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APPLICATION OF AHP METHOD IN ASSESSMENT OF THE INFLUENCE OF ATTRIBUTES ON VALUE IN THE PROCESS OF REAL ESTATE VALUATION[1]

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Abstract

Property valuation in the comparative approach requires the determination of the impact of market characteristics on the formation of prices on the local real estate market. Valuers have a variety of methods for determining weights. Some of them require the collection of a sufficiently large database of information on transactions. However, this is not always possible. In the absence of sufficient data, alternative approaches, including an expert approach, may be used. The goal of the article is the proposal of an expert approach at the stage of assessing the influence of attributes on the value of the real estate. The AHP (Analytical Hierarchy Process) method will be used. On its basis, pairwise comparisons of the importance of attributes will be done by experts (valuers). By means of the AHP method, the weights of each attribute will be obtained and, subsequently, the influence of each attribute on the real estate value will be assessed. Research will be done on the basis of 318 real estates in Szczecin.

Key words: *real estate valuation, AHP method, real estate market*.

JEL Classification: *C44, R30*.

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1. Introduction

Property valuation in the comparative approach requires the identification of market characteristics that significantly affect the diversity of transaction prices in the real estate market. As R. Gaca points out, these features have a twofold character. On the one hand, these are "features which are the determinants of the selection of a set of similar properties, i.e. features distinguishing this set within the broadly understood market and differentiating features, also referred to as market features". (GACA 2017, pp. 3-4). Identification of these features and determination of their impact on the formation of transaction prices on the market is a very important element of the valuer's workshop.

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According to the Interpretative Note "Application of the Comparative Approach in Property Valuation", the basic way to determine the impact of individual property attributes on prices, i.e. the weights of market characteristics, is its percentage share in the difference between the maximum price and the minimum price from the set of updated transaction prices of similar properties, which are the basis for valuation. The note also points to four possible ways of determining the weights of the characteristics. One of these methods is the analysis of data on prices and market characteristics of similar properties traded on the market for the purpose of valuation of the real estate market. The issue of market characteristics in the comparative approach can be divided into two subproblems. The first one is the problem of identifying features and the second one is the problem of determining their impact on transaction prices. KUCHARSKA-STASIAK (2000) points to the problem of identifying market features, rightly defining this stage as crucial. The main aim of the article will be the second sub-problem, i.e., determining the impact of features on prices. On the example of two sets of information about land plots, the use of the Analytical Hierarchical Process for determining the weights of market characteristics of real estate will be presented and tested. The smaller of these sets, with 30 observations, will be used to verify the effectiveness of the AHP method in valuations carried out in a comparative approach. The second set of 318 observations will be used to verify the effectiveness incorporating the presented method in mass valuation.

2. Literature review

Issues related to the analysis of relations between real estate prices and various factors is a huge research area, which has been discussed by many authors over the years. Researchers are looking for links between property prices and a variety of factors at both macroeconomic (e.g. ÉGERT, MIHALJEK 2007) and microeconomic levels (see FERLAN et al. 2017). The latter is particularly important from the point of view of property valuation. The search for factors influencing the value of property has a long history (e.g. BRIGHAM 1965). One of the characteristics of the real estate market is its local character. As a result, analyses and scientific work are carried out using examples of different markets (see CHOY et. al. 2007, GWAMMA et. al. 2015, ZYDROŃ et. al. 2016). The studies also pay attention to the influence of the method of measuring the characteristics of properties on their value (BOYLE, TAYLOR 2001). Determination of a set of attributes significantly influencing the formation of prices on local real estate markets, in consequence, leads to the determination of their impact strength, i.e. the determination of weights. The traditional method of weighting is based on the *ceteris paribus* principle. Its basic stage is to search, within the collected database of information on similar real estate transactions, for pairs of objects differing only in terms of one market feature. As far as alternative methods of determining market weights are concerned, studies (see BARAŃSKA 2018; GACA, SAWIŁOW 2014 and others) and oftentimes practical applications propose the use of statistical methods based on correlation coefficients or regression models. The presented proposals are based on the shares of correlation coefficients or structural parameter estimates of regression models assessing the strength of the relationship between real estate characteristics and transaction prices in sums of calculated or estimated coefficients. Pearson product-moment, rank and other correlation coefficients are proposed. The relative ease of calculation of these proposals makes them increasingly popular among valuers. However, the statistical approach to determining the weights of market characteristics also has drawbacks. These include the need to have adequate databases on market transactions, which is not possible to obtain in all local markets. Another problem is the uncritical application of statistical methods to data that do not meet the theoretical assumptions of the method. The second category includes the question of the measurement scale of market characteristics. The problem of weak measurement scales in the area of determining the impact of property attributes on their prices was discussed by Doszyń (2017). The author proposes the use of the Kendall correlation coefficient. Foryś and Gaca also draw attention to the problem of weak measurement scales in the real estate market (FORYŚ, GACA 2016). It is also proposed to use non-parametric statistical methods for the determination of weights of market features (GACA 2018). Statistical analyses in the context of real estate characteristics are also used more widely. There are proposals of methods for the selection of features (BARAŃSKA 2004), and research on the similarity of real estate (ZYGA 2013) has been conducted. A counter-proposal to statistical methods are expert methods, which are a kind of response to the issues related to the lack of sufficient data on transactions on local markets and the problem of measurement scales. Real estate in the process of valuation are described by several market characteristics. Since there are many attributes that are considered to assess the real estate value, multiple-criteria decision making techniques ought to be applied. There are many expert multiple-criteria decision making techniques, such as (NERMEND 2017):

- AHP (*Analytical Hierarchy Process*),
- ANP (*Analytic Network Process*),
- REMBRANDT,
- DEMATEL (*DEcision MAking Trial and Evaluation Laboratory*),
- ELECTRE (*ELimination Et Choix Traduisant la REalité*),
- PROMETHEE (*Preference Ranking Organization METHod for Enrichment of Evaluations*).

The AHP method was created by Thomas L. Saaty in the 1970s. It assumes that all criteria (attributes) are independent (BRUNELLI 2015, p. 17). The criteria can be divided into sub-criteria, thus they have the hierarchical structure. The extension of the AHP method, also proposed by Thomas L. Saaty, was the ANP method. It assumes that there can be interaction and dependence between the criteria (attributes) (SAATY, VARGAS 2006, p. 7). The REMBRANDT method is also based on pairwise comparisons between criteria (attributes), but these do not have a hierarchical structure (NERMEND 2017, p. 258). The DEMATEL method converts the interrelations between the criteria (real estate attributes in the case of this research) into cause and effect groups (SI et al. 2018, p. 2). The ELECTRE-group methods (ELECTRE I, ELECTRE Iv, ELECTRE II, ELECTRE III, ELECTRE IV, ELECTRE IS and ELECTRE TRI) are based on the outranking method and allow for the choosing, ranking or sorting of alternatives (NOWAK 2004, pp. 36-37). The PROMETHEE method does not aim to find the "right" or "best" decision, but rather one that suits the goal best. It also allows for the identification and quantification of conflicts and synergies, clusters of decisions and select main alternatives (BRANS, MAERCHAL 2005). Some of the above-mentioned methods have already been applied on the widely understood real estate market or urban analysis. For example, the PROMETHEE method was used in the evaluation of urban regeneration processes in Northern Italy (BOTTERO et al. 2018). The DEMATEL method was applied in analysis of relationships between the real estate attributes and divided them into the causal and effect ones (GOŁĄBESKA 2018). The AHP method, the use of which is the main focus of the presented research, was used, among others, in the process of evaluating real estate for purchase purposes (BALL, SRNIVASAN 1994), or for forecasting the value of real estate (YALPIR 2014). In the context of the presented study, it is worth noting the attempts to use AHP to determine the weights of market characteristics (KOZIOŁ-KACZOREK 2012), the author also proposes supplementing the AHP method with elements of goal programming (KOZIOŁ-KACZOREK 2014). A similar approach, based on a comparison of the obtained weights with some counter-proposals, was presented in the paper (KRYVOBOKOV 2005). Here, the effectiveness of the AHP method was also confirmed, in this case on the example of the local real estate market located in Ukraine. Studies also point to the possibility of using the AHP method in combination with the capabilities of GIS tools (KARAKAYACI 2015). The mentioned studies are limited to determining weights and comparing them with a competitive method. The presented study goes a step further and uses the weights set by different methods of real estate valuation in both the individual and mass approach. In addition to the comparison of market weights alone, a comparison of valuation results is also presented, which gives a more complete picture of the effectiveness of the AHP method.

3. Data and Methods

The survey covered 318 land plots located in Szczecin. All plots were used for housing purposes and constituted a part of the properties owned by the Szczecin City Municipality, for which annual fees for perpetual usufruct of land were updated. Figure 1 shows the spatial distribution of valuation objects. These plots were valued by a group of property valuers who carried out the valuation in the comparative approach, and the results of the valuations obtained by the experts will constitute a point of reference to the results obtained in the presented study. Part of the set of land parcels valued was drawn to a separate set of representative land plots. A total of 30 so-called representatives were drawn. These land plots constitute a set, for which a comparison of valuations with weights determined by two methods, including AHP, will be carried out. In the next stage, the values of these land plots formed the basis for calculating the coefficients of the market value (according to the mass valuation algorithm in use) of the three sub-areas into which the valuation objects were classified. In the article, the AHP method will be used for the estimation of weights assessed to attributes describing each plot of land. Valuated land plots were described by means of the following attributes:

x_1 – area: 1 – unfavourable, 2 – average, 3 – favourable;
x_2 – utilities access: 1 – unfavourable, 2 – average, 3 – favourable;
x_3 – communication accessibility: 1 – unfavourable, 2 – average, 3 – favourable;
x_4 – surroundings quality: 1 – onerous, 2 – unfavourable, 3 – average, 4 – favourable;
x_5 – plot shape attractiveness: 1 – unfavourable, 2 – average, 3 – favourable.

As the residential, undeveloped land plots were valuated, the area (measured in square metres) was treated as small, medium and large. The real estate area was considered small if its area was not higher than 500 m². An area between 500-1200 m² was considered medium and above 1200 m² – high. Generally, the higher area, the lower the unit value of real estate is, with some exceptions. In the case of this research, if the land plot was designated for low residential development, the area was considered as having a negative impact on the value of 1 m². If the plot was designated for high residential development, the area was assumed to have a positive impact on the value of 1 m².

The "plot shape attractiveness" attribute was determined on the basis of a plot shape. It was assumed that a rectangle with a side length ratio of 3:2 was the optimum plot shape. Having the data concerning a plot circumference, its surface was calculated, assuming its rectangular shape, and it was compared with the actual surface (the actual surface was divided by the surface obtained with the assumption of a 3:2 rectangle). If the ratio was greater than 0.9, then the value of the attribute was 3. If it fell within the range of 0.5-0.9 – the value of the attribute was 2. If it was less than 0.5, the attribute assumed the value of 1 (DMYTRÓW et al. 2018).

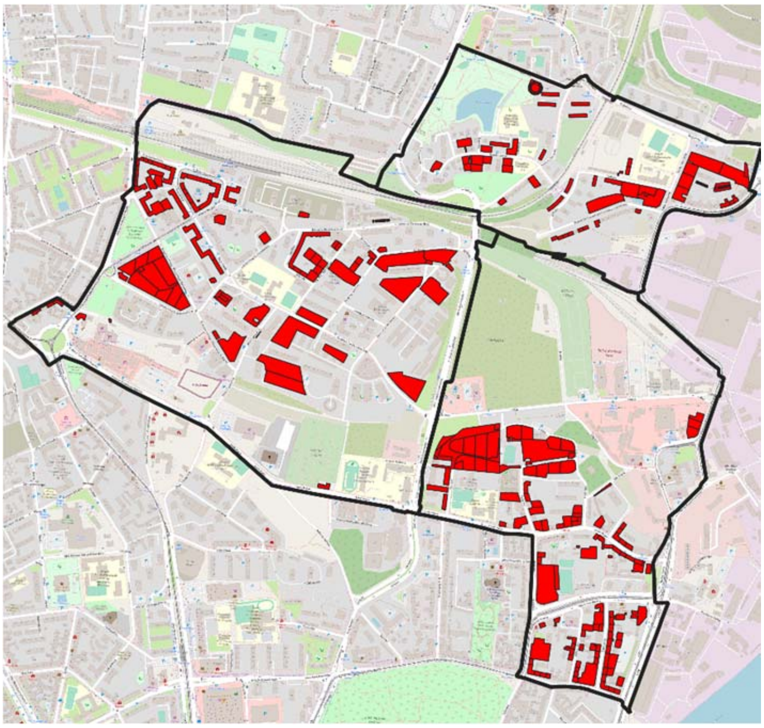

Fig. 1. Land plots being the subject of valuation. *Source*: own elaboration.

The common feature of all expert methods is that pairwise comparisons are made between criteria (in our case – real estate attributes). The AHP method was performed on the basis of a survey directed to four experts (valuers). Each question in the survey compares two attributes (Attribute 1 vs Attribute 2). Answers are based on the 9-point Saaty scale. The expert chooses, which attribute dominates over the other one. If the expert decides that the attributes are indifferent, then the value 1 is conceded. If one attribute moderately dominates over the other, then the admitted value is 3. If the domination is strong, then the value is 5. If the domination of one attribute over the other attribute is very strong, then the expert assigns the value of 7. If the domination is extreme, then the value is 9. The values of 2, 4, 6 and 8 denote indirect situations (BRUNELLI 2015, p. 15). For example, if the expert chooses the value of 2, he/she hesitates between 1 and 3, etc. If the expert decides that attribute 1 strongly dominates over attribute 2 then the pair attribute 1 – attribute 2 will have the value of 5 and the pair attribute 2 – attribute 1 will have the value 1/5. Such comparisons are done for every pair of attributes. Obtained values are placed in a pairwise comparison matrix (the AHP matrix):

$$A = \begin{bmatrix} 1 & c_{12} & \cdots & c_{1l} & \cdots & c_{1K} \\ \frac{1}{c_{12}} & 1 & \cdots & c_{2l} & \cdots & c_{2K} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \frac{1}{c_{1l}} & \frac{1}{c_{2l}} & \cdots & \ddots & \cdots & c_{kK} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{c_{1K}} & \frac{1}{c_{2K}} & \cdots & \frac{1}{c_{kK}} & \cdots & 1 \end{bmatrix}, \tag{1}$$

where:
k, l – compared attributes ($k, l = 1, 2, \ldots, K; k \neq l$),
K – number of attributes,
c_{kl} – value assigned by the expert (valuer) by comparing the k-th and l-th attributes.

In order to check if the expert's all direct pairwise comparisons are confirmed by all non-direct comparisons, the consistency of matrix A is analyzed. This is done by means of the *consistency index* (CI):

$$CI = \frac{\lambda_{\max} - K}{K-1}, \tag{2}$$

where $\lambda_{\max}$ – maximum real eigenvalue of matrix A.

On the basis of equation (2), the *consistency ratio* (CR):

$$CR = \frac{CI}{R}, \tag{3}$$

where R is the *random index*, which is an estimation of the average CI obtained from a large enough set of randomly generated matrices of size n (BRUNELLI 2015, p. 25). Estimated values of R with respect to the size of the matrix (n) are presented in Table 1.

Table 1

Values of R

n	3	4	5	6	7	8	9	10
R	0.5247	0.8816	1.1086	1.2479	1.3417	1.4057	1.4499	1.4854

Source: (BRUNELLI 2015, p. 25).

If the obtained value of CR for matrix A is not higher than 0.1, then it can be accepted that the expert's judgements are consistent. Otherwise the expert is asked to review his/her to review his/her judgement until the threshold $CR \leq 0.1$ is reached.

If matrix A is consistent the weights of attributes can be estimated. This is done in several steps. In the first step, the elements of each column are summed:

$$c_l = \sum_{k=1}^{K} c_{kl}, \; l = 1, 2, \ldots, K, \tag{4}$$

where c_l – sum of elements of the l-th column.

Next, each element of matrix A is divided by the appropriate element of c_l:

$$z_{kl} = \frac{c_{kl}}{c_l}, \; k, l = 1, 2, \ldots, K. \tag{5}$$

In the final step, mean values of z_{kl} are calculated for each row:

$$w_k = \frac{\sum_{l=1}^{K} z_{kl}}{K}, \; k = 1, 2, \ldots, K. \tag{6}$$

Obtained values of w_k will be the weights of each attribute. They will always satisfy the condition: $\sum_{k=1}^{K} w_k = 1$. The procedure of calculating weights, described by formulas (1) – (6) was applied for all four experts. For further analysis, mean weights from the values calculated for all experts were applied.

When the values of weights of attributes are determined, they are applied to determine the influence of every state of each attribute on the unit value of the real estate and will be used in valuation done by means of the following algorithm:

$$\hat{v}_{ji} = wwr_j \cdot v_b \cdot \prod_{k=1}^{K} \prod_{p=1}^{k_p} (1 + a_{kpi}), \; i = 1, 2, \ldots, n, \; j = 1, 2, \ldots, J \tag{7}$$

where:

$\hat{v}_{ji}$ – market (or cadastral) value of i-th real estate in j-th location's attractiveness zone,
v_b – value of 1 m² of the cheapest plot in the appraised area,
a_{kpi} – impact of p-th state of k–th attribute in i-th real estate,
wwr_j – market value ratio in j-th location's attractiveness zone,
n – number of real estates,
J – number of attractiveness zones.

Assessment of the impact of the p-th state of k-th attribute in i-th real estate is as follows:

$$1 + a_{kpi} = e^{\ln\left(\frac{v_{\max}}{v_{\min}}\right) \cdot u_{kpi}}, \tag{8}$$

where:

$v_{\max}$ – maximum value of real estate in the set of representatives,
$v_{\min}$ – minimum value of real estate in the set of representatives,
u_{kpi} – influence of weight of k-th attribute calculated as follows:

$$u_{kpi} = w_k \cdot t_{kp}^{(i)}, \tag{9}$$

where:

w_k – weight of k-th attribute obtained by means of the formula (6),
$t_{kp}^{(i)}$ – transition to the p-th state of k-th attribute (which is possessed by the i-th real estate) calculated as follows:

$$t_{kp}^{(i)} = \frac{\bar{v}_{rp}^{(i)}}{\bar{v}_{r1}}, \tag{10}$$

where:

$\bar{v}_{rp}^{(i)}$ – mean real value of 1 m² of all real estates having the same p-th state of k-th attribute, as the analysed i-th real estate has,
$\bar{v}_{r1}$ – mean real value of 1 m² of all real estates having the worst (1st) state of the k-th attribute.

Procedure of assessing the impact of the p-th state of the k-th attribute in the i-th real estate, given by formulas (8) – (10) is the authors' own approach. The reason for using the $\frac{v_{\max}}{v_{\min}}$ ratio is to consider the range of real estates valuated by equation (7), and equations (9) – (10) are used for the estimation of the impact of particular states of attributes. It is worth noting that the $v_{\min}$ value in equation (8) is not the same, as the v_b value in equation (7). The former is a real value, existing in the set of representatives, while the latter is the theoretical cheapest value for the worst states of attributes, in the cheapest location attractiveness zone, set by the appraiser.

After estimation of $1 + a_{kpi}$, the hypothetical values of the representative real estates were appraised by means of the following formula:

$$\hat{v}_{hi} = v_b \cdot \prod_{k=1}^{K} \prod_{p=1}^{k_p} \left(1 + a_{kpi}\right), i = 1, 2, \ldots, n, j = 1, 2, \ldots, J \tag{11}$$

where $\hat{v}_{hi}$ – hypothetical value of i-th representative land plot valuated by the algorithm,

Next, the value of wwr_i is calculated for each representative real estate:

$$wwr_i = \frac{v_{ri}}{\hat{v}_{hi}}, \tag{12}$$

where v_{ri} –value of i-th representative real estate valuated by the expert

The wwr_j value for every attractiveness zone is calculated by means of the following formula:

$$wwr_j = \sqrt[l]{\prod_{i=1}^{l} wwr_i}, \tag{13}$$

where l is the number of real estates in the j-th attractiveness zone. The values of wwr_j can be understood as a so-called "location premium". They represent the influence of location on the value of real estate.

Having estimated the wwr_j values, and $1 + a_{kpi}$ values, obtained for each state of all five attributes given at the beginning of this chapter, they are substituted into equation (7) to obtain the $\hat{v}_{ji}$ values.

Valuations done by the algorithm were compared with the individual valuations done by the experts. The basic error measure calculated for every valuated real estate was the *percentage error* (*PE*):

$$PE_{ji} = \frac{\hat{v}_{ji} - v_{ji}}{v_{ji}} \cdot 100\%, \tag{14}$$

where:

$\hat{v}_{ji}$ – market (or cadastral) value of *i*-th real estate in *j*-th location's attractiveness zone (valuated by the algorithm),

v_{ji} - market (or cadastral) value of *i*-th real estate in *j*-th location's attractiveness zone (valuated by the expert).

The closer the value of *PE* is to 0%, the better. Next, the *mean percentage error* (*MPE*) was calculated:

$$MPE = \frac{\sum_{i=1}^{n} PE_{ji}}{n}. \tag{15}$$

The optimum value of *MPE* is 0%; in such case valuations are unbiased. If the value of *MPE* is larger than 0%, it means that the valuations done by the algorithm are overestimated. Otherwise, they are underestimated .

The final error measure was the *mean absolute percentage error* (*MAPE*):

$$MAPE = \frac{\sum_{i=1}^{n} |PE_{ji}|}{n}. \tag{16}$$

The optimum value of *MAPE* is 0%. In such case, all valuations done by the algorithm would be exactly the same as the valuations done by the experts.

4. Empirical results

The survey, on which the AHP method was based, consisted of 15 questions. Every question had the same structure. For every pair of attributes this was formulated as: which attribute, i.e. attribute 1 or attribute 2, had a greater impact on the real estate value of 1 m^2? Instead of using the full 9-point Saaty scale, the valuers suggested using a shorter, 4-point scale. The answers were as follows (in parentheses are points measuring the dominance of attribute 1 over attribute 2):

- attribute 1 has a definitely greater impact (4),
- attribute 1 has a noticeably greater impact (3),
- attribute 1 has a slightly greater impact (2),
- the attributes are indifferent (1),
- attribute 2 has a slightly greater impact (1/2),
- attribute 2 has a noticeably greater impact (1/3),
- attribute 2 has a definitely greater impact (1/4).

Four experts (valuers) were surveyed. On the basis of the surveys, weights for each attribute were calculated. The pairwise comparison matrices for all valuers were consistent (the consistency ratios did not exceed the value 0.1). After obtaining weights for each valuer, mean weights for each attribute were estimated. In the next stage, the results of valuations carried out with the use of scales obtained using the AHP method and scales determined by one of the methods often used in practice based on correlation analysis (referred as benchmark) were compared (BARAŃSKA 2018). The analysis was conducted on the basis of transaction prices for undeveloped land plots intended for residential development located in the northern part of Szczecin. The temporal scope of the market analysis covered the years 2016-2017. Correlation coefficients between variants of these characteristics and unit transaction prices have been calculated for particular market characteristics. The share of the absolute value of each Pearson correlation coefficient in the sum of all coefficients was the weight of the feature. A comparison of the weights set in the proposed approach and the benchmark approach is shown in Figure 2.

As can be seen, the weights of the different market characteristics do not differ diametrically. The greatest differences were observed in terms of plot area and communication accessibility. For the first of these features, the weight obtained with the AHP method was lower than for the benchmark. In the second one, the AHP weight was higher. For the other characteristics, the differences in weights were small.

In order to determine how much the differences in weights affect the results of property valuations, the following experiment was carried out. The value of 30 land plots was determined, coming from a larger base used in the accompanying studies on the mass valuation of properties. The set of transactional prices consisted of 64 items. The minimal market unit price was 133.70 PLN/m², maximal 695.77 PLN/m². The valuation was carried out in a comparative approach, using the method of comparison in pairs. In the database of transaction prices, for most of the valued land plots, one could find 3-5 plots which did not differ from each other by variants of market features. As a result of this situation, the method of comparison in pairs would boil down to calculating the average value from unit transaction prices. The weights of the market characteristics would not affect the outcome as there would be no correction due to differences between properties. For the purpose of the study, in order to avoid the described effect, instead of the three properties most similar to the valued one, random real estates were selected from the set of recorded transactions. Table 2 shows the results of the valuations with weights of market features from both analyzed methods. Differences in the value of one square meter ranged from - 40 PLN to 28 PLN. This translated into relative differences between -9.4% and 5.2%. The median relative difference was only - 0.58%. The distribution of relative differences is shown in Figure 3. The obtained results indicate that the AHP method used to determine market characteristics weights gives results of valuations at a level similar to those in which benchmark weights were applied.

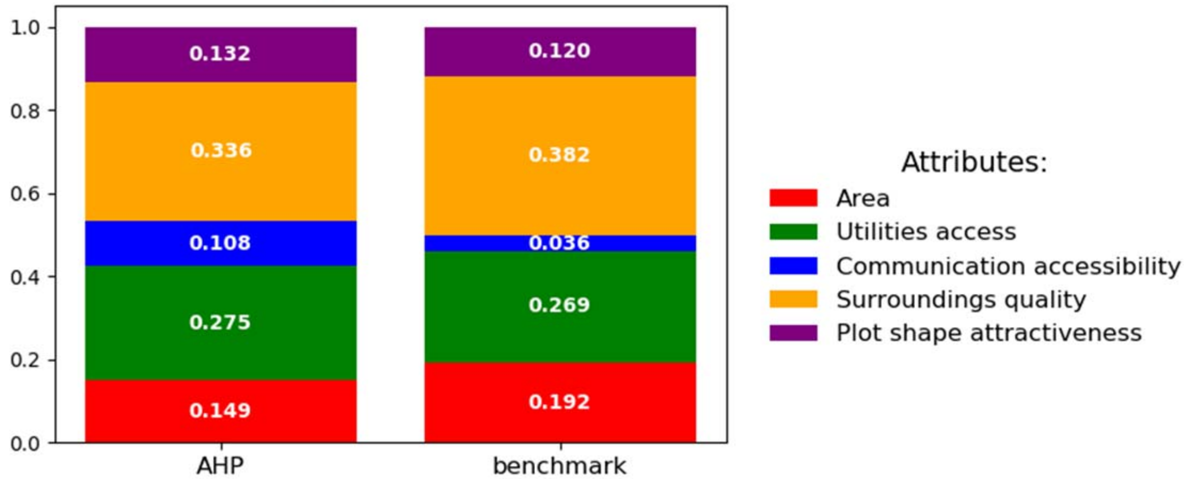

Fig. 2. Weights of attributes set by means of the AHP method and correlation analysis (benchmark). *Source*: own elaboration.

Table 2

Results of valuation on the basis on weights obtained by the AHP and benchmark methods

Plot number	Attributes					Value (PLN/m²)	
	Area	Access to Utilities	Communication accessibility	Quality of surroundings	Plot shape attractiveness	AHP	benchmark
1	2	3	2	3	3	574.20	565.13
2	1	3	2	3	3	520.19	523.23
3	3	3	3	3	3	638.45	637.45
4	3	3	3	1	2	461.58	474.50
5	1	3	3	3	2	496.60	516.62
6	1	3	3	3	3	530.43	553.65
7	1	3	2	3	3	520.19	523.23
8	3	3	2	3	1	560.55	532.97
9	2	3	2	3	3	574.20	565.13
10	3	3	3	4	3	709.97	700.41
11	1	3	2	3	3	520.19	523.23
12	1	3	3	3	3	530.43	553.65
13	3	3	3	3	1	570.79	563.39
14	1	3	3	4	3	601.95	616.61
15	1	3	3	3	3	530.43	553.65
16	3	3	3	3	2	604.62	600.42

Plot number	Attributes					Value (PLN/m²)	
	Area	Access to Utilities	Communication accessibility	Quality of surroundings	Plot shape attractiveness	AHP	benchmark
17	1	3	3	3	3	530.43	553.65
18	3	3	3	3	3	638.45	637.45
19	1	3	3	1	3	387.39	427.73
20	1	3	3	3	2	496.60	516.62
21	1	3	3	3	3	530.43	553.65
22	1	3	3	3	2	496.60	516.62
23	3	3	3	3	3	638.45	637.45
24	1	3	2	3	2	486.36	486.20
25	1	3	3	4	2	568.12	579.58
26	1	3	3	2	1	391.25	416.63
27	1	3	2	3	3	520.19	523.23
28	3	3	3	4	2	676.14	663.38
29	2	3	3	1	3	441.40	469.63
30	2	3	3	4	1	588.30	584.45

Source: own elaboration.

Table 3

Comparative plots of land used for individual valuations

Plot number	Area	Access to Utilities	Communication accessibility	Quality of Surroundings	Plot shape attractiveness	Unit transactional price (PLN/m²)
1	3	3	3	3	2	613.50
2	1	3	2	3	3	554.66
3	3	3	3	2	1	455.93

Source: own elaboration.

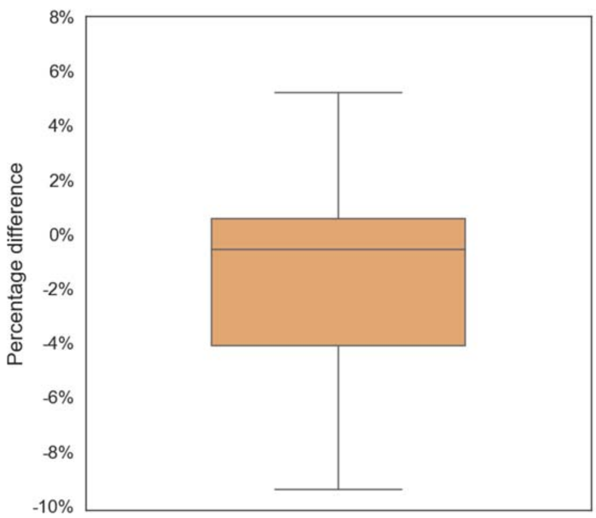

Fig. 3. Percentage errors of valuation on the basis of weights obtained by means of the AHP method.
Source: own elaboration.

In the third stage of the analysis an algorithm depicted by means of equation (7) was used to valuate 318 real estates. Two valuations were carried out. The first one was done on the basis of mean weights of attributes obtained by means of the AHP method, with the second one on the basis of Pearson product moment correlation coefficients. Then obtained results were compared with individual valuations done by the experts. Calculated error measures are presented in Table 4.

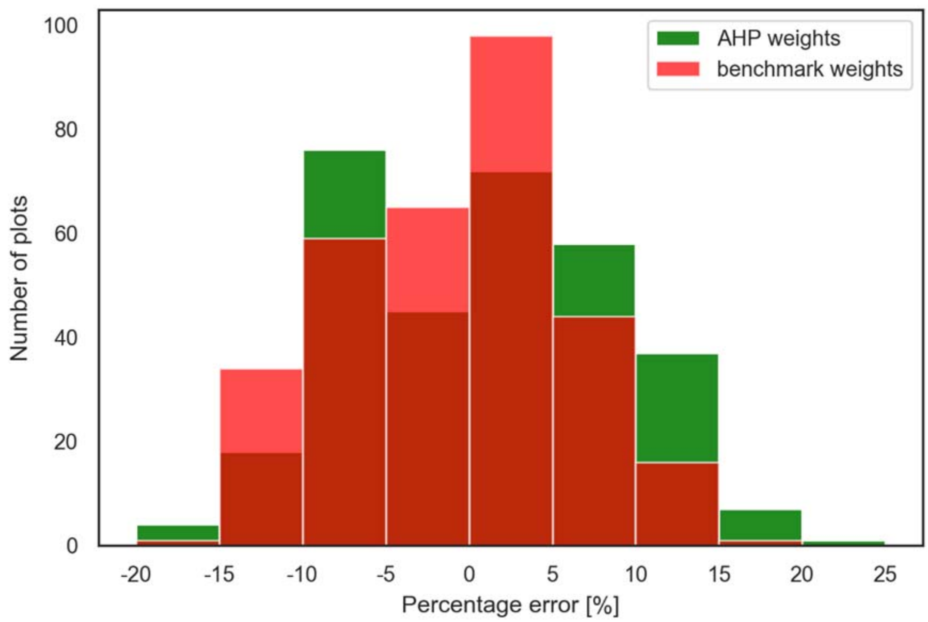

Fig. 4. Distribution of valuation percentage errors with attribute weights obtained by the AHP and benchmark methods. *Source*: own elaboration.

Table 4

Valuation error measures

Valuation basis	*min PE*	*max PE*	*MPE*	*MAPE*	*±5% PE*	*±10% PE*	*±15% PE*
AHP	-14.99%	17.14%	-0.62%	5.46%	51.26%	83.33%	99.69%
Benchmark	-16.66%	17.16%	-0.63%	5.49%	51.26%	83.65%	99.37%

Source: own elaboration.

Both methods of estimating attribute weights gave good, similar results of valuation. Slightly better results were obtained by using the AHP method. Valuation percentage errors did not exceed -15% and 17% for attribute weights obtained by means of the AHP method (-17.14% and 17.16% respectively for the benchmark method). In general, valuations were slightly underestimated. Mean underestimation for the AHP method was 0.62%, and for the benchmark – 0.63%. On average, valuations done by the algorithm deviated from the individual valuations by less than ±5% (for both methods; the AHP method was characterized by a slightly smaller error). The distribution of *PE* shows that, for both methods, valuation errors of most land plots were small and very small (Fig. 4).

For both methods of estimation of attribute weights (AHP and benchmark), the difference between valuations done by the algorithm and experts within the range of ±5% occurred in the case of 51.26% real estates. The range ±10% was obtained for over 83% of real estates (benchmark method was characterized by a slightly better result). Percentage errors of valuations of almost all real estates (99.69% in case of the AHP method and 99.37% in case of the benchmark method) fell within the range ±15%. All these measures confirm that both methods of estimating the influence of attributes on the unit value of real estate and the algorithm gave very good results with very small values of error measures.

5. Discussion and conclusions

The article presents the application of the AHP method in the assessment of weights of attributes and the comparison of the obtained results with weights obtained by means of the correlation analysis (this method was used as the benchmark). Obtained weights were subsequently used to estimate the influence of attributes in the process of valuation. Weights obtained by both methods were quite similar. In order to verify the usefulness of the AHP method, two stages of valuation were done. In the first stage, by means of comparative approach, using the method of comparison in pairs, values of 30 land plots were determined. The results show that differences between values obtained by both methods were relatively small – median percentage difference was only 0.58%, which can be considered very small. Maximum differences were -9.4% and 5.2%. In most cases (18 out of 30) unit market value estimated with attribute' weights obtained with the AHP method was smaller than the benchmark. The mean relative differences of values were also higher in such cases. For negative differences, the mean was -3.5%, and only 1.2% for positive ones. This does not mean that the presented method of determining weights underestimates the value of the property. This result is closely related to the characteristics of real estate in the valued set. In order to test the presented method of weight calculation, more extensive studies have been planned. Different types of properties with different characteristics will be taken into consideration.

In the second stage of analysis, both methods were applied in the valuation of 318 land plots, previously valuated individually by experts (valuers). The results show that valuation based on attribute weights obtained by means of the AHP method was very similar to that based on the benchmark method. Also, values obtained by both methods were very similar to these assessed individually by the valuers. Valuations done by both approaches were slightly underestimated (valuations obtained by the algorithm were lower than valuations done by the appraisers). Maximum valuation errors in plus and in minus were far below 20%. Values obtained by the algorithm differed from these set by the appraisers by less than 5.5% on average. It is worth noting that more than half of the land plots had valuation errors that fell within the range of ±5% and almost all of them – ±15%. Valuation errors were, in both cases, very small, and slightly better results were obtained using the AHP method for estimating the influence of attributes. The differences, however, are negligible in most cases, and it is hard to state which method is better on the whole.

The obtained results prove that the AHP method is a very useful tool in assessing the weights of attributes and, subsequently, the estimation of their influence on the values of land plots, and can be used in both individual or mass valuation. Future research within this area will include application of other expert methods (DEMATEL, ELECTRE or PROMETHEE) for estimating the influence of attributes on the unit value of land plots.

6. References

BALL J., SRNIVASAN V.C., 1994, *Using the Analytic Hierarchy Process in House Selection*, Journal of Real Estate Finance and Economics, no. 9, pp. 69-85, DOI: 10.1007/BF01153589.

BARAŃSKA A., 2004, *Wybór cech nieruchomości do modelowania matematycznego wartości rynkowej na przykładzie kilku baz nieruchomości gruntowych* (*Selection of real estate features for mathematical modelling of the market value exemplified by several databases of ground real estates*) [in Polish], Geodezja : półrocznik Akademii Górniczo-Hutniczej im. Stanisława Staszica w Krakowie, Vol. 10, No. 1, pp. 31-38.

BARAŃSKA A., 2018, *Linear and nonlinear correlation in calculation of weights of real estate features*, in SGEM 18th international multidisciplinary scientific geoconference : 2 July – 8 July, 2018, Albena, Bulgaria: conference proceedings. Vol. 18, Geoinformatics, geodesy and mine surveying. Iss. 2.2, Informatics, geoinformatics and remote sensing — Sofia, DOI: 10.5593/sgem2018/2.2/S09.109.

BOTTERO M., D'ALPAOS C., OPPIO A., 2018, *Multicriteria Evaluation of Urban Regeneration Processes: An Application of PROMETHEE Method in Northern Italy*, Advances in Operations Research, vol. 2018, pp. 1-12, DOI: 10.1155/2018/9276075.

BOYLE K.J., TAYLOR L.O., 2001, *Does the Measurement of Property and Structural Characteristics Affect Estimated Implicit Prices for Environmental Amenities in a Hedonic Model?*, Journal of Real Estate Finance and Economics, No. 22:2/3, pp. 303-318, DOI: 10.1023/A:10078559.

BRANS J.P., MARESCHAL B., 2005, *Multiple Criteria Decision Analysis: State of the Art Surveys*, eds. J. FIGUEIRA, V. MOUSSEAU and B. ROY, Springer, New York, pp. 163-195, DOI: 10.1007/b100605.

BRIGHAM E.F., 1965, *The Determinants of Residential Land Values*, Land Economics, Vol. 41, No. 4, pp. 325-334, DOI: 10.2307/3144665.

BRUNELLI M., 2015, *Introduction to the Analytic Hierarchy Process*, SpringerBriefs in Operations Research, DOI: 10.1007/978-3-319-12502-2.

CHOY L.H.T., MAK S.W.K., HO W.K.O., 2007, *Modeling Hong Kong real estate prices*, Journal of Housing and the Built Environment, No. 22, pp. 359–368, DOI: 10.1007/s10901-007-9089-2.

DMYTRÓW K., GNAT S., KOKOT S., 2018, *Próba uwzględnienia kształtu nieruchomości gruntowych jako atrybutu w procesie zalgorytmizowanej wyceny* (*An attempt to include the shape of land properties as an*

attribute in the algorithmic valuation proces) [in Polish], Problemy Rynku Nieruchomości: Biuletyn Stowarzyszenia Rzeczoznawców Majątkowych Województwa Wielkopolskiego, No. 2 (50), s. 4-10.
DOSZYŃ M., 2017, *Statistical Determination of Impact of Property Attributes for Weak Measurement Scales*, Real Estate Management and Valuation, vol. 25, no. 4, pp. 75-84, DOI: 10.1515/remav-2017-0031.
ÉGERT B., MIHALJEK D., 2007, *Determinants of House Prices in Central and Eastern Europe*, Comparative Economic Studies, 49: 367, DOI: 10.1057/palgrave.ces.8100221.
FERLAN N., BASTIC M., PSUNDER I., 2017, *Influential Factors on the Market Value of Residential Properties*, Inzinerine Ekonomika-Engineering Economics, No. 28(2), pp. 135–144, DOI: 10.5755/j01.ee.28.2.13777.
FORYŚ I., GACA R., 2016, *Application of the Likert and Osgood Scales to Quantify the Qualitative Features of Real Estate Properties,* Folia Oeconomica Stetinensia, Vol. 16, Issue 2, pp. 7-16, DOI: 10.1515/foli-2016-0021.
GACA R., 2017, *Cechy nieruchomości identyfikacja, topologia, pomiar* (*Characteristics of real estate identification, topology, measurement*) [in Polish], Rzeczoznawca Majątkowy 3, pp. 3-12.
GACA R., 2018, *Parametric and Non-Parametric Statistical Methods in the Assessment of the Effect of Property Attributes on Prices*, Real Estate Management and Valuation, vol. 26, no. 2, pp. 83-91, DOI: 10.2478/remav-2018-0018.
GACA R., SAWIŁOW E., 2014, *Zastosowanie współczynników korelacji rang Spearmana do ustalania wag cech rynkowych nieruchomości,* (*Application of Spearman's Rank Correlation Coefficient for Establishing Ranks of Real Estate Characteristics*) [in Polish] Rzeczoznawca Majątkowy no. 82, pp. 24-30.
GOŁĄBESKA E., 2018, *The Dematel Method in the Analysis of the Residential Real Estate Market in Bialystok*, Real Estate Management and Valuation, vol. 26, no. 1, pp. 16-25, DOI: 10.2478/remav-2018-0002.
GWAMNA E.S., WAN YUSOFF W.Z., ISMAIL M.F., 2015, *Determinants of Land Use and Property Value*, Advanced Science Letters, Volume 21, Number 5, pp. 1150-1153(4), DOI: 10.1166/asl.2015.6065.
KARAKAYACI Z., 2015, *Using of analytic hierarchy process on evaluating the affecting factors in the value of farmlands*, Bulgarian Journal of Agricultural Science, no. 21 (4), pp. 719–724.
KOZIOŁ-KACZOREK D., 2012, *Hierarchizacja cech nieruchomości z zastosowaniem analitycznego procesu hierarchicznego* (*Hierarchization of real estate features using an analytical hierarchical proces*) [in Polish], Studia i Materiały Towarzystwa Naukowego Nieruchomości, no. 20 (1), pp. 165-174.
KOZIOŁ-KACZOREK D., 2014, *The Use of Combined Multicriteria Method for the Valuation of Real Estate,* Optimum : studia ekonomiczne, no. 5 (71), pp. 208-218, DOI: 10.15290/ose.2014.05.71.16.
KRYVOBOKOV 2005, *Estimating the weights of location attributes with the Analytic Hierarchy Process in Donetsk, Ukraine*, Nordic Journal of Surveying and Real Estate Research, no. 2 (2), pp. 5-29.
KUCHARSKA-STASIAK E., 2000, *Wartość rynkowa nieruchomości* (*Market value of real estate*) [in Polish], Twigger, Warszawa.
NERMEND K., 2017, *Metody analizy wielokryterialnej i wielowymiarowej we wspomaganiu decyzji* (*Methods of multi criteria and multidimensional analysis in decision making*) [in Polish], Wydawnictwo Naukowe PWN, Warszawa.
NOWAK M., 2004, *Metody ELECTRE w deterministycznych i stochastycznych problemach decyzyjnych* (*ELECTRE methods in deterministic and stochastic decision problems*) [in Polish], Decyzje, No. 2, pp. 35-65.
SAATY T.L, VARGAS L.G., 2006, *Decision making with the Analytic Network Process. Economic, Political, Social and Technological Applications with Benefits, Opportunities, Costs and Risks*, Springer, New York, DOI: 10.1007/0-387-33987-6.
SI S.L., YOU X.Y., LIU H.C., ZHANG P., 2018, *DEMATEL Technique: A Systematic Review of the State-of-the-Art Literature on Methodologies and Applications*, Mathematical Problems in Engineering, vol. 2018, Article ID 3696457, 33 pages, DOI:10.1155/2018/3696457.
Yalpir S., 2014, *Forecasting residential real estate values with AHP method and integrated GIS*, In conference proceedings of People, Buildings and Environment, an international scientific conference, Kroměříž, Czech Republic, pp. 694-706.
ZYDROŃ A., KAYZER D., Adamowicz K., 2016. *Attributes affecting the value of undeveloped real property based of communes of Komorniki and Murowana Goślina*, Ekonomia i Środowisko, No. 4, pp. 133-142.
ZYGA J., 2013, *Connection Between Similarity and Estimation Results of Property Values Obtained by Statistical Methods*, Real Estate Management and Valuation, Vol. 24, No. 3, pp. 5-15, DOI: 10.1515/remav-2016-0017.