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PRINCIPLES AND CRITERIA FOR USING STATISTICAL PARAMETRIC MODELS AND CONDITIONAL MODELS FOR VALUATION OF MULTI-COMPONENT REAL ESTATE[1]

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Abstract

The complexity of multi-component real properties results from the possibility of identifying various components in legal, physical or functional terms. The possibility of distinguishing various functional elements of real properties, combined with the specificity resulting from their market properties, is problematic when applying the comparative approach to real estate valuation. In this case, the valuation procedure can be implemented using statistical models: the parametric model or the conditional one.

This research paper demonstrates the construction of the parametric and conditional models taking into account the geometric and pricing attributes of multi-component real estate. The authors paid attention to adjusting the models to the available market data. They also specified the conditions for the use of statistical models in the real estate valuation process. Based on the analytical and accounting considerations, the estimation criteria for the parametric model and the conditional model were defined, which allow the correct application of these models at the stages of the real estate market analysis and the real estate valuation process.

Key words: *conditional model, parametric model, statistical models, real estate valuation, statistical analysis of the market, multi-component real estate.*

JEL Classification: *C00, C13, C19, C52.*

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1. Introduction

The complexity of real properties, analyzed both from the legal point of view by identifying their components, as well as in terms of physical or functional aspects, allows to identify multi-component ones (PARZYCH, BYDŁOSZ 2008), as exemplified by developed land properties (ADAMCZEWSKI 2006; ADAMCZEWSKI, HOPFER 2008; ADAMCZYK 2010; ADAMCZYK, DĄBROWSKI 2010; ADAMCZYK, JASIOŁEK 2012; BIEDA et al. 2016; CZAJA, PARZYCH 2008; PARZYCH, BYDŁOSZ 2017; PARZYCH 2007; PARZYCH 2009a, 2011; PARZYCH, CZAJA 2015), agricultural and forest land (PARZYCH 2008, 2009a, 2009b, PARZYCH AND CZAJA 2015), as well as properties with non-uniform land use contained in local zoning plans (BIEDA 2017, 2018; ADAMCZYK et al. 2019).

The specificity of this type of real estate requires taking all their attributes into account in the process of the market analysis and for their valuation (CELLMER et al. 2014). However, this task is significantly hampered by the complexity and heterogeneity of such real properties (DAWIDOWICZ et al. 2004) as well as by the diversity and inaccuracy of their attributes (RENIGIER-BIŁOZOR et al. 2017).

The necessary data on the multi-component real estate market is information on the geometrical parameters and pricing attributes of a given property. In the context of market analysis and property valuation, this information must relate to marketable real estate for which transaction prices are also known (CZAJA 2001).

The geometric attributes are the parameters resulting from the geometric qualities of the objects. Most frequently, these are the areas or cubic capacities of the building structures, and the areas of the land parcels on which these structures are located. In the case of buildings and structures, the geometrical parameters are determined based on the technical documentation or as-built surveys. The quantity describing the area should correspond to the definition of the total area, because the usable floor space is defined differently, depending on the standard adopted for the calculations (ŁUCZYŃSKI, KOTARBA 2017).

Those real estate attributes that significantly affect the volatility of property prices should be considered to be the pricing attributes (ZYGA 2012). Determination of the pricing attributes results from the analysis of the real estate qualities through a prism of their attractiveness to a potential buyer. The selection of the pricing attributes depends on the type of market and should take its local character into account. Location, technical and utility parameters are most often considered, the most important of which include: property location, transport accessibility, the standard of buildings, technical and utility condition of buildings, and the possibility of changing the manner of their use.

The transaction price is set between the independent parties thereto, which is the result of the negotiations between the seller and the buyer (CHEN et al. 2017). This price should be derived from the prices observed on the local real estate market and should be determined adequately to the geometric attributes and attractiveness of the pricing attributes. In some cases, the transaction price may not reflect the attributes of the real property. For such cases to be captured, knowledge of the terms of the transaction is required in the first place (BRZEZICKA, WIŚNIEWSKI 2012, 2015; KUCHARSKA-STASIAK 2014).

A multi-component real property, marketed as a sales offer, has an asking price determined by the seller. The level of the asking price may follow from the analysis of the local real estate market carried out personally by the seller or by the professionals involved in rendering their services in the real estate sector. The professionals engaged in the real estate market, including certified real estate appraisers, may use the market information on unit prices of the real estate components, especially for multi-component real properties. The captured information on the unit prices of the components for the multi-component real estate is the result of the thorough analysis of the transactions on the local market. Determination of the asking price based on the seller's own analyses of the transaction prices of multi-component real estate may lead to distortions, being the consequence of a weak connection between the price itself and the pricing as well as geometric attributes. In addition to the above factors regarding the level of the asking price, the final transaction price is affected by the negotiating abilities of both the buyer and the seller.

When determining the value of multi-component real estate, the initial stage of market analysis may be based on the analysis of the transaction data regarding the market turnover of one-component real estate. Depending on the type of the components present in multi-component real estate, the land parcels of specific land use, commercial and residential premises or industrial buildings may be considered as individual components. Based on the analysis of one-component properties, their unit prices are determined at the initial stage, which are further used to determine the trend of price changes in time and to identify the pricing attributes. The forecast of the unit market value ($\widetilde{w}$) of the analyzed properties precedes the market analysis and valuation of multi-component real properties.

Real estate valuation methods in the comparative approach are based on a selected database of the real properties representative of the analyzed market (ADAMCZEWSKI 2006; CZAJA 2001; ŹRÓBEK, BEŁEJ 2000). The pairwise comparison method and the mean price adjustment method have different conditions regarding the selection of real estate for the database used for comparison purposes. The selection of the real estate for the database in the method of statistical analysis of the market is much broader and depends on the statistical valuation procedure used. Particular attention should be paid to the adaptation of statistical models and algorithms of their valuation to the existing market information on real estate. The method of statistical analysis of the market provides a wide range of possible applications of various statistical methods, including e.g. parametric valuation models and conditional valuation models (CZAJA 1997).

The use of the parametric and conditional models for the market analysis and valuation of real estate consisting of many functional components is particularly justified. These models take into account both the geometry of the property constituents in the form of geometric attributes, as well as the pricing attributes in relation to the entire property. The parametric valuation model for the multi-component real estate may be used for broad real estate markets, where the number of properties making up the database for valuation is greater than the total number of the geometric and pricing attributes included in the model. The structure of the conditional model and the estimation of the market parameters apply to those local markets where the number of the sold properties which are comparable to the one being valued is smaller than the number of all the attributes considered in the model.

The estimation of the parameters of both statistical models is preceded by the estimation of the unit market values of the components of multi-component real estate. The determination of the unit market values of the components of multi-component real estate can be carried out based on the analysis of the unit prices of one-component properties or according to the relationship between the unit market rental rates and capitalization rates.

In the parametric models, the adjustments to the approximate unit values of the real estate components and the pricing factors for the attributes are taken into account as the parameters, and there are also random deviations to the transaction prices or their differences.

In the conditional models, the place of the parameters is occupied by the random adjustments to the approximate unit values of the real estate components, as well as the random adjustments to the approximate values of the pricing factors of the considered attributes.

In the parametric and conditional models, the following denotations were used for the analyzed values:

- C_T - the transaction price of the multi-component real estate,
- $\widetilde{w}_k$ - the approximate unit market value representing the k-th component of multi-component real estate,
- S_k - the area or volume of the k-th component of multi-component real estate,
- C_S - the price estimated for multi-component real estate based on the approximate unit market values $\widetilde{w}_k$,
- $\Delta C = C_T - C_S$ - the difference between the transaction price and the price estimated for multi-component real estate,
- b_k - the adjustment to the approximate unit market value ($\widetilde{w}_k$), for the k-th geometric attribute, in the parametric model,
- a_u - the value of the u-th pricing attribute,
- p_u - the adjustment to the approximate value of the pricing factor, for the u-th discretionary attribute in the parametric model,
- v_i - the random deviation to the price difference $\Delta C = C_T - C_S$ for the i-th transaction,
- β_k - the random adjustment to the approximate unit market value ($\widetilde{w}_k$) for the k-th geometric attribute in the conditional model,
- ρ_u - the random adjustment to the approximate value of the pricing factor for the u-th discretionary attribute in the conditional model.

In the denotations used in the models, the principle was adopted that all the approximate values of the parameters are denoted by the letters of the Latin alphabet. In the parametric model, the random adjustments and the random deviations are marked with Latin letters, while the random adjustments and the random deviations in the conditional model are marked with the letters of the Greek alphabet.

The analytic form of the parametric and conditional equations, which are the basis for estimating the parameters of the multi-component real estate valuation model, can be written using the adopted denotations.

The equation of the parametric valuation model, taking into account only the geometrical attributes of multi-component real estate, takes the following form:

$$S_1(\widetilde{w}_1 + b_1) + \cdots + S_k(\widetilde{w}_k + b_k) = C_T \tag{1}$$

On the other hand, the equation of the conditional valuation model, taking into account only the geometrical attributes of multi-component real estate, takes the form:

$$S_1(\widetilde{w}_1 + \beta_1) + \cdots + S_k(\widetilde{w}_k + \beta_k) = C_T \tag{2}$$

By multiplying the binomials in the dependencies (1) and (2), and then adding the products $S_k \cdot \widetilde{w}_k$, the dependence for the estimated price is obtained:

$$C_S = \sum_k S_k \cdot \widetilde{w}_k \tag{3}$$

Taking into account the dependence (3) and inserting it into the equations (1) and (2), the following form is obtained:

- for the parametric model:

$$S_1 \cdot b_1 + \cdots + S_k \cdot b_k = C_T - C_S = \Delta C \tag{4}$$

- for the conditional model:

$$S_1 \cdot \beta_1 + \cdots + S_k \cdot \beta_k = C_T - C_S = \Delta C \tag{5}$$

The estimation of the values of the adjustments b_k to the approximate values of $\widetilde{w}_k$ is possible based on the system of parametric equations in the form (4). Taking into account only geometrical attributes for the estimation of the parametric pricing model has a limited scope of use due to the inaccuracy of estimating the market parameters.

Obtaining the estimated unit values similar to the market information for the system of conditional equations (5) is possible using the appropriate algorithms for the estimation of the values of the random adjustments β_k to the approximate values of $\widetilde{w}_k$.

In the case of the documented effect of the pricing attributes on the volatility of the prices of the analyzed properties, the parameters concerning the variability of the pricing attributes should be taken into account in equation (4):

$$S_1 \cdot b_1 + \cdots + S_k \cdot b_k + a_1 \cdot p_1 + \cdots + a_u \cdot p_u = \Delta C \tag{6}$$

Simplified proceedings, taking account of the pricing attributes in the model, consist in the use of the weighting for the system of equations (4). The values of the weights should result from the similarity of the considered properties in terms of their pricing attributes.

In the conditional model, the pricing attributes can be taken into account analogically to the parametric model. Having taken the attributes into account, there are random variables ρ_u in equation (5) which constitute the random adjustment to the approximate value of the pricing factor of the u-th attribute:

$$S_1 \cdot \beta_1 + \cdots + S_k \cdot \beta_k + a_1 \cdot \rho_1 + \cdots + a_u \cdot \rho_u = \Delta C \tag{7}$$

Having compiled the system of parametric equations in the form (6), the mutual consistency of market information should be verified and gross errors eliminated. After the initial verification of the system of equations (6), the next step involves the estimation of the values of the parameters b_k and p_u, and a full analysis of variance of the valuation model is performed. The course of proceedings together with the necessary formulas are contained in Section 2.

The system of conditional equations in form (7), as was the case with the system of parametric equations, is subject to initial verification in order to check the mutual consistency of market information. After the possible elimination of errors, the estimation of the values of the random adjustments β_k and ρ_u to the corresponding approximate market values is performed. After the estimation of the model parameters, a full analysis of variance is performed, which is presented in Section 3.

2. Criteria and algorithms for estimation of parametric valuation model for multi-component real estate

Parametric valuation model can be used only if the (n) number of approximation equations in form (6) is greater than the number $(k+u)$ of all the considered attributes for the analyzed multi-component real properties.

The system of equations (6) contained in the analytical form is as follows:

$$\begin{aligned} S_{11}\cdot b_1 + \cdots + S_{k1}\cdot b_k + a_{11}\cdot p_1 + \cdots + a_{u1}\cdot p_u &= \Delta C_1 \\ \dots\dots\dots \quad \dots \quad \dots\dots\dots \quad \dots\dots\dots \quad \dots \quad \dots\dots\dots &= \dots \\ S_{1i}\cdot b_1 + \cdots + S_{ki}\cdot b_k + a_{1i}\cdot p_1 + \cdots + a_{ui}\cdot p_u &= \Delta C_i \\ \dots\dots\dots \quad \dots \quad \dots\dots\dots \quad \dots\dots\dots \quad \dots \quad \dots\dots\dots &= \dots \\ S_{1n}\cdot b_1 + \cdots + S_{kn}\cdot b_k + a_{1n}\cdot p_1 + \cdots + a_{un}\cdot p_u &= \Delta C_n \end{aligned} \tag{8}$$

The analytical form of the system of equations (8) contained in the matrix form is as follows:

$$A = [S \quad a]; \quad X = \begin{bmatrix} b \\ p \end{bmatrix}; \quad \Delta C = \begin{bmatrix} \Delta C_1 \\ \dots \\ \Delta C_n \end{bmatrix} \tag{9}$$

where:

S - the vertical rectangular matrix with the dimension of $(n \times k)$, containing information on the surfaces or volumes S_k of the considered real estate components, that is the values of their geometric attributes,

a - the vertical rectangular matrix with the dimension of $(n \times u)$, containing information on the values of the pricing attributes a_u of the considered real estate,

b - the single-column matrix of the estimated adjustments b_k to the approximate unit market values of the real estate components,

p - the single-column matrix of the estimated adjustments p_u to the approximate values of the pricing factors,

ΔC - the single-column matrix of the constant terms, created from the differences $\Delta C = C_T - C_S$ for each considered real estate.

The basis for the preliminary verification of mutual consistency of the market information may be the values of the constant terms from the system of equations (9). The differences between the transaction prices of the real properties and their prices estimated based on the unit values of the components may become a criterion for such verification. The following inequality should be the criterion of the permissible non-compliance between the transaction prices and the estimated price calculated as the sum of the products $S_k \cdot \widetilde{w}_k$:

$$|\Delta C| \leq 0.25 \cdot C_T \tag{10}$$

The value of the 0.25 coefficient was determined based on analytical and accounting considerations, and it should ensure that the parameters obtained from the estimated valuation model will be close to the values from the analyzed real estate market.

The information on the real properties that do not satisfy the criterion (10) must be verified again. If all the data on the verified property prove to be correct, such a property should be eliminated from the market analysis and valuation database. The errors in the system of equations (9) can relate both to the geometric and pricing attributes as well as to the unit values of the components of the analyzed real properties. Failure to satisfy the criterion may also result from the failure to associate the transaction price with the geometric and pricing attributes.

Finally, after verification according to the criterion (10), the system of approximation equations in the form (6) is as follows:

$$A \cdot X = \Delta C \tag{11}$$

The use of the least sum of squares to solve the system of equations (11) is related to minimizing the quadratic form:

$$F = (\Delta C - A \cdot X)^T \cdot (\Delta C - A \cdot X) = v^T \cdot v = min. \tag{12}$$

The necessary condition for the minimum of the function (12) leads to a system of normal equations. The solution of the system of normal equations allows the values of adjustments b and factors p to be obtained from the analyzed real estate market, that is:

$$\begin{bmatrix} \hat{b} \\ \hat{p} \end{bmatrix} = [(A^T \cdot A)^{-1} \cdot A^T] \cdot \Delta C \tag{13}$$

In order to satisfy the dependence (13), the matrix $(A^T \cdot A)$ must be a non-singular matrix. The usual inverse can be calculated if Matrix A does not have a column defect. In the case of a singular matrix $(A^T \cdot A)$, methods of calculating the other inverse of the matrices should be used.

If the criterion (10) of the compliance between the transaction prices and the estimated prices is met, the estimated values of the adjustments $\hat{b}$ and the factors $\hat{p}$ should describe the market values of the components of the analyzed real estate quite well.

Finally, the most probable unit market values of the components of the multi-component real estate are calculated from the dependence:

$$\hat{w} = \tilde{w} + \hat{b} \tag{14}$$

In the case of gross errors in the market information, the estimated parameters of the valuation model will exhibit large differences relative to the unit market values adopted for the construction of the model. Such a situation may occur when criterion (10) is not met. In some extreme cases, with a small number of approximation equations, the calculated unit values can be virtual, or even negative. Such a parametric model can never be used to estimate the value of multi-component real estate.

The strength and accuracy of the estimation of the parametric valuation model is also dependent on the number of approximation equations in relation to the number of estimated parameters. If the n number of real properties is used for the estimation of the parametric valuation model, and the number of the determined parameters is $(k + u)$, then the number of the overdetermined approximation equations is $(n - k - u)$. This number should be greater than or equal to the number of the geometric attributes, i.e.

$$(n - k - u) \geq k \tag{15}$$

The analysis of variance is preceded by the calculation of random deviations v to the constant terms:

$$v = C_T - A \cdot [\hat{w} \quad \hat{p}]^T \tag{16}$$

The variance for the estimated model is calculated based on the random deviations v and the number of overdetermined approximation equations $(n - k - u)$:

$$\sigma_M^2 = \frac{v^T \cdot v}{n-k-u} \tag{17}$$

The variance-covariance matrix for the estimated parameters is determined in accordance with the following dependence:

$$Cov \begin{bmatrix} \hat{w} \\ \hat{p} \end{bmatrix} = \sigma_M^2 \cdot (A^T \cdot A)^{-1} \tag{18}$$

The second criterion for the selection of market information is based on the factor θ_b that determines the consistency of the estimated parameters $\hat{b}_k$ with the market information. The value of the factor θ_b is calculated according to the following dependence:

$$\theta_b = 1 - \frac{\sum_j (\hat{b}_j)^2}{\sum_j (\tilde{w}_j)^2} \tag{19}$$

The value of the factor θ_b should satisfy the inequality:

$$\theta_b \geq 0.95 \tag{20}$$

If the criterion (20) is not met, it is necessary to go back to the initial market analysis and verify the criterion (10) with a different level of compliance, i.e. $|\Delta C| \leq 0.15 \cdot C_T$, and in the next step, repeat the calculations and analyses.

Positive verification of both criteria allows the estimated parameters to be used for the valuation of multi-component real estate on the local market. During the estimation process of the parameters and also later, in the course of the valuation, it should be remembered that each local real estate market usually results in the different values of the parameters $\hat{w}$ i $\hat{p}$.

The market value for the real property for which the geometric and pricing attributes can be defined by a single-row matrix $[S_W \quad a_W]$, is determined from formula:

$$WR_N = [S_W \quad a_W] \cdot \begin{bmatrix} \hat{w} \\ \hat{p} \end{bmatrix} \tag{21}$$

The variance of the estimated market value is defined by the following dependence:

$$\sigma^2(WR_N) = [S_W \quad a_W] \cdot Cov \begin{bmatrix} \hat{w} \\ \hat{p} \end{bmatrix} \cdot \begin{bmatrix} S_W \\ a_W \end{bmatrix} \tag{22}$$

Taking account of the proposed criteria for the estimation of the parametric valuation model usually results in the standard deviations of the estimated market value at a level which does not exceed 5% of its value, i.e. $\sigma(WR_N) \leq 0.05 \cdot WR_N$. The standard deviation which is mostly assumed for the market information is at the level of 10%, therefore this valuation accuracy should be considered very high.

If the proposed criteria are not taken into account, it is still possible to obtain the most reliable unit values of the real estate components, but this involves the use of algorithms from a higher level of knowledge, which will eliminate the outliers inconsistent with the market parameters in the estimation process. One such algorithm is robust estimation, which allows information burdened with gross errors to be eliminated from the analyzed data sets, or to minimize its effect.

Another algorithm that can be used for the initially unverified information databases is the structure of the parametric model with the extended range of random variables which adjust the market information burdened with errors in the estimation process.

3. Criteria and algorithms for estimation of conditional valuation model for multi-component real estate

The conditional valuation models are characterized by a smaller number of analyzed multi-component real properties with known transaction prices than the total number of their geometric and pricing attributes.

The preliminary market analysis should, first of all, verify the compliance of the real estate transaction prices and the prices estimated based on the unit values of the components and of the geometric attributes. The differences between these prices, being the constant terms, form the basis for defining the criterion of permissible price compliance. These differences are mainly related to the inaccuracy in the initial estimation of the unit values $\tilde{w}_k$ and the inaccuracy in the definition of the geometrical attributes. The permissible absolute value ΔC_W should be assumed in the same way as in the dependence (10), i.e.

$$|\Delta C_W| \leq 0.25 \cdot C_T \tag{23}$$

Failure to satisfy the criterion (23) should lead to the verification of both the geometrical attributes and the unit values of the components of the analyzed properties. If no errors are identified, the property that does not satisfy the criterion should be removed from the market analysis and valuation database. The positive verification of the database, in accordance with the criterion (23), allows for the estimation of the conditional valuation model and the analysis of the variance of the model.

If exclusively geometric attributes are taken into account in the model, the following dependence is obtained:

$$S_1 \cdot \beta_1 + \cdots + S_k \cdot \beta_k = C_T - C_S = \Delta C_W \tag{24}$$

In this case, the estimated values of the random adjustments β_k complement the approximate unit values $\tilde{w}_k$ of the real estate components, hence their model value $\hat{w}_k$ is as follows:

$$\widehat{w}_k = \widetilde{w}_k + \hat{\beta}_k \tag{25}$$

After the changed values $\widetilde{w}_k^*$ are adopted, there is a linear change in the values of the constant terms ΔC_W^*, which results in the estimated new values of the adjustments $\hat{\beta}_k^*$ approximately satisfying the dependence $\widehat{w}_k \simeq \widetilde{w}_k^* + \hat{\beta}_k^*$. This means that the system of conditional equations (24) defines a very strong estimation model, and therefore all linear transformations of this system of equations yield the similar values of $\widehat{w}_k$ that can be called model values. This attribute results from the process of the estimation of the adjustments $\hat{\beta}_k$, which is based on the Lagrange function that includes the condition $\sum_k \hat{\beta}_k^2 = min$. The model values of $\widehat{w}_k$ obtained based on the various $\hat{\beta}_k^*$ have two main drawbacks:

- the model values may not correspond to the unit market values whatsoever,
- from each set of adjustments $\hat{\beta}_k^*$, completely divergent standard deviations of the estimated values are obtained, as well as of the estimated market value of the real estate being valued.

In order for the random adjustments $\hat{\beta}_k$ to satisfy the market and accuracy conditions at the fixed values of $\widetilde{w}_k$, the value of the following factor should be determined:

$$\theta_\beta = 1 - \frac{\sum_j (\hat{\beta}_j)^2}{\sum_j (\widetilde{w}_j)^2} \tag{26}$$

The factor θ_β reflects a part of the volatility of the unit market prices $\widetilde{w}_k$ explained by the random adjustments $\hat{\beta}_k$, hence its value should be equal to or greater than 0.95, i.e.

$$\theta_\beta \geq 0.95 \tag{27}$$

Failure to satisfy the criterion (27) means that the transaction prices of the analyzed real estate are also dependent on the values of the pricing attributes. In this case, the estimation of the conditional valuation model should take into account the conditional equations including both the geometric and pricing attributes.

Having verified the market information, the system of conditional equations (7) can be contained in the following analytical form:

$$\begin{array}{c} S_{11} \cdot \beta_1 + \cdots + S_{k1} \cdot \beta_k + a_{11} \cdot \rho_1 + \cdots + a_{u1} \cdot \rho_u = \Delta C_{W1} \\ \dots\dots\dots \quad \dots \quad \dots\dots\dots \quad \dots\dots\dots \quad \dots \quad \dots\dots\dots = \dots \\ S_{1i} \cdot \beta_1 + \cdots + S_{ki} \cdot \beta_k + a_{1i} \cdot \rho_1 + \cdots + a_{ui} \cdot \rho_u = \Delta C_{Wi} \\ \dots\dots\dots \quad \dots \quad \dots\dots\dots \quad \dots\dots\dots \quad \dots \quad \dots\dots\dots = \dots \\ S_{1n} \cdot \beta_1 + \cdots + S_{kn} \cdot \beta_k + a_{1n} \cdot \rho_1 + \cdots + a_{un} \cdot \rho_u = \Delta C_{Wq} \end{array} \tag{28}$$

where the number of conditions q must be less than the total the number of geometric and pricing attributes, i.e. $q < (k + u)$.

The estimation of the random adjustments $\hat{\beta}_k$ and $\hat{\rho}_k$ the following matrices into account:

$$B = [S \quad a]; \quad \beta^T = [\beta_1 \quad \dots \quad \beta_k]; \quad \rho^T = [\rho_1 \quad \dots \quad \rho_u]; \quad \Delta C_W^T = [\Delta C_{W1} \quad \dots \quad \Delta C_{Wq}] \tag{29}$$

where:

S - the rectangular matrix with the dimension $(q \times k)$, containing the information on the areas or volumes S_k of the analyzed real estate components, i.e. their geometric attributes,

a - the rectangular matrix with the dimension of $(q \times u)$, containing the information on the values of the pricing attributes a_u of the analyzed real estate,

β - the single-column matrix of the estimated random adjustments β_k to the approximate unit market values $\widetilde{w}_k$ of the real estate components,

ρ - the single-column matrix of the estimated random adjustments ρ_u to the approximate or zero values of the pricing factors,

ΔC_W - the single-column matrix of the constant terms with the dimension $(qx1)$, comprising the differences $\Delta C_W = C_T - C_S$ for each analyzed real estate creating a condition.

In creating the conditions, the random nature of the estimated values β_k and ρ_u requires the observance of the principle of the values of the pricing attributes being of similar order as the values of the geometrical attributes.

Taking into account the matrix denotations, the system of conditional equations (28) can be contained in the matrix form:

$$B \cdot \begin{bmatrix} \beta \\ \rho \end{bmatrix} = \Delta C_W \tag{30}$$

Considering the Lagrange function to find the conditional extremes for the system of Equations (30), the quadratic form for the random adjustments takes the following form:

$$\psi = \begin{bmatrix} \beta \\ \rho \end{bmatrix}^T \cdot \begin{bmatrix} \beta \\ \rho \end{bmatrix} + \Gamma \left(B \cdot \begin{bmatrix} \beta \\ \rho \end{bmatrix} - \Delta C_W \right) = min. \tag{31}$$

The first derivatives relative to the random adjustments β_k and ρ_u, and the Lagrange coefficients Γ equalized to zero, yield a system of equations, the solution to which takes the form:

$$\begin{bmatrix} \hat{\beta} \\ \hat{\rho} \end{bmatrix} = [B^T \cdot (B \cdot B^T)^{-1}] \cdot \Delta C_W \tag{32}$$

Finally, the most probable unit market values of the components of multi-component real estate are calculated from the following dependence:

$$\widehat{w}_k = \widetilde{w}_k + \hat{\beta}_k \tag{33}$$

For the pricing attributes, it is not possible to estimate their approximate values of the factors, therefore, the estimated values of $\hat{\rho}_u$ are ultimately the pricing factors on the local real estate market.

The analysis of variance is preceded by the calculation of the sum of squares of the random adjustments. The sum of the adjustments can also be calculated from the dependence:

$$\beta^T \cdot \beta + \rho^T \cdot \rho = \Delta C_W^T \cdot (B \cdot B^T)^{-1} \cdot \Delta C_W \tag{34}$$

The variance for the estimated model is calculated based on the sum of squares of the random adjustments and the number of the conditional equations:

$$\sigma_M^2 = \frac{\beta^T \cdot \beta + \rho^T \cdot \rho}{q} \tag{35}$$

The variance-covariance matrix for the estimated random adjustments is determined by the following relationship:

$$Cov \begin{bmatrix} \hat{\beta} \\ \hat{\rho} \end{bmatrix} = \sigma_M^2 \cdot [B^T \cdot (B \cdot B^T)^{-1} \cdot B] \tag{36}$$

The second criterion for selecting the market information is based on the factor θ_β that defines satisfying the market and accuracy conditions for the estimated parameters $\hat{\beta}_k$ with predetermined values of $\widetilde{w}_k$. The value of factor θ_β is calculated according to the dependence (26) and should satisfy the inequality (27). The positive verification of the criterion (27) allows the estimated parameters to be used for the valuation of multi-component real estate on the local real estate market.

The market value for the estimated real property for which the geometric and pricing attributes can be defined by the single-row matrix $[S_W \quad a_W]$ is determined from the formula:

$$WR_N = S_W \cdot \widetilde{w} + [S_W \quad a_W] \cdot \begin{bmatrix} \hat{\beta} \\ \hat{\rho} \end{bmatrix} = [S_W \quad a_W] \cdot \begin{bmatrix} \widehat{w} \\ \hat{\rho} \end{bmatrix} \tag{37}$$

The variance of the estimated market value due to the inaccuracy of the estimation of the random quantities contained in the covariance matrix (36) is given by formula:

$$\sigma^2(WR_N) = [S_W \quad a_W] \cdot Cov \begin{bmatrix} \hat{\beta} \\ \hat{\rho} \end{bmatrix} \cdot \begin{bmatrix} S_W \\ a_W \end{bmatrix} \tag{38}$$

Taking into account the proposed criteria and conditions in the estimation of the conditional valuation model usually leads to standard deviations of the estimated market value at a level not exceeding 5% of its value, i.e. $\sigma(WR_N) \leq 0.05 \cdot WR_N$. For most market information, the 10% standard deviation is assumed, and therefore this accuracy of the real estate valuation should be considered high.

4. Summary

The valuation of multi-component real estate is an extremely demanding procedure that can be implemented in a comparative approach by using the parametric model or the conditional model.

However, the models used in the method of statistical market analysis cannot be used uncritically. The aim of this research paper was to identify the conditions that would facilitate the correct use of these models.

In order to make sure that the obtained parameters are close to the market values, the constant terms determined in the parametric model and the conditional model cannot be greater than 25% of the corresponding transaction price. Real properties that do not satisfy this criterion may be burdened with gross errors. The information on these properties should be verified and, if no errors are identified in the data, they ought to be removed from the database for market analysis and valuation.

However, even if the above condition is met, it is necessary to specify the factor that will determine the conformity of the estimated parameters with the market information before the estimated model is used for the real estate valuation. The accuracy of the selection of market information for estimating parameters of the valuation model, determined in accordance with the formulas described in this research paper, should be greater than or equal to 0.95.

If the estimation of the valuation models is performed taking both of the proposed criteria into consideration, then the standard deviation of the estimated market value for the specific real estate being valued should not exceed 5% of its value. Such accuracy of real estate valuation can be considered very high, since 10% standard deviation is assumed for most market information.

5. References

ADAMCZEWSKI Z., 2006, *Elementy modelowania matematycznego w wycenie nieruchomości. Podejście porównawcze (Elements of mathematical modeling in real estate appraisal. Comparative approach)*,. Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa.

ADAMCZEWSKI Z., HOPFER A., 2008, *Trend udziału gruntu i budynku w wartości nieruchomości zabudowanej (Relationship of Separated Land and Building Value in Built-Up Real Estate Valuation),* Studia i Materiały Towarzystwa Naukowego Nieruchomości, vol. 16, no. 3, pp. 19-32.

ADAMCZYK T., BIEDA A., PARZYCH P., 2019, *Appraisal of real estate with various functions in the context of sustainable development*, Journal of Applied Engineering Sciences, vol. 9(22), iss. 1, art. no. 247, pp. 7-18

ADAMCZYK T., DĄBROWSKI J., 2010, *Algorytm estymacji rynkowych wartości wskaźników kosztu wytworzenia i zużycia łącznego obiektów budowlanych posiadających różne okresy eksploatacji (Estimate Algorithm of Development Cost and Combine Wear Cost Indicators Associated with the Market Value of Real Properties with Different Exploitation Time Periods),* Studia i Materiały Towarzystwa Naukowego Nieruchomości, vol. 18, no. 1, pp. 83-93.

ADAMCZYK T., JASIOŁEK J., 2012, *Podejście kosztowe w modelowaniu wartości rynkowej nieruchomości zabudowanych (Modelling of Built-Up Properties Market Value Based on Cost Approach*), Studia i Materiały Towarzystwa Naukowego Nieruchomości, vol. 20, no. 2, pp. 235-246.

ADAMCZYK T., 2017, *Application of the Huber and Hampel M-estimation in real estate value modeling,* Geomatics and Environmental Engineering, vol. 11, no. 1, pp. 15–23.

BIEDA A., 2017, *Parametric model of real estate valuation for land located in different land-use zones,* Geomatics and Environmental Engineering, vol. 11, no. 4, pp. 17-33.

BIEDA A., 2018, *Conditional Model of Real Estate Valuation for Land Located in Different Land Use Zones.* Real Estate Management and Valuation, vol. 26, no. 1, pp. 122-130.

BIEDA A., BIEDA A., ADAMCZYK T., PARZYCH P., 2016, *The procedure for the valuation of real properties developed with passive and energy-efficient houses,* 16th International Multidisciplinary Scientific Geoconference, Conference proceedings, vol. 2, pp. 633–640.

BRZEZICKA J., WIŚNIEWSKI R., 2012, *Behawioralne aspekty rynku nieruchomości (Behavioral Aspects of the Real Estate Market),* Studia i Materiały Materiały Towarzystwa Naukowego Nieruchomości, vol. 20, no. 2, pp. 21-31.

BRZEZICKA J., WIŚNIEWSKI R., 2015, *Kierunki ewolucji badań rynku nieruchomości (The Directions of the Real Estate Market Research Evolution)*, Świat Nieruchomości, no. 2(92), pp. 43-48.

BYDŁOSZ J., PARZYCH P., DĄBROWSKI J., 2011, *The possibilities of real estates market development in Poland in connection with INSPIRE directive*, Geomatics and Environmental Engineering, vol. 5, no. 1, pp. 15-23.

CZAJA J., 1997, *Modele statystyczne w informacji o terenie (Statistical models in land information)*,. Wydawnictwo Akademii Górniczo-Hutniczej, Kraków.

CZAJA J., 2001, *Metody szacowania wartości rynkowej i katastralnej (Methods of Estimating Market and Cadastral Property Value),* Wydawnictwo KOMP-SYSTEM, Kraków.
CZAJA J., PARZYCH P., 2008, *Zaawansowane modele statystyczne wyceny nieruchomości zurbanizowanych (The Urbanized Estates Valuation Advanced Statistical Models for Mortgage Securing),* Studia i Materiały Towarzystwa Naukowego Nieruchomości, vol. 16, no. 3, pp. 7-18.
CELLMER R., BEŁEJ M., ŹRÓBEK S., KOVAC M., S., 2014, *Urban land value maps-a methodological approach.* Geodetski vestnik, vol. 58, no. 3, pp. 535-551.
CHEN Z., HU Y., ZHANG C. J., LIU Y., 2017, *An Optimal Rubrics-Based Approach to Real Estate Appraisal*, Sustainability, vol. 9, no. 6, pp. 1-19.
DAWIDOWICZ A., RADZEWICZ A., RENIGIER-BIŁOZOR M., 2014, *Algorithm for purposes of determining real estate markets efficiency with help of land administration system,* Survey review, no. 46(336), pp. 189-204.
ŁUCZYŃSKI R., KOTARBA P., 2017, *Analysis of selected standards of floor area measurements on the example of office premises,* Geographic Information Systems Conference and Exhibition "GIS ODYSSEY 2017", Conference proceedings, pp. 235-246.
KUCHARSKA-STASIAK, E. (2014). *Behawioralne aspekty w wycenie nieruchomości (Behavioral Aspects of the Real Estate Valuation)*, Zarządzenie i Finanse, no. 4, pp. 247-261.
PARZYCH P., 2007, *Modelowanie wartości nieruchomości zurbanizowanych (The Modeling of Urban Real Estates' Values),* Studia i Materiały Towarzystwa Naukowego Nieruchomości, vol. 15, no. 3-4, pp. 167-176.
PARZYCH P., 2008, *Warunkowe modele estymacji jednostkowych cen elementów nieruchomości (The conditional estimation models for unit estates' elements)*, Studia i Materiały Towarzystwa Naukowego Nieruchomości, vol. 16, no. 2, pp. 93-100.
PARZYCH P., 2009a, *Modele estymacji wartości rynkowej lub katastralnej nieruchomości zurbanizowanych, rolnych i leśnych (Estimation models of the market of cadastral value of urbanized, agricultural and forest estates),* AGH Uczelniane Wydawnictwa Naukowo-Dydaktyczne, Kraków.
PARZYCH P., 2009b, *Warunkowy model estymacji jednostkowych cen dla nieruchomości leśnych (The conditional model of unit prices estimation for forest estates).* Studia i Materiały Towarzystwa Naukowego Nieruchomości, vol. 17, no. 1, pp. 47-57.
PARZYCH P., 2011, *Modelling of urban estates' values*. Geomatics and Environmental Engineering, vol. 5, no. 4, pp. 63-72.
PARZYCH P., BYDŁOSZ J., 2007, *The Algorithm of Urban Estates Valuation.* FIG Working Week 2007, Conference proceedings, pp. 1-9.
PARZYCH, P., BYDŁOSZ, J., 2008, *Problems of real estates' valuation for taxing purposes,* FIG Working Week 2008, Conference proceedings, pp. 1-6.
PARZYCH P., CZAJA J., 2015, *Szacowanie rynkowej wartości nieruchomości (Estimation of real estates market value).* Wydawnictwo AGH, Kraków.
RENIGIER-BIŁOZOR M., BIŁOZOR A., WISNIEWSKI R., 2017, *Rating engineering of real estate markets as the condition of urban areas assessment,* Land Use Policy, vol. 61, pp. 511-525.
ZYGA J., 2012, *Istota podobieństwa w procedurach wyceny nieruchomości (Essence of similarity in real rstate appraisal arocedures),* Rzeczoznawca Majątkowy, vol. 75, no. 3, pp. 22-26.
ŹRÓBEK S., BEŁEJ M., 2000, *Podejście porównawcze w szacowaniu nieruchomości (The comparative approach in estimation of property),* Educaterra Sp. z o.o., Olsztyn.