

# Measuring Inefficiency of the Czech Labour Market<sup>1</sup>

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## Abstract:

This paper aims to quantify the performance of the Czech regional labour markets and to reveal the most influential economic factors standing behind its dynamics in the last fifteen years. Investigated labour markets are described using matching function approach. The successful matches are treated as an output of production process, where the unemployed are paired with vacancies. Efficiency of this matching process plays an important role in determining unemployment outflows. Using stochastic frontier model approach, dynamics of quantified efficiency terms is revealed and differences among regions are evaluated. The model specification includes a fixed effect term, where individual effect terms and inefficiency terms are estimated jointly. The stochastic frontier is estimated using monthly and quarterly regional panel data of 77 districts for the period 1999-2014. Matching efficiency of the Czech regional labour markets is negatively influenced people who have been unemployed for a long time and by the unemployed aged over 50 years. Although all districts were able to operate at their stochastic frontiers of matching, an upward trend in the inefficiency has been found within the investigated period. These tendencies are accompanied by rising disparities among the regions. Low levels of estimated matching inefficiency do not necessary mean the low unemployment in the corresponding districts.

**Key words:** Matching efficiency, Matching function, Regional labour markets, Stochastic frontier model, Panel data, Czech Republic

**JEL Classification:** R23, J41, C23, E24

## Introduction

Labour market in the Czech Republic has experienced significant changes in the last fifteen years. General unemployment rate of people from 15 to 64 years of age, which is published by the Czech statistical office, reached 7.9 percent in January 1999, and 6 percent at the end of November 2014. During this period, one could observe the unemployment rate of 4.3 percent in the months of 2008 or 9.3 percent in 2000. Of course, the most important part of this variability may be explained by business cycles. But on the other hand, strong disparities have appeared in unemployment rates of the Czech

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regions and districts in this period. As an illustration, let us take a look at regional statistics provided by the Czech Ministry of Labour and Social Affairs. In 2005, the average unemployment rate (computed as a ratio of the unemployed to the population of people from 15 to 64 years of age) in the Czech Republic was 6.59 percent but the regional unemployment rates were 2.64 percent in Praha, 3.85 percent in Beroun, 5.37 percent in the district of Domažlice, 10.3 percent in Hodonín, and 16.49 percent in Most. In the year 2013, unemployment rates of 5.14 percent in Praha, 6.78 percent in Beroun, 6.42 percent in Domažlice, 11.81 percent in Hodonín, and finally, 13.51 percent in Most, can be observed. The average unemployment rate in the Czech Republic was 8.17 percent in 2013. The differences in regional unemployment rates are evident. Moreover, the relative distance in the unemployment measures have been changing as well.

How to explain the differences in regional unemployment rates in the Czech Republic? The answer to this question may be connected to the problem of efficiency of labour markets. Labour market efficiency is one of the most important factors influencing labour market dynamics and its performance. There are many approaches of how to deal with the 'efficiency' concept. Most of them are based on the matching function framework. In this framework, successful labour market matches are treated as an outcome of interactions between unemployed job seeker and vacancies.

The main goal of this article is to quantify the inefficiency of the Czech regional labour markets and to evaluate its development in the last 15 years. Finding the main sources standing behind the regional disparities in efficiency is one of the highly relevant tasks in labour market analysis that allow us to answer many important questions: Are labour markets with higher average unemployment rate less efficient than those with low unemployment rate? What are the effects of unemployment benefits, age structure of unemployed and the length of unemployment on the efficiency? What about the impact of overall economic growth on the labour markets performance?

All the questions mentioned previously may be answered using the estimates of the matching efficiency of the Czech regional labour market. The degree of efficiency, or inefficiency, to be more precise, is estimated using the stochastic frontier panel data model approach with monthly district-level regional data and explicitly treated fixed effects term in the matching function model equation. On the one hand, this approach extends the previous investigations of the efficiency of the Czech labour market carried out by Němec (2013a), Němec (2013b) or by Tvrdůň and Verner (2012). Their results have been based on aggregate labour market statistics. On the other hand, using the data from monthly regional labour market statistics and stochastic frontier panel data model methodology offers a new insight into the outcomes of the Czech labour market in the last 15 years, and extends the detailed analysis of Galuščák and Münich (2007) in a specific way - by dealing with efficiency issues.

Stochastic frontier model approach has been used by Ilmakunnas and Pesola (2003) in their study of regional labour markets in Finland. They used annual data and did not take into account explicitly possible individual fixed effects of the examined regions. Gorter et al. (1997) investigated the efficiency in the Dutch labour market in the Netherlands along the same lines. They observed that the estimated labour market efficiency increases during recession and recovery periods and decreases during economic booms. This interesting feature is considered in this article as well.

The main contribution of this article consists in quantification of inefficiency of the Czech regional labour, evaluating its changes in the last 15 years, and in identification of the main sources of estimated inefficiency disparities among the regions. Using the stochastic frontier model approach, the districts are classified regarding their inefficiency patterns. Matching function parameters and the inefficiency terms are estimated jointly. Inefficiency terms are treated as a function of district specific labour markets factors and common economic factors (like overall economic activity). Robustness of results is checked using the models with monthly and quarterly regional panel data from 1999 to 2014 and providing the estimates on the full sample and the samples covering the periods before and after the economic slowdown starting in 2008.

The structure of this article is as follows: the first section gives a short overview of matching function approach to analyse labour market dynamics. It provides a short introduction to efficiency analysis as well. Stochastic frontier model with individual effects is explained in more detail in the second part of this article. The third part contains data description and methodology of model estimates. The models are then estimated in the fourth part of the paper and the results are interpreted. The final section concludes.

### Matching Efficiency of the Regional Labour Markets

The matching function expresses the interaction mechanism between the unemployed and vacancies. This concept is based on the fact that both the flows of unemployed and the flows of unfilled job vacancies are able to meet each other. The matching function can be thus viewed as a standard production function with two inputs: the unemployed and the vacancies. New matches are an outcome of this matching process. In this contribution, the regional labour markets are represented by a Cobb-Douglas matching function in log-linear form:

$$\log M_{it} = \alpha_i + \beta_{\log U} \log U_{it} + \beta_{\log V} \log V_{it} + \epsilon_{it}, \quad (1)$$

where the subscripts  $i = 1, \dots, N$  denote the districts and  $t = 1, \dots, T$  represents the time period. Parameters  $\alpha_i$  are treated as the district specific fixed effects. The term  $\epsilon_{it}$  is a stochastic factor discussed below. The number of successful matches,  $M_{it}$ , is influenced by the number of unemployed,  $U_{it}$ , and by the number of unfilled vacancies,  $V_{it}$ . Parameters  $\beta_{\log U}$  and  $\beta_{\log V}$  denote the matching elasticity of unemployed and matching elasticity of vacancies respectively.

Due to the fact that the matching function can be perceived as a production function, we are able to measure the efficiency of this production process. We can suppose that the firms (or production units) maximise the output for a given level of inputs and available production technology. Any deviations from the optimal production may thus indicate inefficiency in the production process. The empirical problem is how to estimate the production frontiers and the possible inefficiency given observable data. In general, frontier analysis follows the idea that there exists a function mapping inputs and outputs resulting to the set of all optimal production plans. We can denote them as the production possibility frontier. After estimating it we could evaluate the inefficiency as the deviations of observable production from the potential level.

Treating the production possibility frontier as a deterministic frontier is connected with data envelopment analysis or free disposable hull technique. There is no possibility of observations exceeding the frontier (efficiency above 100 percent). These non-parametric techniques are not robust with regards to the outliers. On the other hand, stochastic frontier analysis is one of the stochastic parametric methods allowing the presence of super-efficient observations. It is thus robust to outliers. Using stochastic frontier analysis (model), we try to find the highest achievable level of production where most of observations may be found below the estimated production frontier. Estimated differences are thus the corresponding technical inefficiency terms.

The aforementioned form of matching function in the equation (1) may be extended and modified in many ways. Ilmakunnas and Pesola (2003) implemented regional and labour force characteristics directly into the matching function by the means of other explanatory variables. The resulting efficiency was thus a linear function of regional fixed effects and various regional characteristics. In their view, the term  $\epsilon_{it}$  was treated purely as white noise process. Similar approach may be found in the work of Gorter et al. (1997). Galuščák and Münich (2007) enhanced the basic matching function form by the flow factors (i.e. unemployment and vacancy inflows realized during the time period).

Stochastic frontier model approach tries to model the stochastic term  $\epsilon_{it}$  as consisting of combination of random variations in the matching process and the region specific inefficiency term. Regional and labour force characteristics are then implemented directly into this inefficiency term. This approach was used by Ilmakunnas and Pesola (2003). But they did not include the fixed (or random) region effects. In this paper, the inefficiency of the Czech regional labour market is estimated using fixed effect panel stochastic model. This model approach is able to capture region specific individual effects, basic matching function characteristics and time-varying regional inefficiency terms at once. Equation (1) could be enhanced by the time effect as well. But, incorporating the time-specific variables, which are common to all regions, might lead to some identification issues. Time-specific effect in the matching function cannot be distinguished easily from the time-specific factors incorporated into the inefficiency term. Time effects are used to control for business cycle dynamics. For these purposes, business cycle variables (like GDP growth or growth of industrial production) are implemented into the inefficiency term.

### **Stochastic Frontier Model with Panel Data**

Panel data models are powerful tools to identify relationships among variables in cases where lack of observations for individual cross-sectional units does not allow obtaining efficient estimates of model parameters. Moreover, using panel data models, we are able to control unobservable individual heterogeneity in our sample.

Working with panel data structure is necessary in stochastic frontier models framework. It is really hard to estimate stochastic frontier within a model using the data for a single cross-sectional unit only. Unobservable individual effects thus play an important role in estimation of panel stochastic frontier model. In many applications, these individual

effects may be connected to inefficiency term. All time-invariant individual heterogeneity across observed cross-sectional units is treated as a part of estimated inefficiency.

The pioneering work on stochastic frontier models (using the last mentioned approach) was written by Aigner et al. (1977). After estimating stochastic frontier, it is possible to conditionally compute inefficiency terms on estimated residuals. These estimates may be used to determine the factors standing behind the inefficiency by the means of ordinary regression. Wang and Schmidt (2002) showed that this two-stage procedure leads to heavily biased results. As a consequence, it is crucial to estimate the stochastic frontier and the determinants of inefficiency simultaneously. Battese and Coelli (1993) specified a model where inefficiency determinants form a part of stochastic frontier model framework. Their framework assumes that all production units face the same production possibility frontier. Greene (2005) suggested an alternative approach where all time-invariant heterogeneity across production units was removed before estimating the stochastic frontier and the inefficiency factors. His approach was capable of correcting the likely overestimated inefficiency terms resulting from the methodology of Battese and Coelli (1993). Amsler et al. (2009) offer a detailed review of the history and development of stochastic frontiers models.

In this article, we take advantage of an alternative approach that is used to remove cross-sectional heterogeneity from the estimates of inefficiency. Wang and Ho (2010) proposed a consistent methodology to deal with individual effects and efficiency terms separately. This approach is used in presented paper. Wang and Ho (2010) specify a stochastic frontier model as follows:

$$y_{it} = \alpha_i + \mathbf{x}_{it}\boldsymbol{\beta} + \epsilon_{it}, \quad (2)$$

$$\epsilon_{it} = v_{it} - u_{it}, \quad (3)$$

$$v_{it} \sim N(0, \sigma_v^2), \quad (4)$$

$$u_{it} = h_{it} \cdot u_i^*, \quad (5)$$

$$h_{it} = f(\mathbf{z}_{it}\boldsymbol{\delta}), \quad (6)$$

$$u_i^* \sim N^+(\mu, \sigma_u^2), \quad (7)$$

where  $i = 1, \dots, N$  and  $t = 1, \dots, T$ . In this model specification,  $\alpha_i$  is the individual fixed effect for the unit  $i$ ,  $\mathbf{x}_{it}$  is a  $1 \times K$  vector of explanatory variables,  $v_{it}$  is a random error term with zero mean,  $u_{it}$  is a stochastic variable measuring inefficiency, and  $h_{it}$  is a positive function of a  $1 \times L$  vector of non-stochastic determinants of inefficiency,  $\mathbf{z}_{it}$ . Constant term is excluded from explanatory variables and from determinants of inefficiency. It should be noted that the notation  $N^+$  means truncated normal distribution for positive values only. The realized values of the variable  $u_i^*$  are positive. In case of  $\mu = 0$  the variable  $u_i^*$  follows half-normal distribution.

Wang and Ho (2010) showed how to remove the fixed individual effect from the model. Their procedure allows estimating all the model parameters. Of course, the individual effect term may be recovered from the final parameter estimates. Wang and Ho (2010)

presented two possible approaches to model transformation: first-differencing and within-transformation. Both methods are asymptotically equivalent, which was proven by Wang and Ho (2008). First-differencing and within-transformation are standard methods used in panel models applications to remove individual effects before estimating the key model parameters. But, these approaches are more complicated in nonlinear models like stochastic frontier models.

The first-differencing approach is applied in the empirical part of this article. It may be thus useful to discuss this method in greater detail. As the first step, one first has to define differences of corresponding variables as  $\Delta w_{it} = w_{it} - w_{it-1}$  and the stacked (column) vector of  $\Delta w_{it}$  for given  $i$  and  $t = 2, \dots, T$  as  $\tilde{\mathbf{w}}_i = (\Delta w_{i2}, \Delta w_{i3}, \dots, \Delta w_{iT})'$ . Assuming that the function  $h_{it}$  is not constant, i.e. the vector  $\mathbf{z}_{it}$  contains at least one time-varying variable, the model in its first-difference form may be expressed as:

$$\tilde{\mathbf{y}}_i = \tilde{\mathbf{x}}_i \boldsymbol{\beta} + \Delta \tilde{\boldsymbol{\epsilon}}_i, \tag{8}$$

$$\tilde{\boldsymbol{\epsilon}}_i = \tilde{\mathbf{v}}_i - \tilde{\mathbf{u}}_i, \tag{9}$$

$$\tilde{\mathbf{v}}_i \sim MN(0, \boldsymbol{\Sigma}), \tag{10}$$

$$\tilde{\mathbf{u}}_i = \tilde{\mathbf{h}}_i u_i^*, \tag{11}$$

$$u_i^* \sim N^+(\mu, \sigma_u^2), \tag{12}$$

where  $i = 1, \dots, N$ . First-differencing procedure, described by the equations (8)-(12), leads to the correlations of differenced error terms in many applications of panel data models. As an example, one can consider the dynamic panel data model and Arellano-Bond estimator derived by Arellano and Bond (1991). In the case of the first-differencing approach proposed by Wang and Ho (2010), one can observe correlations of  $\Delta v_{it}$  within the  $i$ th panel. Resulting covariance matrix of the multivariate normal distribution of  $\tilde{\mathbf{v}}_i = (\Delta v_{i2}, \dots, \Delta v_{iT})'$  is

$$\boldsymbol{\Sigma} = \begin{bmatrix} 2\sigma_v^2 & -\sigma_v^2 & 0 & \dots & 0 \\ -\sigma_v^2 & 2\sigma_v^2 & -\sigma_v^2 & \dots & 0 \\ 0 & -\sigma_v^2 & 2\sigma_v^2 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & -\sigma_v^2 \\ 0 & 0 & \dots & -\sigma_v^2 & 2\sigma_v^2 \end{bmatrix}.$$

This  $(T - 1) \times (T - 1)$  matrix has the elements  $2\sigma_v^2$  on the diagonal and  $-\sigma_v^2$  on the off-diagonals. Knowing the exact form of the covariance matrix is essential for the efficiency of maximal likelihood estimates discussed later. Moreover, one does not have to use any kind robust covariance matrix to correct final estimates and their standard errors.

The main point of Wang and Ho (2010) is that the truncated normal distribution of the part of inefficiency term,  $u_i^*$ , is not affected by this transformation. This fact allows us to derive the likelihood function of the model. To be more specific, the marginal log-likelihood function for the  $i$ th cross-sectional unit is

$$\begin{aligned} \ln L_i = & -\frac{1}{2}(T-1)\ln(2\pi) - \frac{1}{2}\ln(T) - \frac{1}{2}(T-1)\ln(\sigma_v^2) \\ & - \frac{1}{2}\tilde{\epsilon}_i'\Sigma^{-1}\tilde{\epsilon}_i + \frac{1}{2}\left(\frac{\mu_*^2}{\sigma_*^2} - \frac{\mu^2}{\sigma_u^2}\right) + \ln\left(\sigma_*\Phi\left(\frac{\mu_*}{\sigma_*}\right)\right) \\ & - \ln\left(\sigma_u\Phi\left(\frac{\mu}{\sigma_u}\right)\right), \end{aligned} \tag{13}$$

where

$$\begin{aligned} \mu_* &= \frac{\frac{\mu}{\sigma_u} - \tilde{\epsilon}_i'\Sigma^{-1}\tilde{h}_i}{\tilde{h}_i'\Sigma^{-1}\tilde{h}_i + 1/\sigma_u^2}, \\ \sigma_*^2 &= \frac{1}{\tilde{h}_i'\Sigma^{-1}\tilde{h}_i + 1/\sigma_u^2}, \\ \tilde{\epsilon}_i &= \tilde{y}_i - \tilde{x}_i\beta. \end{aligned}$$

As for the notation,  $\Phi(\cdot)$  is the cumulative distribution function of a standard normal distribution. Log-likelihood function of the model may be obtained by summing the above function over all cross-sectional-units,  $i = 1, \dots, N$ . The model parameters are estimated by maximizing the log-likelihood function of the model.

For practical purposes, one wishes to estimate observation-specific technical inefficiency. Wang and Ho (2010) approximated this kind of inefficiency as a conditional expectation  $E(u_{it}|\tilde{\epsilon}_i)$  evaluated at estimated values of  $\tilde{\epsilon}_i$ :

$$E(u_{it}|\Delta\tilde{\epsilon}_i) = h_{it} \left[ \mu_* + \frac{\phi\left(\frac{\mu_*}{\sigma_*}\right)\sigma_*}{\Phi\left(\frac{\mu_*}{\sigma_*}\right)} \right],$$

where  $\phi(\cdot)$  represents the density function of standard normal distribution. This estimator is a modified estimator of inefficiency terms which uses differenced error terms stacked into the vector  $\Delta\tilde{\epsilon}_i$ , instead of  $\epsilon_{it}$  as the conditional term. The original estimator of inefficiency based on model residuals  $\epsilon_{it}$  was derived by Jondrow et al. (1982).

The main advantage of the modified approach to computing inefficiency lies in the fact that the vector  $\tilde{\epsilon}_i$  contains all information of individual unit in the sample and does not depend on individual effect term,  $\alpha_i$ . Wang and Ho (2010) argue that the individual effect term has the variance of higher order in the case of small time dimension of the sample (variance of order  $1/T$ ) in comparison to the variance of  $1/((N-1)T)$  for parameters estimator,  $\hat{\beta}$ . The modified estimator of inefficiency is based on the parameter estimates  $\hat{\beta}$ , and is thus more efficient than the standard one. Wang and Ho (2010) derived the expression for individual fixed effects terms. However, the modified version of inefficiency estimator does not contain the individual effect terms which can be thus omitted in practical applications. Technical efficiency may be obtained in accordance with other studies. Battese and Coelli (1993) and Battese and Coelli (1995) proposed the expression  $\exp(-u_{it})$ .

## Data and Methodology

The stochastic frontier model of the Czech regional labour markets is estimated using the monthly and quarterly data set covering a sample of 77 districts from January 1999 to June 2014. When compared to models of other authors, the models proposed estimate inefficiency of the labour markets using “high” frequency data set. Galuščák and Münich (2007) worked with quarterly Czech regional data only, Iilmakunnas and Pesola (2003) and Gorter et al. (1997) focused on annual data of regions in Finland and the Netherlands, respectively. The reason is that aggregation may lead to some loss of information. Moreover, proving the relationship among the variables using the monthly data should provide us with more efficient estimates.

The original labour market data come from the database of the Ministry of Labour and Social Affairs (MLSA). This database covers the monthly and quarterly data from regional Employment offices. In our empirical analysis, the models are estimated using three groups of data types. The first group covers the core data for matching function specification:

- Number of registered successful matches for each district in the corresponding month (source MLSA);
- Number of registered unemployed at the beginning of the month (source MLSA);
- Number of registered vacancies at the beginning of the month (source MLSA).

The second group of the data represents the district specific labour market fundamentals:

- Number of registered unemployed receiving the unemployment benefits in the corresponding month (source MLSA);
- Number of registered unemployed of age 50 and older in the corresponding quarter (source MLSA);
- Number of the registered unemployed who have been unemployed for more than 12 months in the corresponding quarter (source MLSA).

The third data group captures the overall economic conditions and economic activity in the Czech Republic:

- Index of industrial production (base year 2010 = 100, source Czech National Bank and International Financial Statistics);
- Quarterly real gross domestic product (source Czech National Bank).

All the data are seasonally unadjusted. Seasonal pattern of the variables constitutes an integral part of the stochastic frontier model. Treating the seasonality as a source of inefficiency is in accordance with the econometric methodology advocated by Kalman (1979). In his opinion, an adequate model should incorporate all behavioural aspects of the data. From this point of view, seasonal adjustment of the variables done by researcher outside model framework may be misleading and may influence the final results. Seasonal behaviour is removed by averaging using the final model estimates. In this way, one can obtain overall tendencies in inefficiency development.

Stochastic frontier models are estimated using monthly and quarterly data. Number of successful matches and number of vacancies are computed as monthly averages due to

lack of quarterly counterparts of these statistics. Time series of the index of industrial production is based on index provided by the Czech National Bank. Only values starting in 2000 were available. The values for 1999 were thus computed using the industrial production index provided by International Monetary Fund in its International Financial Statistics.

All the district specific labour market variables are expressed relatively to the pool of unemployed people in the corresponding month or quarter. It means that ratios of these variables have been used. Models consist of quarterly dummies as well. The first quarter of the year is the basic quarter (category).

The estimates are carried out using the monthly and quarterly data. To be more specific, the models are formulated in accordance with the equations (2)-(7) as:

$$\begin{aligned}
 y_{it} &= \log M_{it}, \\
 \mathbf{x}_{it} &= (\log U_{it}, \log V_{it}), \\
 \boldsymbol{\beta} &= (\beta_{\log U}, \beta_{\log V}), \\
 h_{it} &= |\delta_{Benefit} Benefit_{it} + \delta_{\Delta IPP} \Delta IPP_t \\
 &\quad + \delta_{\Delta IPP_{-1}} \Delta IPP_{t-1} + \delta_{Age50+} UAge50_{it} \\
 &\quad + \delta_{Ulong} ULong_{it} + \delta_{Q2} Q_{2t} + \delta_{Q3} Q_{3t} + \delta_{Q4} Q_{4t} |
 \end{aligned}$$

for the model with monthly data, and

$$\begin{aligned}
 h_{it} &= |\delta_{Benefit} Benefit_{it} + \delta_{\Delta GDP} \Delta GDP_t \\
 &\quad + \delta_{\Delta GDP_{-1}} \Delta GDP_{t-1} + \delta_{Age50+} UAge50_{it} \\
 &\quad + \delta_{Ulong} ULong_{it} + \delta_{Q2} Q_{2t} + \delta_{Q3} Q_{3t} + \delta_{Q4} Q_{4t} |
 \end{aligned}$$

for the model with quarterly data. The specification of inefficiency function  $h_{it}$  does not contain time trend. Unlike the specification of inefficiency function  $h_{it}$  proposed by Němec (2014a) and Němec (2014b), no time trend is used and instead of that, regional labour market specifics are implemented. This approach should provide a more reliable and detailed view of the sources of labour market inefficiency. The model variables are denoted as follows:

- number of registered successful matches,  $M_{it}$ , in the corresponding month (averaged in the quarterly models);
- number of unemployed at the beginning of the month,  $U_{it}$  (averaged in the quarterly models);
- number of vacancies at the beginning of the month,  $V_{it}$  (averaged in the quarterly models);
- ratio of registered unemployed receiving the unemployment benefits in the corresponding month,  $Benefit_{it}$  (averaged in the quarterly models), to the total unemployed;
- ratio of the registered unemployed of age 50 and older in the corresponding quarter,  $UAge50_{it}$ , to the total number of unemployed people;

- ratio of the registered unemployed who have been unemployed for more than 12 months in the corresponding quarter,  $ULong_{it}$ , to the total number of unemployed people;
- seasonal (quarterly) dummies for the 2<sup>nd</sup>, 3<sup>rd</sup> and 4<sup>th</sup> quarter ( $Q_{2t}, Q_{3t}, Q_{4t}$ );
- monthly growth of the industrial production,  $\Delta IPP_t$ , and quarterly growth of real gross domestic product,  $\Delta GDP_t$ .

Subscripts  $i$  denote a district and  $t$  corresponds to the month or quarter. Moreover, to check the robustness of the results and to evaluate the possible changes in parameters and inefficiency terms, the estimates are based on a full sample of the years from 1999 to 2014, and on the sample covering both the pre-crisis period of 1999-2007 and the period of 2008-2014, respectively. The results will be used for finding the basic tendencies and distributional inefficiency changes among the districts.

As for the estimation techniques, parameter estimates were obtained by nonlinear optimization techniques in Matlab 2013b, applied on the log-likelihood of the models defined as the sum of marginal log-likelihoods from the equation (13). Standard deviation of the parameter estimates are based on inverted negative Hessian of log-likelihood evaluated at the maximal likelihood estimates. All first and second derivatives were computed numerically within the optimization procedure. The algorithm converged very well in all cases.<sup>3</sup> For computational purposes, the variance parameters were parameterised as  $\log \sigma_v^2$  and  $\log \sigma_u^2$  respectively. Parameter  $\mu$  is defined as  $\mu = 0$ . This calibration leads to a half-normal representation of the model.

### Efficiency Estimates of the Czech Regional Labour Markets

Parameter estimates of the matching function and the inefficiency term for the Czech regional labour markets are presented in Table 1 and Table 2. The models have been estimated using the monthly and quarterly data using both the full sample and restricted samples.

The results in Table 1 do not confirm the empirical findings presented by Ilmakunnas and Pesola (2003) who claim that with regional data it may be more likely to find increasing returns in matching. The Czech regional labour market proves the diminishing returns in matching. The elasticity of matches to vacancies,  $\beta_{\log(v)}$ , is extremely low. It means that the vacancy creation is not a sufficient condition to diminish unemployment. This kind of conclusion may be justified by the hypothesis that new vacancies do not correspond to qualification structure of the unemployed. Using the full sample estimates, we reach the value of 0.109. It means that only 10 percent of new monthly vacancies may be matched with the unemployed. This effect is lower using the restricted sample estimates of the pre-crisis period. It seems that the economic slowdown that started at the end of 2009 led to a more efficient utilization of the unfilled vacancies. Estimated elasticity matching to the unemployed,  $\beta_{\log(u)}$ , shows that approximately half of the

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<sup>3</sup> Data and programme files are available from the author upon a request.

new unemployed might be able to find a new job immediately. Comparison of the estimates of 0.569 and 0.522 in the pre-crisis and post-crisis period indicates modest changes in matching elasticity of unemployed. Unfortunately, these changes tend to worsen the matching conditions of regional labour markets.

**Table 1 Parameter estimates – stochastic frontier model (monthly data)**

Parameter	1999 – 2014	1999 – 2007	2008 – 2014
$\beta_{\log(U)}$	0.493 (0.013)	0.569 (0.016)	0.522 (0.026)
$\beta_{\log(V)}$	0.109 (0.005)	0.044 (0.005)	0.083 (0.011)
$\delta_{Benefit}$	-0.338 (0.016)	0.024 (0.012)	-0.365 (0.037)
$\delta_{\Delta IPP}$	-2.874 (0.049)	-3.238 (0.025)	-2.953 (0.090)
$\delta_{\Delta IPP_{-1}}$	-1.987 (0.034)	-1.448 (0.105)	-1.859 (0.086)
$\delta_{U_{Age50+}}$	2.073 (0.055)	1.034 (0.042)	0.526 (0.022)
$\delta_{U_{Long}}$	0.353 (0.029)	0.303 (0.027)	1.748 (0.010)
$\delta_{Q_2}$	-0.380 (0.021)	-0.526 (0.016)	-0.571 (0.007)
$\delta_{Q_3}$	0.081 (0.014)	0.111 (0.005)	0.014 (0.017)
$\delta_{Q_4}$	0.416 (0.013)	0.350 (0.010)	0.472 (0.017)
$\log(\sigma_v^2)$	-2.761 (0.012)	-3.382 (0.016)	-2.458 (0.018)
$\log(\sigma_u^2)$	-0.940 (0.163)	-0.920 (0.162)	-0.911 (0.163)

Source: Own calculations (standard errors in parenthesis, grey shaded parameter estimates are statistically insignificant at 5 percent level of significance).

Inefficiency of the regional labour markets is strongly influenced by region (district) specific factors and common economic factors. The ratio of the registered unemployed receiving unemployment benefits has a positive impact on efficiency (i.e. it reduces inefficiency level) using the post-crisis and full sample data. The value of the corresponding parameter  $\delta_{Benefit}$  means that a ten-percent increase of the ratio of unemployed receiving benefits tends to lower the inefficiency by 3.3 percent for the full sample estimates (we can treat the inefficiency term of 1 as a sign of absolute inefficiency). This surprising result may be explained by stating that these unemployed people are mostly short-term unemployed who are willing to get themselves a job as soon as possible. This behaviour leads to increasing matching creation. As Table 1 suggests, this effect is prevailing in the period of economic slowdown, where the unemployed prefer getting a worse job rather than staying in the pool of the unemployed. It is clear that pre-crisis value of 0.024 suggests the opposite behaviour of the unemployed, which was caused by the fact that the labour market had not been so tight. Of course, the positive influence on inefficiency is not so evident.

The ratio of the registered unemployed of age 50 and older,  $\delta_{U_{Age50+}}$ , and the ratio of the long-term unemployed,  $\delta_{U_{Long}}$ , both have a negative effect on matching efficiency. Using the full sample data shows that the effect of the older unemployed prevails the effect of long-term unemployed. On the other hand, the estimates based on restricted

samples indicate the changes of their relative strength. The predominant role of the older unemployed in the pre-crisis period was replaced by the increasing influence of the long-term unemployed. It is a logical outcome of the retirement process. The older unemployed in the pre-crisis period were leaving the labour market after the economic slowdown in 2008 (and consecutive years). The role of long-term unemployed was thus more important in the determining of labour markets inefficiency. Ten percent increase of ratio of the long-term unemployed may cause 17 percent (0.17) rise in inefficiency.

As for the parameters of economic activity,  $\delta_{\Delta IPP}$  and  $\delta_{\Delta IPP_{-1}}$ , we can see that the marginal effects of the monthly growths of industrial production in the last two periods diminish the matching inefficiency. Positive economic development supports the vacancies creation and lowers the unemployment inflows. These vacancies can be filled immediately. The structure of the skills of the unemployed registered at the employment office seems to be not a problem at the regional level. As pointed out by Polasek and Sellner (2013), the economic growth may be induced by many other factors connected to openness of the Czech economy.

The inefficiency within the year tends to be accompanied by important seasonal patterns, especially by a positive effect on the matching function outcomes in the second quarters. Quarterly dummies show a substantial jump in the last quarter of the year (compared with the first quarter).

Higher variability of the inefficiency term,  $\sigma_u^2$ , in comparison to the white noise process variability,  $\sigma_v^2$  (i.e.  $\sigma_u^2/\sigma_v^2$ ), contributes to the satisfying identification of the stochastic frontier model as stated by Wang and Ho (2010).

**Table 2 Parameter estimates – stochastic frontier model (quarterly data)**

Parameter	1999 – 2014	1999 – 2007	2008 – 2014
$\beta_{\log(U)}$	0.449 (0.018)	0.478 (0.018)	0.404 (0.042)
$\beta_{\log(V)}$	0.054 (0.007)	0.074 (0.006)	-0.001 (0.015)
$\delta_{Benefit}$	-0.287 (0.064)	-0.009 (0.014)	-0.386 (0.019)
$\delta_{\Delta GDP}$	-2.268 (0.496)	-2.050 (0.008)	-1.380 (0.032)
$\delta_{\Delta GDP_{-1}}$	-4.630 (1.043)	-2.489 (0.015)	-5.988 (0.014)
$\delta_{U_{Age50+}}$	0.595 (0.129)	1.713 (0.010)	0.908 (0.073)
$\delta_{U_{Long}}$	0.404 (0.088)	-0.332 (0.020)	0.586 (0.071)
$\delta_{Q_2}$	-0.360 (0.075)	-0.131 (0.013)	-0.704 (0.006)
$\delta_{Q_3}$	0.471 (0.106)	0.397 (0.008)	0.434 (0.008)
$\delta_{Q_4}$	0.346 (0.077)	0.401 (0.013)	0.234 (0.012)
$\log(\sigma_v^2)$	-3.434 (0.021)	-4.573 (0.028)	-3.157 (0.033)
$\log(\sigma_u^2)$	0.634 (0.425)	-0.573 (0.186)	1.178 (0.184)

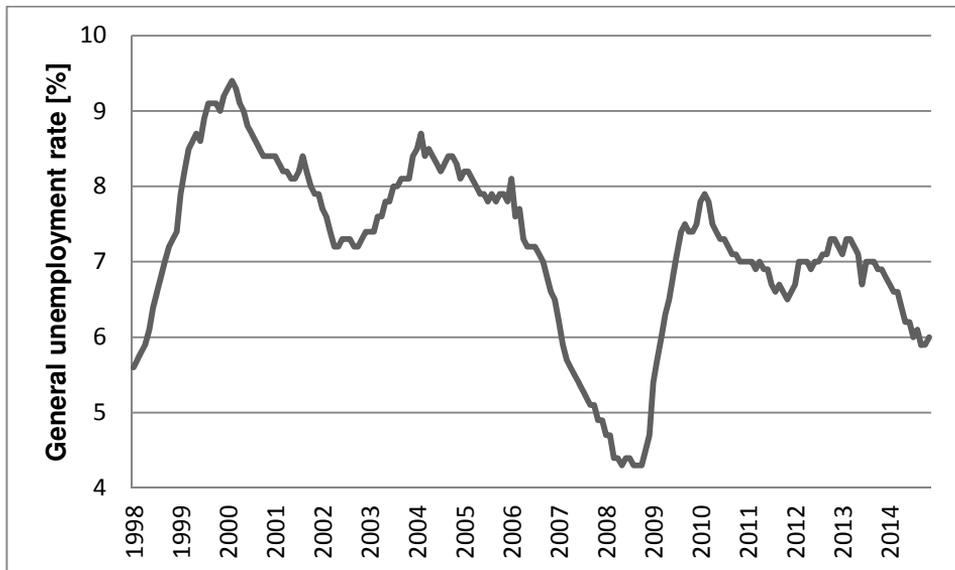
Source: Own calculations (standard errors in parenthesis, grey shaded parameter estimates are statistically insignificant at 5 percent level of significance).

Table 2 shows stochastic frontier model estimates which use quarterly data. The results are very similar with regard to the estimated coefficients signs. As for some remarkable

difference, higher influence of the GDP growth on the inefficiency term can be observed. Moreover, the lagged GDP growth contributes to diminishing inefficiency with a remarkable extent. This property could not be detected using the monthly data only. Compensating changes in the parameter at seasonal dummy,  $\delta_{Q_3}$ , can be viewed as other consequences of the GDP growth effects. The long-term and the older unemployed influence the inefficiency in a more balanced way. There is a surprisingly negative effect of the long-term unemployed on the inefficiency in the period from 1999 to 2007. Unemployment rate declined (see Figure 1) and the labour market experienced an extremely low tightness. As a result, for people who have been unemployed for a long time, but are younger, finding a job was easier than for the older ones. This explanation may be proved by the estimates mentioned above.

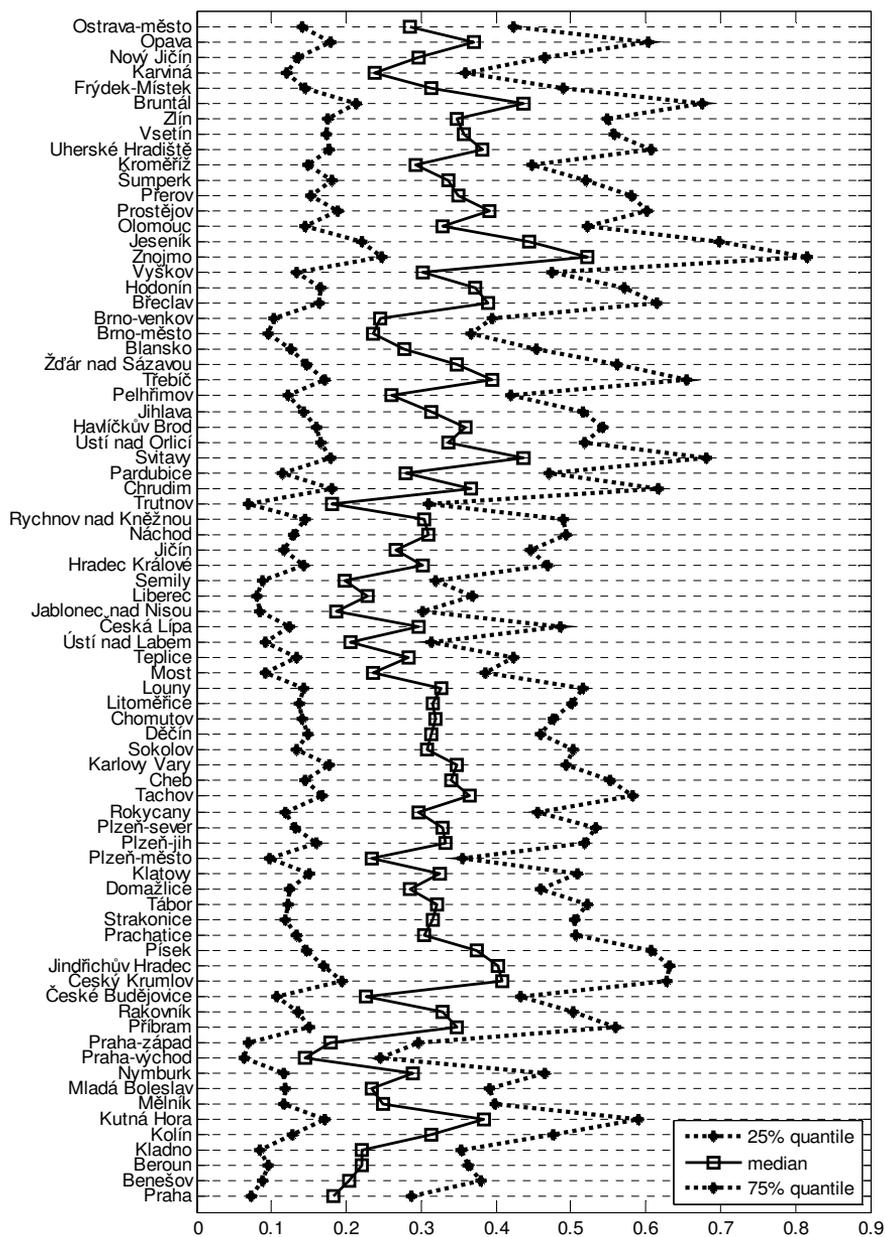
Estimated parameters in stochastic efficiency variability have changed dramatically. Standard values obtained previously using the monthly data may be found for the pre-crisis sample only. Estimates resulting from full data set show statistically insignificant parameter of variance,  $\log(\sigma_u^2)$ . It means that the estimated variance,  $\sigma_u^2$ , could equal one.

**Figure 1 Unemployment rate in the Czech Republic**



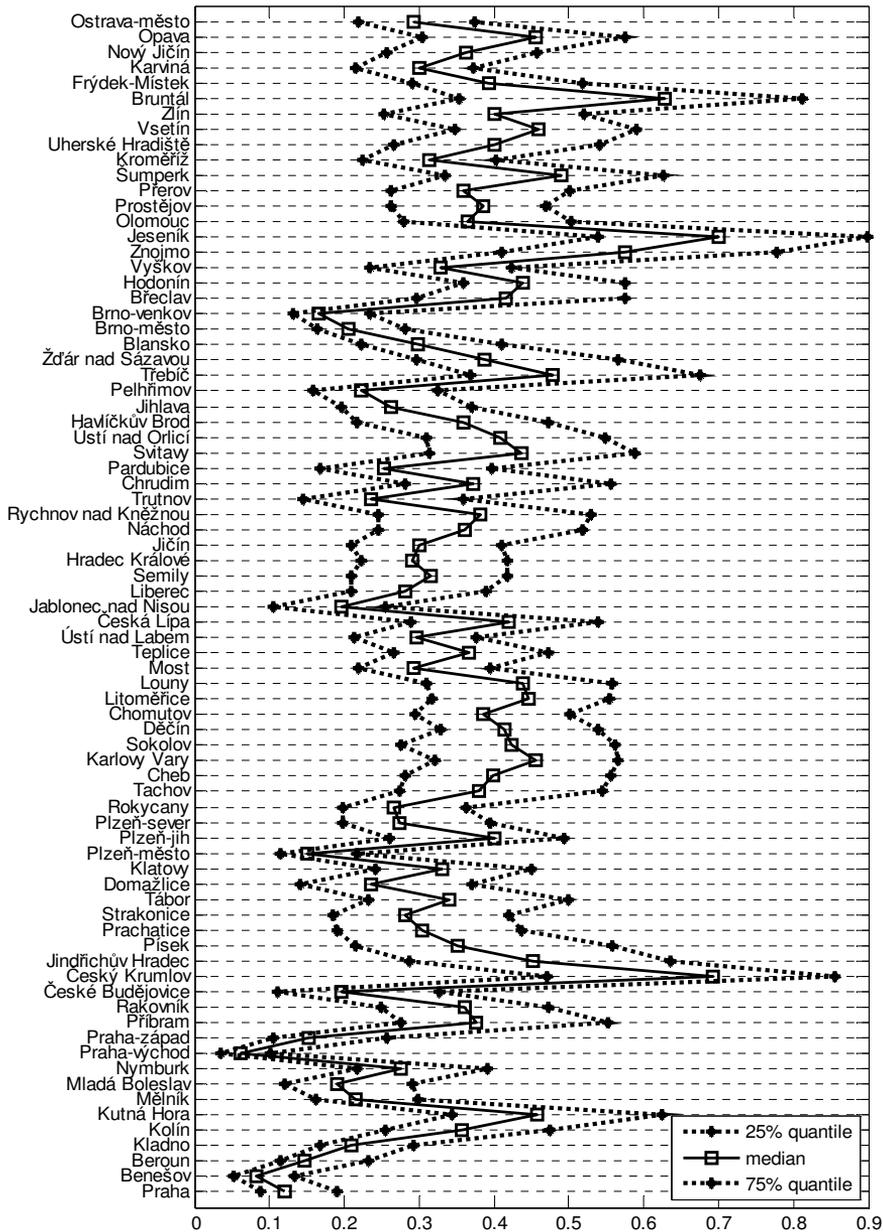
Source: Own calculations.

Figure 2 Inefficiency range (full sample 1999-2014, monthly data)



Source: Own calculations.

Figure 3 Inefficiency range (full sample 1999-2014, quarterly data)



Source: Own calculations.

Estimated distribution of the monthly inefficiency among the Czech districts is depicted in Figure 2. The presented interquartile range of inefficiency terms distributions for all

77 districts is based on the estimates using the full sample period. Minimum inefficiency values for each district (which are not presented here) are almost zero for all investigated labour markets. All investigated districts thus can match the unemployed with the vacancies at the full rate. Of course, it is often caused by seasonal factors in the second quarter of the year. There are some districts with exceptionally good efficiency of matching (e.g. Praha, Benešov, Trutnov or Jablonec nad Nisou) and some districts showing bad efficiency performance (e.g. Jeseník, Znojmo or Bruntál).

It may be surprising that districts such as Karviná or Ústí nad Labem are among the well performing districts, and it is necessary to say that the two districts are not to be classified to come from regions with low unemployment properties. But, it should be stressed that low inefficiency does not automatically mean low unemployment. It expresses the potential for new created matches which can be constituted by the interaction between unemployed and available vacancies.

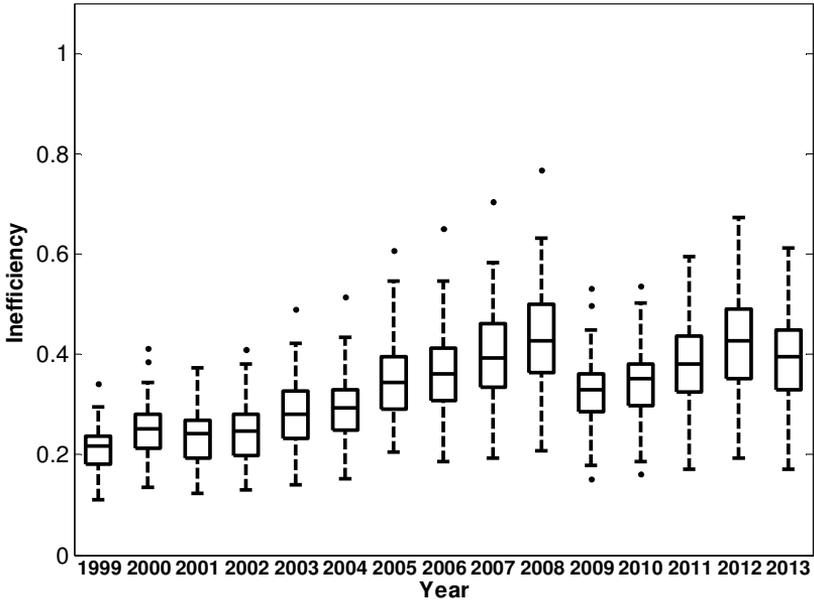
From this point of view, these results imply that the potential of labour market is utilized quite well. There may be an appropriate structure of unemployed people and vacancies, unobserved characteristics of the unemployed support their willingness to actively search for a job, and finally, the surrounding regions may offer other possibilities for employing unemployed job applicants (this spatial dependency is not implemented in estimated models so far). The unfavourable efficiency outcomes of the Jeseník, Znojmo or Bruntál districts may be thus explained in a similar way.

*Similar inefficiency patterns may be found in quarterly estimates which are presented in Figure 3. Quarterly estimates show sharper disparities in inefficiency distribution. The best way to illustrate the development of the labour market inefficiency and its variability is to compute the yearly averages for each district. These aggregated results are presented in Figure 4 and*

Figure 5. The revealed tendencies are almost the same for the monthly and quarterly estimates.

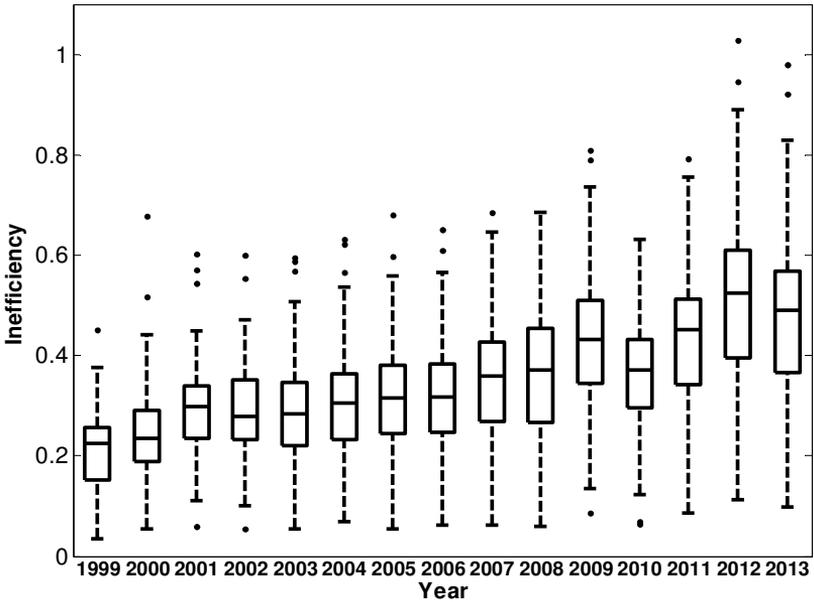
In the period from 1999 to 2007, the Czech regional labour markets may be described as labour markets with rising inefficiency patterns and rising heterogeneity among them. These tendencies are more significant when monthly estimates are used (see Figure 4). When looking at the development of unemployment rate in the Czech Republic, one cannot conclude that rising inefficiency tends to be accompanied by higher unemployment. The increase before 2007 is a result of very low tightness at the Czech labour market. The overall positive economic conditions made it problematic for companies to find appropriate workforce. The structure of unfilled vacancies could not match the structure of the unemployed. 2008 and 2009 are connected with the beginning of the global financial crisis and worldwide economic slowdown. This exogenous shock forced firms to think about their employment policies. Facing the uncertainty of expected length and amplitude of the economic slowdown, they were not willing to dismiss their employees immediately. Instead of that they reduced their vacancy creation. Available stock of vacancies was thus reduced. From this point of view, successful matches operate closer to production frontier (in comparison with the precedent years). The years from 2010 to 2012 reversed the positive efficiency patterns of the matching process due to prevailing economic uncertainty.

**Figure 4 Average inefficiency distributions (full sample 1999-2014, monthly data)**



Source: Own calculations.

**Figure 5 Average inefficiency distributions (full sample 1999-2014, quarterly data)**



Source: Own calculations.

Table 3 and Table 4 show the changes in the inefficiency of Czech labour market in a more compact way. Presented standard deviations illustrate the overall rise of inefficiency disparities among the Czech districts.

**Table 3 Average inefficiency and its variability among districts (monthly data)**

Year	Mean	Std. deviation	Year	Mean	Std. deviation
1999	0.213	0.045	2007	0.395	0.096
2000	0.248	0.054	2008	0.430	0.104
2001	0.239	0.052	2009	0.325	0.072
2002	0.248	0.056	2010	0.342	0.075
2003	0.286	0.066	2011	0.377	0.085
2004	0.293	0.067	2012	0.421	0.097
2005	0.349	0.079	2013	0.390	0.090
2006	0.366	0.085	1999 - 2013	0.328	0.102

Source: Own calculations.

**Table 4 Average inefficiency and its variability among districts (quarterly data)**

Year	Mean	Std. deviation	Year	Mean	Std. deviation
1999	0.210	0.077	2007	0.350	0.126
2000	0.245	0.098	2008	0.361	0.131
2001	0.288	0.103	2009	0.426	0.140
2002	0.290	0.110	2010	0.360	0.117
2003	0.290	0.109	2011	0.429	0.136
2004	0.307	0.110	2012	0.515	0.167
2005	0.314	0.113	2013	0.481	0.156
2006	0.316	0.112	1999 – 2013	0.346	0.148

Source: Own calculations.

The differences across the regions were rising since 1999. We can observe some differences in the results depending on the time frequency of the model. But the basic tendencies are very similar. These results do not indicate that the estimated labour market inefficiency may rise during periods of recession and recovery while it decreases during the economic booms. Regarding the fact that Gorter et al. (1997) used annual data, it should be noted that this contradiction is not conclusive.

**Table 5 Labour market inefficiency across districts (monthly data, yearly averages)**

District	99–13	99–07	08–13	District	99–13	99–07	08–13
Praha	0.193	0.182	0.209	Liberec	0.241	0.221	0.271
Benešov	0.253	0.236	0.278	Semily	0.213	0.188	0.249
Beroun	0.240	0.218	0.272	Hradec Králové	0.314	0.277	0.370
Kladno	0.233	0.212	0.265	Jičín	0.296	0.266	0.340
Kolín	0.323	0.303	0.353	Náchod	0.325	0.286	0.382
Kutná Hora	0.400	0.359	0.462	Rychnov nad Kněžnou	0.333	0.298	0.385
Mělník	0.270	0.244	0.308	Trutnov	0.198	0.179	0.227
Mladá Boleslav	0.264	0.221	0.329	Chrudim	0.412	0.382	0.458
Nymburk	0.309	0.282	0.349	Pardubice	0.305	0.266	0.364
Praha-východ	0.163	0.153	0.177	Svitavy	0.459	0.397	0.553
Praha-západ	0.199	0.185	0.218	Ústí nad Orlicí	0.354	0.320	0.405
Příbram	0.372	0.329	0.437	Havlíčkův Brod	0.371	0.339	0.419
Rakovník	0.343	0.306	0.399	Jihlava	0.341	0.293	0.412
České Budějovice	0.272	0.237	0.324	Pelhřimov	0.287	0.241	0.356
Český Krumlov	0.431	0.398	0.481	Třebíč	0.429	0.385	0.494
Jindřichův Hradec	0.427	0.378	0.501	Žďár nad Sázavou	0.373	0.339	0.424
Písek	0.409	0.359	0.483	Blansko	0.304	0.267	0.360
Prachatice	0.336	0.302	0.387	Brno-město	0.242	0.213	0.285
Strakonice	0.335	0.295	0.395	Brno-venkov	0.264	0.232	0.311
Tábor	0.340	0.291	0.415	Břeclav	0.410	0.354	0.494
Domažlice	0.309	0.258	0.385	Hodonín	0.383	0.343	0.442
Klatovy	0.341	0.303	0.398	Vyškov	0.322	0.286	0.375
Plzeň-město	0.235	0.216	0.262	Znojmo	0.548	0.500	0.619
Plzeň-jih	0.360	0.314	0.427	Jeseník	0.477	0.431	0.547
Plzeň-sever	0.355	0.314	0.417	Olomouc	0.348	0.311	0.402
Rokycany	0.306	0.282	0.343	Prostějov	0.415	0.365	0.489
Tachov	0.395	0.367	0.437	Přerov	0.376	0.320	0.460
Cheb	0.363	0.326	0.419	Šumperk	0.362	0.324	0.420
Karlovy Vary	0.351	0.331	0.382	Kroměříž	0.312	0.280	0.359
Sokolov	0.332	0.293	0.390	Uherské Hradiště	0.405	0.349	0.489
Děčín	0.316	0.280	0.369	Vsetín	0.378	0.325	0.456
Chomutov	0.323	0.292	0.370	Zlín	0.369	0.322	0.439
Litoměřice	0.336	0.306	0.382	Bruntál	0.460	0.418	0.522
Louny	0.343	0.305	0.400	Frýdek-Místek	0.326	0.287	0.385
Most	0.252	0.219	0.300	Karviná	0.245	0.219	0.283
Teplice	0.290	0.274	0.315	Nový Jičín	0.312	0.282	0.356
Ústí nad Labem	0.215	0.196	0.245	Opava	0.404	0.352	0.481
Česká Lípa	0.322	0.292	0.367	Ostrava-město	0.288	0.260	0.329
Jablonec nad Nisou	0.203	0.179	0.239				

Source: Own calculations.

**Table 6 Labour market inefficiency across districts (quarterly data, yearly averages)**

District	99–13	99–07	08–13	District	99–13	99–07	08–13
Praha	0.131	0.111	0.160	Liberec	0.284	0.255	0.328
Benešov	0.102	0.084	0.131	Semily	0.304	0.242	0.397
Beroun	0.159	0.129	0.204	Hradec Králové	0.298	0.251	0.368
Kladno	0.215	0.187	0.256	Jičín	0.301	0.240	0.392
Kolín	0.350	0.308	0.412	Náchod	0.369	0.299	0.473
Kutná Hora	0.466	0.408	0.553	Rychnov nad Kněžnou	0.386	0.312	0.497
Mělník	0.219	0.188	0.267	Trutnov	0.250	0.209	0.311
Mladá Boleslav	0.196	0.150	0.264	Chrudim	0.395	0.334	0.486
Nymburk	0.279	0.238	0.341	Pardubice	0.276	0.222	0.357
Praha-východ	0.069	0.059	0.085	Svitavy	0.447	0.361	0.578
Praha-západ	0.178	0.147	0.225	Ústí nad Orlicí	0.420	0.362	0.507
Příbram	0.398	0.325	0.508	Havlíčkův Brod	0.349	0.295	0.430
Rakovník	0.356	0.282	0.466	Jihlava	0.274	0.217	0.359
České Budějovice	0.232	0.176	0.314	Pelhřimov	0.234	0.177	0.320
Český Krumlov	0.652	0.570	0.774	Třebíč	0.490	0.417	0.600
Jindřichův Hradec	0.464	0.379	0.590	Žďár nad Sázavou	0.412	0.341	0.519
Písek	0.401	0.306	0.543	Blansko	0.305	0.246	0.392
Prachatice	0.317	0.259	0.405	Brno-město	0.214	0.183	0.260
Strakonice	0.300	0.241	0.389	Brno-venkov	0.171	0.136	0.222
Tábor	0.357	0.278	0.475	Břeclav	0.421	0.342	0.539
Domažlice	0.249	0.175	0.361	Hodonín	0.451	0.399	0.529
Klatovy	0.340	0.273	0.440	Vyškov	0.322	0.269	0.402
Plzeň-město	0.158	0.133	0.195	Znojmo	0.588	0.499	0.722
Plzeň-jih	0.387	0.310	0.503	Jeseník	0.684	0.595	0.816
Plzeň-sever	0.285	0.233	0.363	Olomouc	0.371	0.322	0.446
Rokycany	0.266	0.232	0.317	Prostějov	0.372	0.306	0.470
Tachov	0.392	0.329	0.486	Přerov	0.376	0.301	0.488
Cheb	0.406	0.333	0.515	Šumperk	0.490	0.421	0.594
Karlovy Vary	0.453	0.415	0.509	Kroměříž	0.302	0.263	0.360
Sokolov	0.416	0.346	0.521	Uherské Hradiště	0.407	0.335	0.515
Děčín	0.423	0.364	0.511	Vsetín	0.453	0.387	0.551
Chomutov	0.390	0.331	0.479	Zlín	0.390	0.335	0.472
Litoměřice	0.438	0.382	0.523	Bruntál	0.604	0.516	0.736
Louny	0.423	0.371	0.502	Frýdek-Místek	0.391	0.337	0.472
Most	0.297	0.251	0.366	Karviná	0.297	0.266	0.343
Teplice	0.366	0.325	0.426	Nový Jičín	0.352	0.312	0.411
Ústí nad Labem	0.293	0.256	0.348	Opava	0.445	0.372	0.555
Česká Lípa	0.412	0.358	0.493	Ostrava-město	0.289	0.256	0.338
Jablonec nad Nisou	0.189	0.159	0.233				

Source: Own calculations.

Table 5 and Table 6 contain a detailed view on average inefficiency estimates for all Czech districts. The aggregate regional inefficiency changes are presented in a straightforward way. All investigated districts have experienced the rise in their inefficiency. A comparison of the inefficiency estimates to the unemployment rates does not confirm a direct connection between unemployment rate and inefficiency of the matching process.

In the introductory section of this paper, regional unemployment rates for selected districts and their changes have been mentioned. As an example, let us take a look at the districts of Domažlice and Most. Unemployment rate in the Domažlice district reached 5.37 percent in 2005 and 6.42 percent in 2013. Average inefficiency rose from 0.258 to 0.385 when using the monthly estimates and from 0.175 to 0.361 when using the more distinctive quarterly data. Unemployment rate in the Most district was 16.49 percent in 2005 and 13.51 percent in 2013. Average inefficiency increased from 0.219 to 0.300 when using the monthly estimates, and from 0.251 to 0.366 when using the quarterly data. Low matching inefficiency does not lead to lower unemployment automatically. Its dynamics may only help to improve the tendencies of unemployment dynamics. Bad news is that regions with relatively satisfactory matching inefficiency and high unemployment rate cannot improve their performance through the factors influencing the effective matching process. Overall economic growth or diminishing ratio of long-term unemployed would have only a little effect on the matching experience there.

## Conclusion

This paper presented an alternative approach to measure the efficiency of the matching process on the Czech regional labour markets. Obtained results show that the stochastic frontier model approach is able to capture some interesting patterns of these labour markets controlling individual fixed effects of examined districts and possible time-varying changes in the inefficiency terms. The model estimates uses full sample displays increasing tendency of matching inefficiency in all districts with strong seasonal patterns. These tendencies are accompanied by rising disparities among the regions although low inefficiency does not necessarily mean low unemployment in the investigated districts. The differences across the regions were rising since 1999. Surprisingly, the estimated labour market inefficiency does not indicate that it may rise during the recession and recovery period while it decreases during the economic booms.

In the period from 1999 to 2007, the Czech regional labour markets may be described as labour markets with rising inefficiency patterns and rising heterogeneity among them. The increase before 2007 was a result of very low tightness at the Czech labour market. The structure of unfilled vacancies could not match the structure of the unemployed. 2008 and 2009 are connected with the beginning of economic slowdown. Firms were not willing to dismiss their employee immediately. Instead of that they reduced their vacancy creation. The successful matches operated closer to the production frontier. From 2010 to 2012, the positive efficiency patterns of the matching process reversed due to prevailing economic uncertainty.

Matching function of the Czech regional labour markets may be characterized by diminishing returns in matching. The elasticity of matching to vacancies is extremely low within the whole period of 1999-2014. These results are similar to those presented by

Galuščák and Münich (2007) or Münich et al. (1999). Empirical findings presented by Ilmakunnas and Pesola (2003) that with regional data it may be more likely to find increasing returns in matching are not confirmed in the case of the Czech Republic. Moreover, it means that the vacancy creation is not a sufficient condition to diminish unemployment. The main reason for that is that new vacancies do not correspond to the qualification structure of the unemployed.

Inefficiency of the regional labour markets is strongly influenced by district specific factors and common economic factors. The ratio of the registered unemployed receiving the unemployment benefits has a positive impact on efficiency. It seems that that these unemployed people are mostly short-term unemployed who are willing to find themselves a job as soon as possible. The main sources of inefficiency may be connected with the ratio of the unemployed in the age of 50 and older, and with the ratio of the long-term unemployed. In the pre-crisis period, older unemployed people were actually leaving the labour market after the economic slowdown in 2008 (and consecutive years). In determining of labour markets inefficiency, the role of the long-term unemployed was thus more important. All regional labour markets were able to operate at their matching function frontiers due to seasonal factors.

Some key results may be found when taking advantage of the estimates of the matching inefficiency using the pre-crisis data (prior to 2008) and the data covering the period of global economic slowdown that started in 2008. Overall matching efficiency is lower during the period of economic slowdown. As pointed out by Ilmakunnas and Pesola (2003), this conclusion has strong policy implications: supporting job creation through new vacancies is less efficient in the period of economic recession. Unfortunately, regions with relatively satisfactory matching inefficiency and high unemployment rate cannot improve their performance through factors influencing the effective matching process. Overall economic growth or diminishing ratio of the long-term unemployed has little effect on the matching experience there.

It will be of great importance in further research to focus on model outcomes using the aggregate yearly data that allow including more region-specific variables. Münich et al. (1999) suggested that TRANSLOG matching function be used as an alternative to the standard Cobb-Douglas specification. Moreover, spatial properties of labour markets dynamics should be investigated, i.e. efficiency terms should incorporate the influence of neighbouring districts. This kind of model enhancements could provide us with more detailed and precise view of the sources of labour market efficiency.

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