira et al. [25]. However, it generally gives the close values to the measured data at about 333.15 K given in [25]. In addition, the model overestimates the viscosity data of palm oil biodiesel (methyl ester)-diesel fuel-palm oil ternary blends at approximately 20–25 °C and 65–90 °C, underestimates them at about 25–60 °C, but presents close results to the experimental measurements [26] at about 60 °C. Moreover, if an algorithm is not tested with enough data (for example the experimental values given in [25] and [26]) or if it is desired to obtain a linear model with using nonlinear data, it cannot be modelled precisely and generalized. This is named with the under-fitting problem and causes the

algorithm to make a false decision as it increases its flexibility [32]. Therefore, the qualitatively unstable behaviors (i.e. oscillating) were determined for the viscosity estimation from ANN, and ANN results in great differences between measured data and predicted values for all ternary blends given in [25], [26]. On the other hand, the relative errors at some measurement points computed from ANN are relatively lower, compared to the rational model and Gupta et al. model, since ANN forces, like an interpolation polynomial, curves to pass from the experimental data which is qualitatively not meaningful. However, when all errors are considered, it can be said that ANN qualitatively does not give a reasonably accurate prediction for all ternary blends given in [25], [26]. Table 4 and Table 5 list dynamic viscosity values of soybean oil biodiesel-diesel fuel-soybean oil [25] and palm oil biodiesel-diesel fuel-palm oil [26] ternary blends, errors between measured data and computed values from the rational model, Gupta et al. model and ANN, and regression parameters of rational model and Gupta et al. model. As seen in these tables, the highest relative errors were determined from Eq. (3), Eq. (4) and ANN as 0.8723 %, 12.2179 % and 4.9827 %; 1.9851 %, 26.5442 % and 35.5216 % for soybean oil biodiesel-diesel fuel-soybean oil [25] and palm oil biodiesel-diesel fuel-palm oil [26] ternary blends, respectively. Moreover, the lowest R values of Eq. (3) and Eq. (4) were computed as 0.9998 and 0.9971; 0.9994 and 0.9852 for the ternary blends including soybean oil [25] and palm oil [26], respectively. According to the regression results given in Table 4 and Table 5, the dynamic viscosity data of [25] and [26] were also quantitatively the best correlated by means of the rational model, compared to Gupta et al. model and ANN.

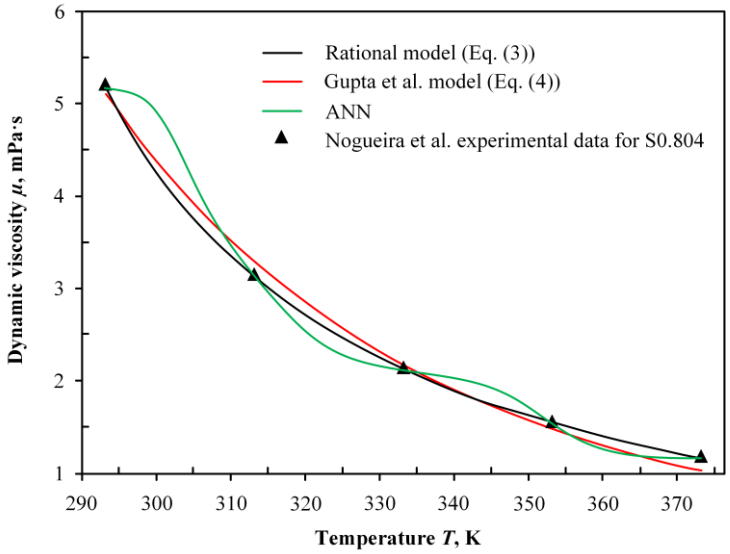

Fig. 3. Variation in dynamic viscosity of S0.804 vs. temperature.

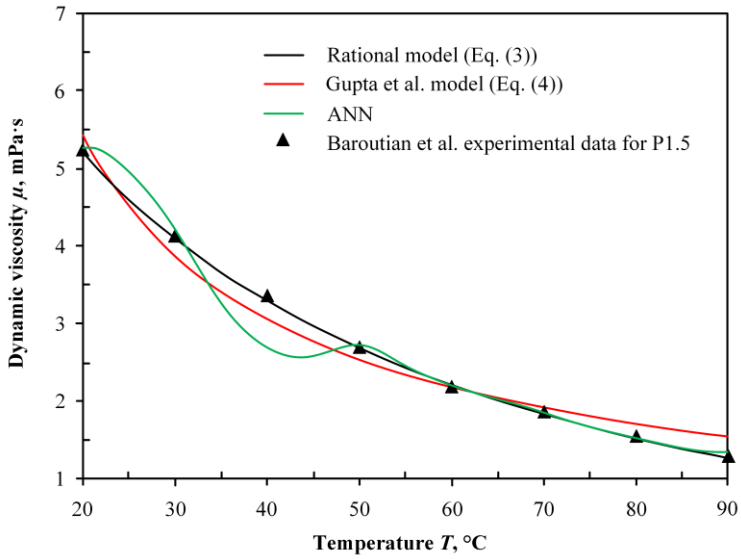

Fig. 4. Variation in dynamic viscosity of P1.5 vs. temperature.

TABLE 4. VISCOSITY DATA OF BLENDS MEASURED BY NOGUEIRA ET AL. [25], RELATIVE ERRORS, AND REGRESSION PARAMETERS

Temperature *T*, K	Measured μ, mPa·s							
	Oil mass fraction *X*, %							
	0.804	0.810	0.838	0.844	0.847	0.878	0.880	0.884
293.15	5.1864	5.0724	4.9359	4.9443	4.7766	4.6204	4.6545	4.5633
313.15	3.1320	3.0823	3.0062	3.0085	2.9173	2.8239	2.8476	2.7940
333.15	2.1184	2.0852	2.0437	2.0388	1.9828	1.9224	1.9362	1.9035
353.15	1.5367	1.5133	1.4801	1.4822	1.4446	1.4028	1.4113	1.3872
373.15	1.1669	1.1470	1.1232	1.1268	1.0994	1.0698	1.0757	1.0543

TABLE 4 (CONTINUED)

Temperature *T*, K	Measured μ, mPa·s					
	Oil mass fraction *X*, %					
	0.912	0.913	0.919	0.943	0.945	0.946
293.15	4.4619	4.3582	4.3478	4.1322	4.1420	4.1425
313.15	2.7326	2.6768	2.6724	2.5490	2.5546	2.5511
333.15	1.8622	1.8284	1.8244	1.7469	1.7508	1.7351
353.15	1.3593	1.3349	1.3314	1.2818	1.2838	1.2625
373.15	1.0368	1.0272	1.0101	0.9809	0.9820	0.9619

TABLE 4 (CONTINUED)

Oil mass fraction *X*, %	Eq.	Regression constants			*R*
		a	b	c	
0.804	(3)	$-5.637e^2$	$3.596e^2$	–1.405	0.9999
0.810		$-5.571e^2$	$3.495e^2$	–1.370	0.9999
0.838		$-5.574e^2$	$3.558e^2$	–1.396	0.9999
0.844		$-5.620e^2$	$3.656e^2$	–1.432	0.9999
0.847		$-5.649e^2$	$3.785e^2$	–1.485	0.9999
0.878		$-5.697e^2$	$3.990e^2$	–1.565	0.9999
0.880		$-5.655e^2$	$3.876e^2$	–1.522	0.9998
0.884		$-5.638e^2$	$3.903e^2$	–1.533	0.9999
0.912		$-5.692e^2$	$4.094e^2$	–1.607	0.9999
0.913		$-5.771e^2$	$4.307e^2$	–1.691	0.9999
0.919		$-5.621e^2$	$4.008e^2$	–1.578	0.9999
0.943		$-5.777e^2$	$4.480e^2$	–1.763	0.9999
0.945		$-5.765e^2$	$4.446e^2$	–1.750	0.9999
0.946		$-5.568e^2$	$4.093e^2$	–1.613	0.9999
0.804	(4)	39.43	–6.654	–	0.9971
0.810		39.16	–6.611	–	0.9974
0.838		38.92	–6.573	–	0.9974
0.844		38.97	–6.582	–	0.9973
0.847		38.65	–6.531	–	0.9973
0.878		38.49	–6.508	–	0.9972
0.880		38.52	–6.513	–	0.9973
0.884		38.43	–6.501	–	0.9974
0.912		38.33	–6.487	–	0.9973
0.913		38.04	–6.441	–	0.9972
0.919		38.16	–6.461	–	0.9975
0.943		37.67	–6.383	–	0.9972
0.945		37.69	–6.387	–	0.9973
0.946		38.17	–6.472	–	0.9976

TABLE 4 (CONTINUED)

Eq.	Relative errors, %						
	Oil mass fraction *X*, %						
	0.804	0.810	0.838	0.844	0.847	0.878	0.880
(3)	0.2113	0.1516	0.1851	0.3413	0.1129	0.1244	0.1042
	0.4715	0.4654	0.1335	0.1390	0.2684	0.2528	0.4447
	0.3285	0.4529	0.4118	0.6967	0.5613	0.5471	0.4594
	0.3212	0.3396	0.5831	0.5671	0.4472	0.4490	0.3801
	0.8379	0.8290	0.6524	0.6828	0.6915	0.6778	0.8432
(4)	1.5197	1.8643	1.5552	1.8330	1.4178	1.0322	1.6003
	5.1170	4.3945	4.7473	4.4873	4.8918	5.3874	4.6427
	2.9386	2.4831	2.5682	2.5808	3.0012	3.4692	2.8301
	3.7232	3.9520	3.4584	3.8650	3.3921	2.9757	3.4959
	12.1210	11.9586	11.4284	12.0021	11.4160	11.1056	11.5586
ANN	0.5159	0.9416	1.8358	0.6347	0.7816	2.3234	0.1904
	0.0086	0.3525	0.6740	1.3627	1.4868	0.4395	0.0021
	0.1261	0.2530	0.4749	0.0901	0.2050	1.6133	2.3946
	0.3075	0.3441	0.9830	0.6564	0.2167	0.7365	1.5436
	2.8017	1.2893	0.7728	0.1959	0.5689	1.4455	0.6029

TABLE 4 (CONTINUED)

Eq.	Relative errors, %						
	Oil mass fraction *X*, %						
	0.884	0.912	0.913	0.919	0.943	0.945	0.946
(3)	0.3574	0.2856	0.2098	0.1103	0.0555	0.0051	0.1482
	0.0544	0.1386	0.2328	0.2086	0.2860	0.3133	0.3164
	0.6250	0.6244	0.5774	0.4664	0.4646	0.4198	0.6454
	0.5121	0.5250	0.7741	0.3000	0.3321	0.3248	0.6087
	0.4997	0.6100	0.8723	0.5159	0.6340	0.6406	0.8654
(4)	1.8011	1.6044	2.1121	1.2493	0.8582	1.3617	1.6551
	4.4297	4.7094	4.1849	4.8865	5.4668	4.9217	4.1797
	2.4950	2.8293	2.3688	2.9868	3.6562	3.0913	2.6056
	3.7247	3.4881	3.6819	3.1712	2.6291	3.1173	3.3066
	11.4561	11.4875	12.2179	10.5925	10.4808	10.9102	11.1493
ANN	0.1511	0.1914	1.3131	0.7069	1.1283	0.2747	0.4262
	0.3077	2.2465	0.5180	1.1167	0.0461	0.6533	4.9827
	1.5887	1.1024	0.7522	1.0510	2.2940	0.1074	0.4448
	2.2328	1.9273	4.2272	0.0412	0.2407	2.2420	3.9629
	0.5058	1.4686	4.6326	4.5873	1.9298	2.0464	0.0004

TABLE 5. VISCOSITY DATA OF BLENDS MEASURED BY BAROUTIAN ET AL. [26], RELATIVE ERRORS, AND REGRESSION PARAMETERS

Temperature T, °C	Measured μ, mPa·s					
	Oil mass fraction X, %					
	1.5	2.5	3	4.5	6	9
20	5.1984	5.1900	5.5201	5.7131	6.5013	6.8713
30	4.1011	4.1420	4.3141	4.5132	5.1984	5.4558
40	3.3324	3.3301	3.4611	3.5898	4.1046	4.3941
50	2.6729	2.6800	2.7855	2.8981	3.2842	3.5898
60	2.1743	2.2260	2.2869	2.4156	2.7190	2.9946
70	1.8421	1.8400	1.9008	1.9973	2.2964	2.4638
80	1.5308	1.4980	1.5952	1.6917	1.9169	2.0938
90	1.2810	1.2280	1.3026	1.4091	1.6078	1.8008

TABLE 5 (CONTINUED)

Oil volume fraction X, %	Eq.	Regression constants			R
		a	b	c	
1.5	(3)	$-1.842e^{2}$	$-1.929e^{1}$	$-6.120e^{-1}$	0.9998
2.5		$-1.677e^{2}$	$-1.841e^{1}$	$-5.006e^{-1}$	0.9999
3		$-1.847e^{2}$	$-1.762e^{1}$	$-6.094e^{-1}$	0.9999
4.5		$-1.942e^{2}$	$-1.786e^{1}$	$-6.279e^{-1}$	0.9999
6		$-1.898e^{2}$	$-1.542e^{1}$	$-5.280e^{-1}$	0.9994
9		$-2.005e^{2}$	$-1.584e^{1}$	$-5.187e^{-1}$	0.9999
1.5	(4)	4.175	–0.8294	–	0.9877
2.5		4.186	–0.8323	–	0.9852
3		4.286	–0.8471	–	0.9887
4.5		4.279	–0.8333	–	0.9892
6		4.416	–0.8348	–	0.9882
9		4.388	–0.8080	–	0.9890

TABLE 5 (CONTINUED)

Eq.	Relative errors, %					
	Oil volume fraction X, %					
	1.5	2.5	3	4.5	6	9
(3)	0.1796	0.1287	0.0953	0.2411	0.5306	0.2086
	0.1337	0.5480	0.1193	0.8579	1.6627	0.4772
	1.1375	0.2259	0.4488	0.0488	0.1214	0.1688
	0.6367	1.1002	0.5564	1.0187	1.7873	0.3575
	1.9851	0.1306	0.6348	0.0388	1.3549	0.0941
	0.2182	0.6626	0.1079	0.6001	0.4036	1.5685
	0.2652	0.1484	1.1113	0.8605	0.6591	0.3747
	1.1210	0.3000	0.3239	0.5689	1.3785	1.8575

Eq.	Relative errors, %					
	Oil volume fraction *X*, %					
	1.5	2.5	3	4.5	6	9
(4)	4.2883	4.6989	4.0727	4.0701	4.4106	4.0910
	5.5603	6.3865	5.5447	6.0328	6.9162	5.5261
	8.4467	8.3558	7.7286	7.0436	7.2799	7.0285
	5.1426	5.4266	5.0958	4.3949	3.8134	4.9732
	0.2447	2.1691	0.9471	1.4657	0.2221	1.6897
	4.1219	4.1031	4.5845	4.8051	3.8745	5.4969
	12.1600	14.4203	11.2919	10.7077	11.3129	11.4429
	21.5575	26.5442	23.3492	20.4853	20.2848	17.8123
ANN	1.5427	0.2244	9.8252	0.0650	5.1436	6.9334
	2.2862	0.0755	32.8439	0.0547	0.1334	11.5234
	19.2630	35.5216	0.2123	0.0207	0.0008	0.1243
	1.9748	1.3786	0.6522	2.1989	0.1985	7.7488
	1.4830	1.2872	0.9798	0.2784	0.3507	0.1477
	1.0094	1.0800	0.1093	0.4964	1.3569	0.2949
	0.0435	2.8884	0.8645	0.9934	3.1839	0.5398
	4.5455	8.9542	24.0160	3.7693	1.7313	1.7092

Finally, the all results mentioned above show that the rational model (Eq. (3)) as a function of temperature of blend, previously recommend by the authors to predict viscosity of different biodiesel-diesel fuel binary blends [23] and biodiesel-diesel fuel-alcohol ternary blends [24], obtained the best agreements with the viscosity values of different biodiesel-diesel-vegetable oil ternary blends measured by the authors, Nogueira et al. [25] and Baroutian et al. [26] at various temperatures, compared to Eq. (4) and ANN.

3.1.2. Density Variation

Fig. 5 illustrates the variation in density of WCO2 ternary blend with respect to temperature, and comparison between the authors' experimental data (represented as points) and the values (represented as lines) obtained by using the exponential model [15] (Eq. (5)), linear model [36] (Eq. (6)) and ANN. The models were formulated as:

$$\rho_{\text{blend}} = \text{a} \cdot e^{\text{b}T} + \text{c} \cdot e^{\text{d}T}, \tag{5}$$

$$\rho_{\text{blend}} = \text{a} \cdot T + \text{b}, \tag{6}$$

where T is temperature of blend in K or °C.

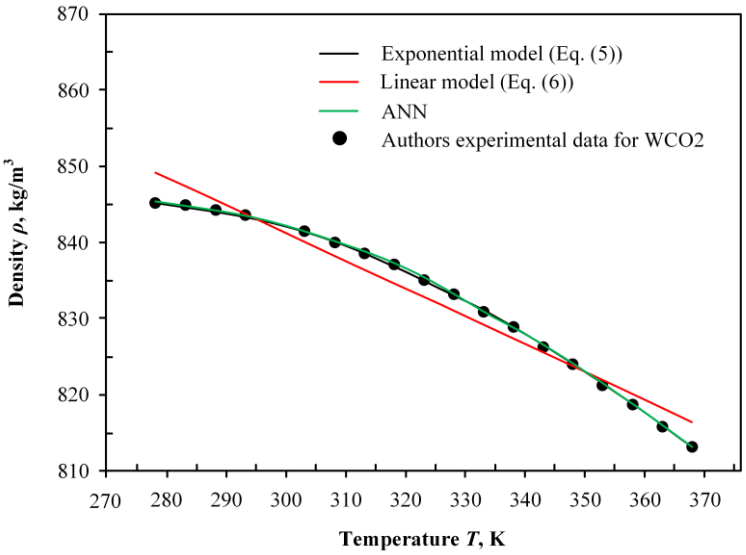

Fig. 5. Variation in density of WCO2 vs. temperature.

The figures including the changes of BF and other ternary blends (WCO4, WCO6, WCO8, WCO10, WCO15 and WCO20) were also given in [35, Fig. 26–32]. These figures indicate that the densities of ternary blends quadratically decrease with increasing temperature. The experimental data measured by the authors have qualitatively the best correlation with the exponential model and ANN while the linear model does not give a reasonably accurate result, as seen in Fig. 5 and [35, Fig. 26–32]. Table 6 shows density values of ternary blends including waste cooking oil biodiesel (ethyl ester)-diesel fuel-waste cooking oil measured by the authors at different temperatures (278.15–368.15 K), percent relative errors, and regression parameters of Eq. (5) and Eq. (6). The highest relative errors coming from Eq. (5), Eq. (6) and ANN were determined as 0.0828 %, 0.4904 % and 0.0962 %. Moreover, the lowest *R* values of Eq. (5) and Eq. (6) were computed as 0.9997 and 0.9796, respectively. These results point out that the exponential model and ANN quantitatively provide very close agreement to the measurements.

TABLE 6. DENSITY DATA OF BLENDS MEASURED BY THE AUTHORS, RELATIVE ERRORS, AND REGRESSION PARAMETERS

Temperature *T*, K	Measured ρ, kg/m^3							
	Oil volume fraction *X*, %							
	0	2	4	6	8	10	15	20
278.15	844.10	845.09	846.49	847.69	848.89	849.69	853.09	856.29
283.15	843.71	844.84	846.24	847.44	848.64	849.44	852.83	856.03
288.15	843.13	844.25	845.65	846.85	848.04	848.84	852.24	855.13
293.15	842.47	843.57	844.97	846.17	847.36	848.16	851.55	854.44
303.15	840.32	841.45	842.85	844.05	845.24	846.04	849.42	852.40
308.15	838.94	840.00	841.46	842.60	843.91	844.91	848.36	851.18
313.15	837.53	838.58	839.97	841.17	842.35	843.15	846.52	849.69

Temperature *T*, K	Measured ρ, kg/m^3							
	Oil volume fraction *X*, %							
	0	2	4	6	8	10	15	20
318.15	835.74	837.07	838.24	839.22	840.69	841.68	845.01	848.02
323.15	833.94	835.02	836.42	837.62	838.79	839.58	842.94	846.09
328.15	831.93	833.23	834.41	835.68	836.85	837.73	841.24	844.04
333.15	829.82	830.88	832.27	833.47	834.63	835.42	838.76	841.90
338.15	827.57	828.85	830.04	831.31	832.47	833.34	836.83	839.42
343.15	825.24	826.31	827.70	828.90	830.05	830.83	834.15	837.07
348.15	822.73	824.00	825.20	826.47	827.62	828.78	831.94	834.70
353.15	820.23	821.25	822.64	822.83	824.97	825.75	829.05	832.05
358.15	817.47	818.75	819.96	821.22	822.36	823.22	826.65	829.42
363.15	814.70	815.79	817.21	818.17	819.60	820.55	823.86	826.43
368.15	811.84	813.16	814.37	815.63	816.76	817.70	821.00	823.76

TABLE 6 (CONTINUED)

Oil volume fraction *X*, %	Eq.	Regression constants				*R*
		a	b	c	D	
0	(5)	1341	$-1.213e^{-3}$	–1786	–0.009919	1.0000
2		1292	$-1.141e^{-3}$	–1982	–0.0109	0.9998
4		1252	$-1.077e^{-3}$	–2235	–0.0119	1.0000
6		1227	$-1.035e^{-3}$	–2571	–0.01283	0.9997
8		1248	$-1.064e^{-3}$	–2313	–0.01213	1.0000
10		1226	$-1.027e^{-3}$	–2565	–0.01284	0.9998
15		1230	$-1.025e^{-3}$	–2603	–0.0129	0.9998
20		1346	$-1.193e^{-3}$	–1866	–0.01019	0.9998
0	(6)	–0.3648	949.6	–	–	0.9801
2		–0.3632	950.2	–	–	0.9797
4		–0.3648	952.1	–	–	0.9802
6		–0.3669	953.9	–	–	0.9798
8		–0.3650	954.5	–	–	0.9801
10		–0.3637	955.0	–	–	0.9797
15		–0.3648	958.7	–	–	0.9796
20		–0.3689	962.9	–	–	0.9798

TABLE 6 (CONTINUED)

Eq.	Relative errors, %							
	Oil volume fraction *X*, %							
	0	2	4	6	8	10	15	20
(5)	0.0335	0.0024	0.0235	0.0133	0.0203	0.0523	0.0212	0.0036
	0.0242	0.0061	0.0258	0.0127	0.0188	0.0505	0.0180	0.0081
	0.0196	0.0023	0.0169	0.0021	0.0293	0.0375	0.0061	0.0347
	0.0314	0.0053	0.0245	0.0090	0.0218	0.0431	0.0106	0.0268
	0.0281	0.0040	0.0247	0.0100	0.0214	0.0411	0.0082	0.0157
	0.0235	0.0091	0.0199	0.0008	0.0187	0.0668	0.0426	0.0001
	0.0359	0.0022	0.0242	0.0122	0.0222	0.0391	0.0063	0.0049
	0.0225	0.0225	0.0196	0.0169	0.0180	0.0654	0.0287	0.0081
	0.0262	0.0033	0.0225	0.0141	0.0240	0.0354	0.0032	0.0009
	0.0221	0.0193	0.0199	0.0219	0.0176	0.0521	0.0387	0.0067
	0.0223	0.0089	0.0178	0.0133	0.0287	0.0299	0.0025	0.0068
	0.0212	0.0166	0.0198	0.0257	0.0176	0.0500	0.0366	0.0318
	0.0250	0.0050	0.0227	0.0218	0.0238	0.0322	0.0002	0.0268
	0.0207	0.0146	0.0192	0.0284	0.0182	0.0828	0.0322	0.0104
	0.0307	0.0064	0.0207	0.1000	0.0270	0.0269	0.0053	0.0146
	0.0212	0.0147	0.0190	0.0293	0.0198	0.0425	0.0273	0.0042
	0.0219	0.0092	0.0194	0.0063	0.0202	0.0513	0.0230	0.0258
	0.0224	0.0178	0.0186	0.0299	0.0207	0.0477	0.0197	0.0020
(6)	0.4775	0.4835	0.4892	0.4904	0.4812	0.4880	0.4854	0.4672
	0.3078	0.2983	0.3033	0.3035	0.2958	0.3035	0.3022	0.2822
	0.1605	0.1533	0.1576	0.1568	0.1516	0.1602	0.1576	0.1721
	0.0224	0.0187	0.0224	0.0205	0.0166	0.0261	0.0245	0.0371
	0.1558	0.1609	0.1589	0.1630	0.1644	0.1531	0.1541	0.1563
	0.2090	0.2048	0.2107	0.2089	0.2233	0.2348	0.2444	0.2299
	0.2588	0.2523	0.2509	0.2573	0.2552	0.2423	0.2430	0.2720
	0.2634	0.2894	0.2626	0.2442	0.2753	0.2841	0.2806	0.2931
	0.2668	0.2620	0.2636	0.2726	0.2670	0.2513	0.2521	0.2837
	0.2451	0.2657	0.2420	0.2607	0.2539	0.2481	0.2674	0.2600
	0.2113	0.2022	0.2046	0.2163	0.2072	0.1899	0.1899	0.2256
	0.1604	0.1769	0.1563	0.1777	0.1675	0.1590	0.1777	0.1505
	0.0995	0.0898	0.0944	0.1088	0.0963	0.0763	0.0757	0.0906
	0.0164	0.0301	0.0127	0.0371	0.0235	0.0485	0.0295	0.0279
	0.0659	0.0835	0.0767	0.1822	0.0764	0.0980	0.0990	0.0689
	0.1807	0.1673	0.1813	0.1552	0.1721	0.1847	0.1690	0.1638
	0.2974	0.3082	0.2953	0.3044	0.2868	0.2891	0.2868	0.3030
	0.4261	0.4093	0.4210	0.3918	0.4120	0.4163	0.4140	0.4042

Eq.	Relative errors, %							
	Oil volume fraction *X*, %							
	0	2	4	6	8	10	15	20
ANN	0.0290	0.0448	0.0201	0.0214	0.0151	0.0094	0.0055	0.0962
	0.0001	0.0088	0.0290	0.0127	0.0043	0.0140	0.0190	0.0524
	0.0047	0.0078	0.0357	0.0135	0.0016	0.0011	0.0131	0.0005
	0.0140	0.0001	0.0278	0.0117	0.0102	0.0174	0.0111	0.0069
	0.0246	0.0192	0.0311	0.0076	0.0191	0.0279	0.0006	0.0219
	0.0027	0.0445	0.0290	0.0112	0.0155	0.0327	0.0364	0.0074
	0.0018	0.0452	0.0002	0.0070	0.0020	0.0205	0.0000	0.0046
	0.0087	0.0006	0.0150	0.0282	0.0032	0.0081	0.0168	0.0177
	0.0178	0.0154	0.0296	0.0093	0.0025	0.0292	0.0162	0.0160
	0.0264	0.0522	0.0261	0.0050	0.0066	0.0196	0.0152	0.0125
	0.0254	0.0141	0.0077	0.0078	0.0068	0.0245	0.0242	0.0106
	0.0173	0.0197	0.0376	0.0093	0.0061	0.0341	0.0209	0.0179
	0.0122	0.0217	0.0317	0.0046	0.0012	0.0146	0.0103	0.0169
	0.0026	0.0076	0.0175	0.0116	0.0003	0.0199	0.0248	0.0039
	0.0137	0.0121	0.0139	0.0095	0.0033	0.0209	0.0160	0.0016
	0.0100	0.0100	0.0246	0.0080	0.0178	0.0052	0.0122	0.0226
	0.0195	0.0201	0.0131	0.0321	0.0063	0.0049	0.0120	0.0215
	0.0495	0.0269	0.0209	0.0184	0.0110	0.0123	0.2278	0.0096

In this study, the density data of ternary blends including soybean oil biodiesel, soybean oil and diesel fuel (S0.804, S0.807, S0.810, etc.) measured by Nogueira et al. [25] were also correlated by the exponential model, linear model and ANN for investigating their prediction capabilities, as seen in Fig. 6 and [35, Fig. 33–45]. Density values diminish linearly as the temperature of the blend increases. The predicted results from the exponential and linear models are in the best agreement with the experimental results in terms of qualitative goodness of fit. However, ANN cannot represent closely the relationship between density and temperature because of insufficient experimental measurements data in [25] mentioned above. The computed values illustrated as lines in Fig. 6 and [35, Fig. 33–45] were forced by ANN to exactly pass the experimental data [25], like the variation in viscosity mentioned above as depicted in Fig. 3 and Fig. 4 and [35, Fig. 8–25]. Therefore, some relative errors in the measurement points computing from ANN are much lower than the rational model. [35, Table 1] presents measured density data given by Nogueira et al. [25] for different temperatures (293.15–373.15 K), percent relative errors coming from the exponential and linear models and ANN, and regression constants and correlation coefficients of these models. The lowest R values of the exponential and linear models were determined as 0.9998 and 0.9997, respectively, while the highest relative errors were determined as 0.0813 %, 0.0919 % and 0.2409 % for Eq. (5) and Eq. (6) and ANN. According to these results, the best match to the density values of ternary blends measured by Nogueira et al. [25] is obtained using the exponential model in terms of quantitative goodness of fit.

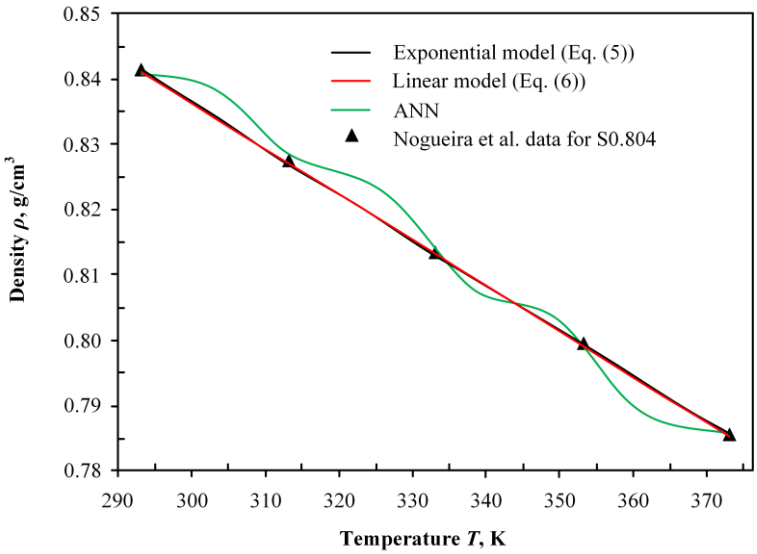

Fig. 6. Variation in density of S0.804 vs. temperature.

In summary, the effect of temperature on density of different ternary blends measured by the authors and Nogueira et al. [25] was qualitatively and quantitatively the best described by the exponential model previously suggested by the authors [15], compared to the linear model and ANN.

3.2. Effect of Oil Content in Blend on Viscosity

Oil content in blend effect on kinematic viscosity of ternary blends measured by the authors at different temperatures (278.15–368.15 K) was illustrated in Fig. 7 and [35, Fig. 46–62]. The change of viscosity vs. oil content was represented by means of the rational model [23], [24], Cragoe model [37] and ANN such as:

$$\nu_{\text{blend}} = (\text{a} + X)/(\text{b} + \text{c} \cdot X), \tag{7}$$

$$\frac{1}{\ln(2000 \cdot \nu_{\text{blend}})} = \frac{X_{\text{biodiesel}}}{\ln(2000 \cdot \nu_{\text{biodiesel}})} + \frac{X_{\text{oil}}}{\ln(2000 \cdot \nu_{\text{oil}})} + \frac{X_{\text{diesel}}}{\ln(2000 \cdot \nu_{\text{diesel}})}. \tag{8}$$

In Eq. (7), X shows oil volume (v/v) or mass fraction (m/m) added to the BF. Similarly, in Eq. (8), $X_{\text{biodiesel}}$, X_{oil} and X_{diesel} are biodiesel, oil and diesel volume (v/v) or mass (m/m) fraction in ternary blends.

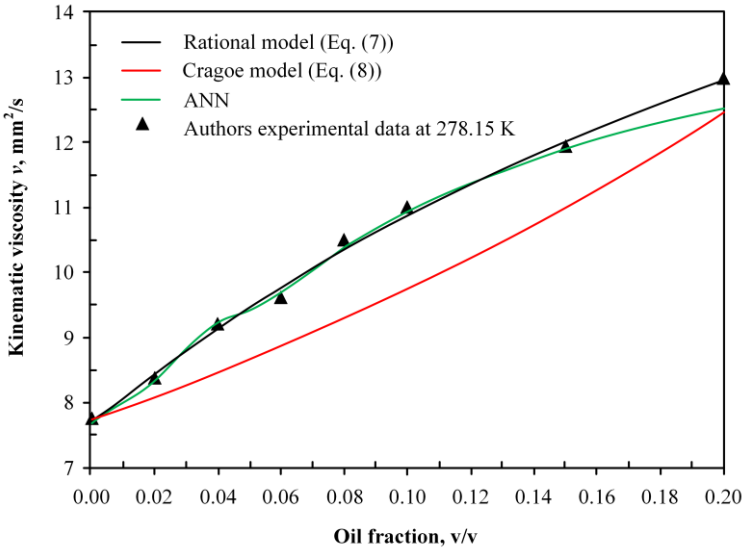

Fig. 7. Variation in kinematic viscosity vs. oil fraction at 278.15 K.

As can be seen in these figures, the curves representing calculated values from Eq. (7) and Eq. (8) exhibit the similar trend: as oil content in blend increases viscosity values about quadratically increase. The rational model (Eq. (7)) fairly well represents the viscosity-oil content relationship for all studied temperature range in terms of the qualitative behavior of the variation. Cragoe model only needs viscosity values of pure fuels (in here diesel, biodiesel and oil) to estimate ternary blends' viscosity as an advantage, however, it is thought that the model estimates with highly large errors in predicting viscosity since the coefficient of Cragoe model (i.e. 2000) is constant. In addition, while ANN is suitable for predicting the viscosity values of many experimental data, some data (especially for 10 %, 15 % and 20 % volume fraction) are not accurately fitted by ANN because of inadequate measured data mentioned above. [35, Table 2] shows experimental viscosity data of base fuel and waste cooking oil ethyl ester-diesel fuel-waste cooking oil ternary blends at different temperatures measured by the authors, relative errors, and regression parameters of Eq. (7) and Eq. (8). The highest relative errors obtained using the rational model, Cragoe model and ANN are 1.9970 %, 73.6994 % and 5.0194 %, and the minimum R value of rational model is 0.9983. These regression results indicate that the rational model (Eq. (7)) also quantitatively presents an excellent match of the experimental point by the authors.

Additionally, the validities of models and ANN were evaluated by fitting to the dynamic viscosity data of palm oil biodiesel-diesel fuel-palm oil ternary blends (including high oil content and equal oil and biodiesel content) measured by Baroutian et al. [26] at different temperatures (20–90 °C), as seen in Fig. 8, Fig. 9 and [35, Fig. 63–76]. There is a quite good agreement between the measured data and estimated values from the exponential model (Eq. (7)) for all studies temperature range, compared to Cragoe model and ANN. [35, Table 3 and Table 7] present the dynamic viscosity data of palm oil biodiesel-diesel fuel-palm oil ternary blends including different oil fraction measured by Baroutian et al. [26], errors and regression parameters of Eq. (7). The maximum relative errors from Eq. (7), Eq. (8) and ANN were determined as 1.4643 % (with the minimum R value of 0.9953), 7.7216 % and 18.5755 %, respectively, for palm oil biodiesel-diesel fuel-palm oil ternary blends including equal oil and biodiesel content [26]. Similarly, the

maximum relative errors from Eq. (7), Eq. (8) and ANN were computed as 2.9742 % (with the minimum *R* value of 0.9915), 23.5901 % and 13.1163 %, respectively, for palm oil biodiesel-diesel fuel-palm oil ternary blends including high oil content [26]. Finally, the rational model as a function of X, previously recommended by the authors in [23], [24], is the best predictor to calculate viscosities of different ternary blends measured by the authors and Baroutian et al. [26] for different temperatures.

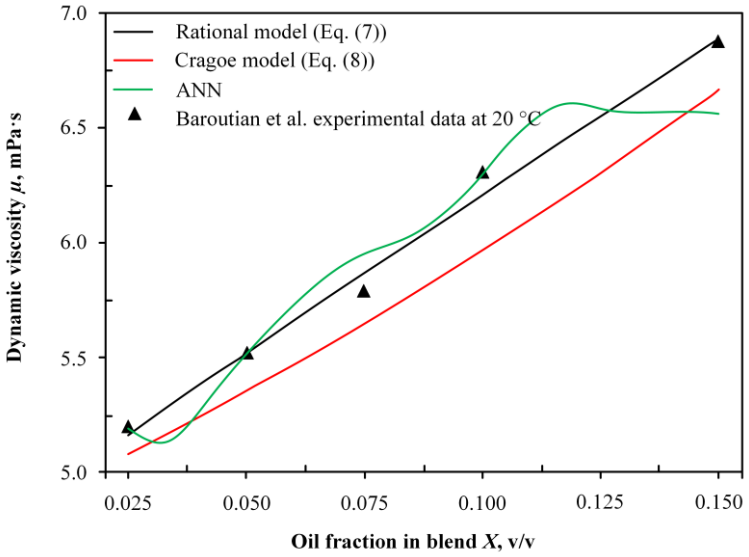

Fig. 8. Variation in dynamic viscosity vs. oil fraction at 20 °C (for equal oil and biodiesel content).

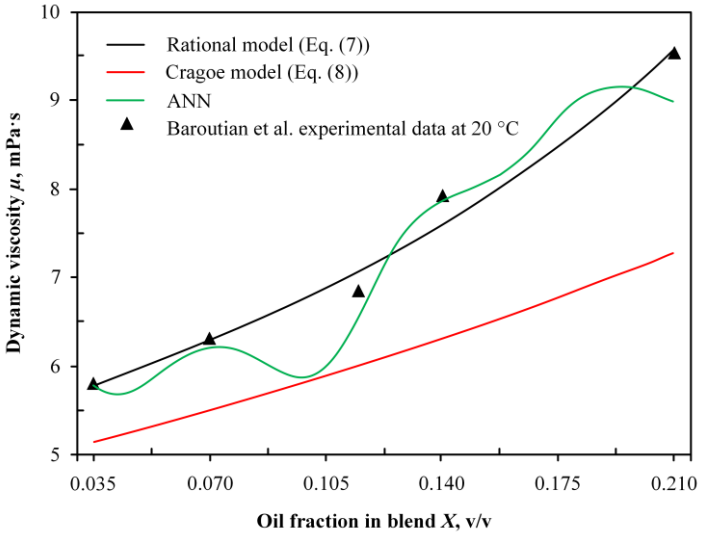

Fig. 9. Variation in dynamic viscosity vs. oil fraction at 20 °C (for high oil content).

TABLE 7. VISCOSITY DATA OF BLENDS (HIGH OIL CONTENT) MEASURED BY BAROUTIAN ET AL. [26] AT DIFFERENT TEMPERATURES, RELATIVE ERRORS, AND REGRESSION PARAMETERS

Oil volume fraction X, %	Measured μ, mPa·s							
	Temperature T, °C							
	20	30	40	50	60	70	80	90
3.5	5.7813	4.4756	3.5490	2.8118	2.2405	1.8586	1.5477	1.4737
7.0	6.2892	4.8073	3.8095	3.0724	2.4773	2.0717	1.7846	1.6158
11.5	6.8397	5.4046	4.3121	3.4987	2.9037	2.3796	2.0688	1.7816
14	7.8987	6.0756	4.7044	3.7776	3.0879	2.5965	2.2856	1.9411
21	9.5186	7.3651	5.5148	4.3749	3.5668	2.9717	2.5661	2.2790

TABLE 7 (CONTINUED)

Temperature T, °C	Eq.	Regression constants			R
		a	b	c	
20	(7)	$6.175e^{6}$	$1.149e^{6}$	$-2.401e^{6}$	0.9915
30		$9.994e^{-1}$	$2.480e^{-1}$	$-4.008e^{-1}$	0.9963
40		$5.048e^{-1}$	$1.585e^{-1}$	$-1.391e^{-1}$	0.9979
50		$2.864e^{-1}$	$1.155e^{-1}$	$-1.077e^{-2}$	0.9989
60		$1.731e^{-1}$	$9.101e^{-2}$	$7.698e^{-2}$	0.9987
70		$1.770e^{-1}$	$1.120e^{-1}$	$8.512e^{-2}$	0.9983
80		$9.845e^{-2}$	$8.030e^{-2}$	$1.873e^{-1}$	0.9972
90		$5.833e^{-1}$	$4.343e^{-1}$	$-4.122e^{-1}$	0.9987

TABLE 7 (CONTINUED)

Eq.	Relative errors, %				
	Oil volume fraction X, %				
	3.5	7.0	11.5	14	21
(7)	0.2943	0.0930	2.9291	2.9742	0.6110
	1.2190	1.1409	2.1231	2.2675	0.2288
	0.9974	1.4271	0.8642	1.4118	0.2520
	0.7115	1.0932	0.4086	0.9793	0.2006
	0.8786	1.7972	0.6453	0.3846	0.2159
	0.7957	1.0744	0.7561	1.4763	0.2720
	0.7262	1.0490	1.3121	2.0605	0.4755
	0.0754	0.2778	1.3063	1.0537	0.1016

Eq.	Relative errors, %				
	Oil volume fraction *X*, %				
	3.5	7.0	11.5	14	21
(8)	10.9687	12.4752	12.1369	20.0577	23.5901
	8.4375	8.8890	11.5897	17.3983	21.6021
	11.6234	11.6349	14.3603	17.2879	18.0458
	8.9654	10.4829	13.6303	15.6365	15.1570
	5.4577	8.4115	14.4985	15.4037	15.2745
	3.8417	7.8966	12.6373	15.9650	15.6636
	0.9694	8.4436	14.1230	18.5060	16.8962
	8.3234	10.9825	12.3688	15.7596	18.0870
ANN	0.2244	1.0478	4.2785	0.5139	5.4458
	0.4527	13.1163	0.1462	0.0382	0.0251
	0.2573	5.3499	0.4165	0.2486	0.3897
	8.4737	0.6704	0.5437	0.4170	0.7998
	1.6703	0.5564	6.7336	0.3651	0.4171
	1.1361	0.3927	0.2709	0.0989	0.0547
	1.6499	3.0356	0.1942	0.0119	0.1183
	2.1403	5.9219	0.3237	0.0470	6.5425

4. CONCLUSIONS

In this study, in order to research the impacts of temperature and oil fraction in blend on the changes of viscosities and densities and the variation in density with respect to temperature, densities and kinematic viscosities of waste cooking oil biodiesel (ethyl ester)-diesel fuel-waste cooking oil ternary blends were measured by the authors at different temperatures (278.15–368.15 K). The density and kinematic viscosity data were correlated by means of rational and exponential models previously given in the authors' studies [15], [23], [24]. Moreover, the models were tested against different density and viscosity data measured by Nogueira et al. [25] and Baroutian et al. [26] by comparing Gupta et al. model, linear model, Cragoe model and ANN recommend before in the existing literature. The rational and exponential equations suggested by the authors can be used to obtain input data for spray, combustion, engine performance and exhaust emission models in thermodynamic or computational fluid dynamic analysis software (KIVA, Fluent, etc.), which shows scientific applicability of the equations. Moreover, the reliable density and viscosity data computed from these equations are required to properly design injection systems, pumps, injectors, reactors, distillation units, separation process, storage tanks and process piping, which indicate their practical applicability. The following conclusions can be deduced from this study:

- The kinematic viscosity and density values of vegetable oil biodiesel-diesel fuel-vegetable oil ternary blends measured by the authors non-linearly and quadratically decrease, respectively, according to the increasing fuel temperature;
- The rational models [23], [24] as a function of temperature and oil blending ratio were developed to estimate viscosity values of different biodiesel-diesel fuel-vegetable oil ternary blends, and it was determined that the predicted values from rational models are

in best agreement with the experimental data measured by the authors, Nogueira et al. [25] and Baroutian et al. [26] compared to Gupta et al. model, Cragoe model and ANN. The rational based models proposed by the authors are formally different from exponential based Gupta et al. and Cragoe models;

- The best estimation of density is obtained from the exponential model [15] as a function of temperature for different biodiesel-diesel fuel-vegetable oil ternary blends measured by the authors and Nogueira et al. [25], compared to linear model;
- Generally, since prediction capability of ANN depends on whether sufficient data are available or not, the predicted values from ANN have high errors for some measurement points. This result can be put forward as a disadvantage of ANN;
- This study shows that the exponential and rational models, previously suggested to estimate density and viscosity of biodiesel-diesel binary blends and biodiesel-diesel-alcohol ternary blends in the authors' previous studies [15], [23], [24], are also the best predictor for estimating the fuel properties of different biodiesel-diesel-vegetable oil ternary blends, which can be said to be an advantage of the models over different regression models and ANN approach.

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Mert Gulum received B. Sc. and Master's degree in Mechanical Engineering at Karadeniz Technical University in 2010 and 2014, respectively. He has been working at Karadeniz Technical University as PhD candidate research assistant since 2012. His research areas include biodiesel production, combustion, fuels, diesel engines and computational fluid dynamic. Some previous publications include:

1. Gulum M., Bilgin A. Measurements and empirical correlations in predicting biodiesel-diesel blends' viscosity and density. *Fuel* 2017:199:567–77. doi:10.1016/j.fuel.2017.03.001
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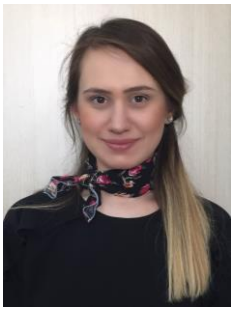
ORCID: http://orcid.org/0000-0002-1792-3499



Funda Kutlu Onay received B. Sc. and M. Sc degrees from the Karadeniz Technical University (KTU), Computer Engineering Department, in 2011 and 2013 respectively. She is currently PhD student at the same university and research assistant in Computer Engineering Department of Amasya University. Her research interests are biomedical signal processing, pattern recognition and human-computer interfaces. Some previous studies as follows:

1. Kutlu Onay F., Kose C. Classification of Functional Near-Infrared Imagining Based Hemodynamic Patterns Recorded at Mental Arithmetic and Resting. *Engineering Sciences* 2018:13(1):27–36.
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Atilla Bilgin received Master's and PhD degree in Mechanical Engineering at 1988 and 1994 from Karadeniz Technical University, respectively. He is working at Karadeniz Technical University as Full Professor Doctor. His research areas include combustion, fuels, engines, computational fluid dynamic, exergy analysis, thermodynamic cycle analysis. Some previous publications are:

1. Cakmak A., Bilgin A. Exergy and energy analysis with economic aspects of a diesel engine running on biodiesel-diesel fuel blends. *International Journal of Exergy* 2017:24(2–4):151–72. doi:10.1504/IJEX.2017.087700
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