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SYNERGISTIC NETWORK CONNECTIVITY AMONG URBAN AREAS BASED ON NON-LINEAR MODEL OF HOUSING PRICES DYNAMICS

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Abstract

The assumption of a strong positive relationship between the specificity of an urban area and the operation of the housing market, which responds to megatrends in the market environment, has been the foundation of the research concept adopted for this study. The study treats the housing markets as complex, adaptive dynamic systems which develop through synergistic network responses. This paper employs a nonlinear critically-damped harmonic oscillator model and phase diagrams to describe the dynamics of housing prices (in the years 2006-2016) in order to demonstrate the synergistic network connections in selected Polish cities. In another important part of the study, the authors propose to employ non-classical dynamic measures, i.e. the absolute time of delay, relaxation time and a long-term level of equilibrium. The study has shown that network connections are strongly synchronized during periods of housing market instability (2006-2007), whereas the process of un-synchronization is observed during a period of stability (2008-2016). Moreover, phase diagrams have been used to demonstrate the similarity of trends in housing prices as well as the shapes of individual trajectories and the existence of multiple points of quasi-equilibrium.

Key words: *urban areas, network connectivity, housing market.*

JEL Classification: *D40, R10, R11, R31.*

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1. Introduction

Particularly intense spatial conflicts, caused by diverse goals (social and economic) of heterogeneous business units, take place in zones of urban space (SKIFTER et al. 2016; ZHU et al. 2018; ROBINSON and GRAHAM 2018). Cities are a multi-layer spatial structure in which people and products of their activities are gathered in not-too-distant places (PARYSEK 2008). Satisfying the housing needs of people is the main function of urban space; the intensity and direction of the urbanization processes is determined by local geospatial conditions and the micro- and macroeconomic situation. According to RENIGIER-BIŁOZOR et al. (2017) the need for housing takes one of the main places in the hierarchy of the importance of needs. It is an integral and indispensable element of meeting needs in virtually every sphere of human life: in the sphere of security needs (for stability), social needs (home, meeting place), needs for recognition and respect (prestige, highlighting social position), and the need for self-realization (proof of independence).

There is a strong positive feedback between the specific nature of an urban area (which arises from the local features of cities in a geospatial, economic, administrative and social sense), and the function of the housing market, which, at the same time, responds to megatrends in the market environment. The housing market is - on the one hand - a factor which stimulates the process of changes of the urban space (increase in house prices vs. developer processes) and, on the other, the actions of space users change the conditions of operation of the market, e.g. changes of zoning functions, construction of linear infrastructure etc. Therefore, urban space is an extremely complex system in which a number of factors and relations associated with space interact (geospatial relations).

This paper presents the results of the examination of the rapid increase in housing prices (in 2006-2007) and subsequent steady decrease in the form of damped oscillations (in 2008-2016). The aim of this paper is to show that the classic paradigm of independent local housing markets is inadequate to how the markets behave because of their strong network links. This assumption is based on an observation of the synergism of trends in housing prices (like in systems of communicating vessels), despite principal differences in the geospatial attributes of the cities under study. This paper presents a concept of applying a nonlinear critically-damped harmonic oscillator model and phase diagrams to describe the dynamics of housing prices in order to demonstrate the synergistic network links in selected cities in Poland.

The paper is structured as follows. After a short introduction into the research (Section 1), a short description of the housing markets is given in Section 2, together with an overview of previous results published in the field and theoretical basis for the performed research. Section 3 presents the methodology of the research, and Section 4 presents the data description, the procedure of applied methodology and a discussion of the obtained results. Section 5 presents conclusions drawn throughout this work.

2. Theoretical basis of conducted research

2.1. Housing market

Housing is one of the areas of the economy which has a significant effect on the level of satisfying social needs, the dynamics of economic processes and effectiveness of developmental activities. Interconnections of housing development and the economy indicate that the former plays a significant role in elevating the level of social, economic and spatial cohesion of the country (CASE et al. 2005; TAYLOR 2007; LIS 2012). Housing is (GALSTER 1996) a special type of commodity because it is spatially immobile, highly durable, highly expensive, multidimensionally heterogeneous and physically modifiable. These characteristics shape attitudes and behaviors toward housing and, in turn, influence neighborhood characteristics, mortgage markets, national housing policies and urban growth and decline. The housing system is made up of interrelations of private and public entities which are engaged in the process of planning, construction, and consumption of housing resources, together with a body of rules set by institutions regulating these relations in a permanent manner (BALL 2013). The price of an apartment is affected by the relationship between housing supply and demand, and house sale and purchase transactions are concluded on the housing market.

2.2. Related research

The mechanisms of housing market operation and spatial-temporal variability of house prices have been studied in detail by researchers representing various branches of science. SCANLON and WHITEHEAD (2011), CELLMER et al. (2016) and SZCZEPAŃSKA (2017) analyzed the effect of macroeconomic factors on housing markets. CARRILLO (2012) specifies and estimates a computationally tractable stationary equilibrium model of the housing market. CAMPBELL and COCCO (2003) studied the dynamics of housing prices in reference to changes in the conditions of financing the housing market. DAVIDOFF (2006) tested the intuition that households whose incomes covary relatively strongly with housing prices should own relatively little housing. DITTMANN (2013) demonstrated the convergence and divergence on local housing markets and similarities between asking and transaction prices of real estate. SZCZEPAŃSKA et al. (2015) analyzed the effect of road traffic noise on the prices of residential properties in European cities. COULSON (2008) applied hedonic models to a housing market to identify the effect of a population decrease on housing prices. TROJANEK and HUDEREK-GLAPSKA (2018) applied the hedonic price model to study the effect of noise generated by Warszawa's Chopin Airport on the prices of apartments in multi-family houses. HECKMAN (2008) analyzed causality on the housing market using MFPs (multivariable fractional polynomials). In many publications, the analysis of the housing market and predictions of property prices have been supported by the use of spatial models (BOWEN et al. 2001; VALENTE et al. 2005; BOURASSA et al. 2010). CELLMER et al. (2014, 2018) proposed integrating statistical and geo-statistical models to map the spatial distribution of average property values and the principles for the delimitation of homogeneous zones in the functional structure of cities. In their fundamental work, DiPasquale and Wheaton (1994) developed a model of housing market dynamics that introduces two market segments (consumption- and investment-related), creating a demand model in a static and dynamic approach. This model assumes that, after the occurrence of significant turbulence in the housing market, so-called partial equilibrium may be restored assuming that the expectations of market participants are reasonable, while, at the same time, it indicates what conditions need to be met in order for housing markets to be more efficient.

2.2. Conceptual framework

The literature review presented in Section 2.2 shows that the following theoretical assumptions prevail in publications on housing:

- the idea of perfect competition and the assumption that economic entities will interact only through a market mechanism (Say's model - the economy as a closed system),
- assumption of the rationality of behavior of business entities (actions of unreasonable participants cancel each other out),
- Walras's theory of general equilibrium (single-point market equilibrium),
- Marshall's theory of partial equilibrium (reductionism - ceteris paribus),
- paradigm of linearity (determinism of phenomena),
- Fama's efficient market hypothesis (prices of goods fully reflect the public information on such goods).

The deterministic image of the phenomena based on the paradigm of linearity, the idea of equilibrium and the hypothesis of an efficient market are the consequences of the long-term stability (in relation to the level of stability of financial markets) of the housing market in Poland. Currently, experience of the global financial crisis as an effect of the mortgage crisis (Mizen, 2008; Ashton, 2009; Roubini, Mihm 2010; Demyanyk, Van Hemert 2011) has resulted in the development of non-classic research trends. They propose looking at the issue in light of limited the rationality of economic entities, multi-point dynamic market equilibrium, dispersed interaction between heterogeneous entities, continuous adaptation of evolving entities and the price- and non-price-related mechanism of market coordination. According to Stiglitz (2002), the classical forces of supply and demand should be taken into account, but only as a part – rather than as a whole – of the analysis, which should be supplemented by issues arising from the asymmetry of information and the role of state institutions in overcoming them. This makes it necessary to use evolutionary models instead of equilibrium models which, in turn, requires taking into account the effect of historical phenomena and observations of the dynamics of the observed processes. More on these issues can be found in: Arthur (1999); Stiglitz (2002); Colander et al., (2004); Witt (2008); Wojtyna (2008); Roubini and Mihm (2010); Jakimowicz (2010) and Bełej (2016).

In Poland, stable increases in housing prices have been observed since the political transformation from a system of a centrally controlled economy to a market economy started in 1989. After Poland's accession to the European Union in 2004, the previously stable trends began to change slightly, which could still be regarded as acceptable deviations from long-term trends. In 2006-2007, the annual housing price growth rate exceeded 100%, which is unique in the 25-year history of the Polish housing market. This trend was reversed in 2008-2016, and housing prices started to drop by 3% to 6% a year. It is the authors' opinion that the rapid changes in real estate prices in Poland in 2006-2007 were not an adjustment of a long-term trend, but defined the structural transformation of the entire housing market, which was trying to reach a new equilibrium level through turbulent movements. In this sense, the classic research assumptions should be expanded considerably.

The authors propose that housing markets be treated as complex adaptive systems with the characteristics of open, dynamic and non-linear systems, which – when influenced by the market environment – evolve in a characteristic way between various states of equilibrium. Due to the variability of stimuli, property markets, as dynamic systems, remain in stable states in the long term, and only periodically go into a state of instability, which can be seen as the exhaustion of opportunities for evolution in their current form and the need to search for a new, alternative path of development.

3. Methodology description

The concept of housing markets as complex, adaptive dynamic systems assumes that they develop by evolution between the states of the system in search of equilibrium. According to Domański (2012), the system is not only expanding in size during the development process, but its structure is also changing, and periods of structural change are intertwined with periods of gradual change. In this study, the nonlinear critically-damped harmonic oscillator model (CDHO) and phase diagrams were applied to diagnose the dualism of structural and gradual changes by demonstrating the principal differences in the dynamics of housing prices during the phase of stability and instability of housing markets. At the same time, the application of the above research methods may allow for the detection of network connections, links, levels of synergy and changes in the hierarchical structure between the urbanized areas under study (in terms of the dynamics of housing prices).

3.1. Nonlinear critically-damped harmonic oscillator model

In the current paper we employ the model of a critically-damped harmonic oscillator (CDHO) to study the time-dependent evolution of housing prices taken as a probe of the behavior of a very specific dynamical system – the housing market. Previously, similar methods were used to study the response of financial markets to shocks and crises concerning price expectations (IDEAND, SORNETTE 2002; SANDOVAL, FRANCA 2011).

The search for similarity between the dynamics of housing prices and the non-linear mathematical function describing the behavior of the harmonic oscillator arises from the non-stationary nature of the phenomenon of rapid changes in housing prices, as after the phase of rapid growth, there appears a phase of asymptotic return to the long-term equilibrium in the form of a series of damped oscillations. Observables are housing prices, which reflect diversity of processes responsible for data variability: from long-term stationary changes to sharp transformations of structural origin.

The model of a critically damped harmonic oscillator was fitted to housing prices by means of non-linear regression. The behaviour of CDHO as a function of time t is described by the following equation:

$$y(t)=\left[A(t-t_0)+B\right]e^{-\left(\frac{t-t_0}{\tau}\right)}+y_{EQ} \tag{1}$$

where: t_0 – is the absolute time delay relative to an arbitrary time-stamp, τ – relaxation time, y_{EQ} – long-term equilibrium level, e – Euler's number ($e \approx 2.72$), and A and B are certain constants determined numerically.

The CDHO model allows the parameters characterizing a given dynamic system to be determined in terms of negative feedback mechanisms, susceptibility to changes and dissipative coupling with the environment, which may be helpful in the comparative analyses of the dynamics of housing prices.

The absolute time delay t_0 determines the time shift between the pre-arranged beginning of the observation and that of rapid changes in a given subsystem, which allows to structure the network of connections in terms of the rate of stimuli propagation throughout the network. Of all the analyzed markets, the one with the lowest value of t_0 was found to be dominant. It was treated as an initiator of changes propagating through the network of connections, which allows to treat real estate markets in a similar way to a system of communicating vessels.

The relaxation time τ is defined as the time after which the value of a dependent variable decreases to about 1/3 of the maximum value, i.e. it is a parameter of the system that determines the time of the disturbance attenuation when the system enters the equilibrium path. On the other hand, the relaxation time also contains information about the link between the system and the environment and is a measure of system inertia. The lower the τ value, the stronger the damping of emerging stimuli, which can affect the long-term stability of the system.

The long-term equilibrium level y_{EQ} determines the level with which the prices converge asymptotically through damped oscillations.

3.2. **Phase diagrams**

One of the possible forms of visualization of the dynamics of the system under study is to plot its trajectory in the classic configuration space, i.e. in the abstract space of states of the system, whose coordinates are directly related to the quantities defining the state of the system. For the analyzed housing markets, this will be the unit transaction price of an apartment. The successive points in the configuration space correspond to successive states of the system in time, and the curve connecting these points is the system evolution path. Therefore, the configuration space illustrates the evolution of the system, treating time as an independent variable. By modifying the coordinates that describe the state of the system, including eliminating time as an independent variable, the trajectory of the system evolution can be alternatively plotted in the phase space. Such a space treats time as a hidden variable (parameter), which allows for illustrating not only the path of evolution itself, but also the nature of the system, e.g. conservative, non-conservative, stable, chaotic, etc. Moreover, the dimension of the phase space is directly associated with the number of degrees of freedom of the system, so the trajectory of a fully-defined deterministic system is an intersection-free curve in the phase space. By its very nature, the analysis of the evolutionary path of the system in the phase space reveals more information about the possible behavior of the system than in the configuration space. The use of phase diagrams to provide a graphical portrayal of the dynamic systems underlying economic models has a long tradition, reaching nearly a hundred years to Marshall's discussion of pure theory of international trade (QUIRK et al. 1968). PATINKIN (1974) discussed the application of phase diagrams proposed by Metzler in an analysis of macroeconomic models. GRADZEWICZ et al. (2010) applied phase diagrams to studying economic cycles in Poland.

4. Data description, results and discussion

4.1. Data description

The research was conducted using time series of housing prices in six Polish cities, considered to be the leading centers in their regions: Warszawa (Mazovia – central region), Krakow (Małopolska – southern region), Poznan (Wielkopolska – western region), Gdansk (Pomerania – northern region), Bialystok (Podlasie – eastern region) and Olsztyn (Warmia and Mazury – eastern region). Table 1 shows the main statistical data for the selected cities.

Table 1

Main statistical data

Town	Area [km²]	Population	Population density people/km²]	Budget expenditure per capita [EUR]
Warsaw	517	1,715,517	3317	1764
Krakow	327	758,463	2321	1315
Poznan	262	550,742	2103	1357
Gdansk	262	460,427	1758	1306
Bialystok	102	294,921	2888	1200
Olsztyn	88	172993	1960	1840

Source: Central Statistical Office.

The average quarter-based prices per 1 m² of housing property on the secondary market, from Q3 2006 to Q3 2016, were used in the study. The data were taken from the Housing Prices Database maintained by the National Bank of Poland.

The analysis of housing price dynamics based on quarterly time series encounters a classic problem of research on housing markets. In particular, time sub-periods, heterogeneous sets of housing transactions, which constitute completely independent (relative to the previous period) data populations (housing prices), are averaged. In effect, the aggregated measures (mean, median) determined in this manner may exhibit large differences (high variability) in particular sub-periods. KOKOT (2017) proposes to reduce this problem significantly by smoothing the time series of housing prices by using moving averages. Following this concept, the time series analyzed were pre-smoothed with a 4253H filter. This method consists in performing several smoothing operations using a moving average with Hanning's time window of various sizes. The procedure allows the amplitude of noise in the original raw data waveform to be significantly reduced and, as a result, while maintaining the original trend of housing price dynamics, it reduces the problem of data inconsistency in different time periods. The application of this procedure to various research areas has been the subject of much scientific debate (VELLEMAN 1977; KARAGIANNIS et al. 2003; KOKOT, BAS 2013; AZMI et al. 2017). Table 2 presents the basic descriptive statistics for time series of housing prices (raw and smoothed with the 4253H procedure), for the examined cities. Figure 1 shows the visualization of the raw data and the results of data smoothing using this procedure.

Table 2

Basic descriptive statistics for time series of housing prices (raw and smoothed with 4253H filter)

	Average	Median	Minimum	Maximum	Standard deviation
Bialystok	4024	4028	2539	4647	420
Bialystok _4253H	4023	4063	2529	4531	410
Gdansk	5371	5383	3926	6267	528
Gdansk_4253H	5359	5439	3926	6136	484
Krakow	6175	6115	5193	7309	391
Krakow_4253H	6182	6116	5391	7015	343
Olsztyn	4285	4269	3126	4938	340
Olsztyn_4253H	4280	4244	3131	4816	332
Poznan	5204	5181	3332	6356	515
Poznan_4253H	5197	5208	3332	6031	479
Warszawa	7808	7601	6232	9137	722
Warszawa_4253H	7804	7659	6232	9005	685

Source: own study.

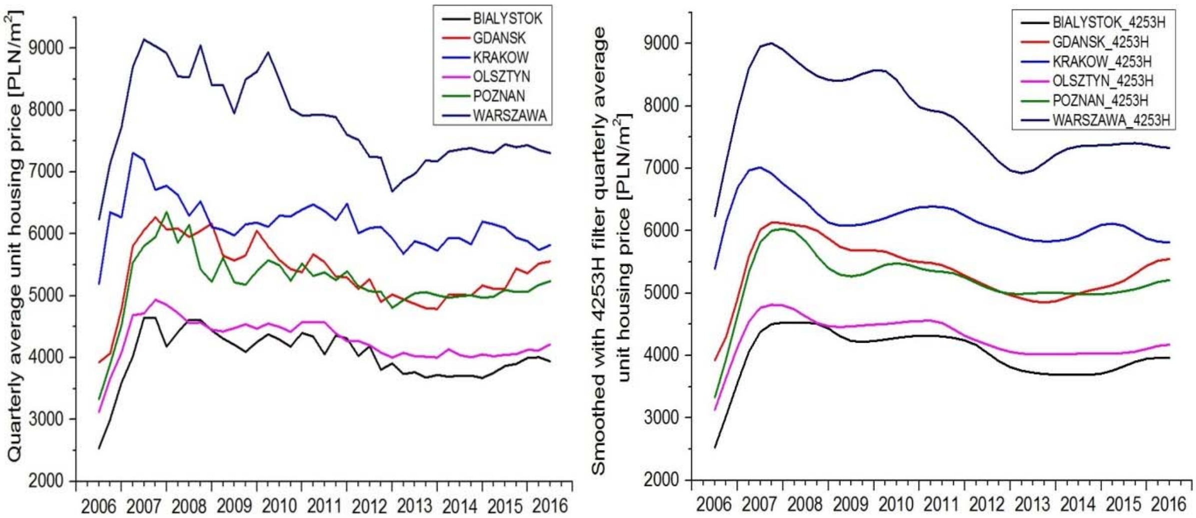

Fig. 1. Plots of transactional housing prices (per square meter) in selected cities in Poland: quarterly averaged (left) and smoothed with 4253H filter (right). *Source:* own study.

An analysis of the dynamics of the time series of housing prices presented in Figure 1 shows a strong synergy of price response in each of the examined Polish cities. This observation is reinforced by an almost uniform feedback from all housing markets (at the end of 2007), when two years of unusually dynamic, commensurate price increases (over 100% per year) were followed by a synergistic shift in trends and long-term steady declines in housing prices. The authors believe that the almost homogeneous response of the housing markets under study, both during a period of instability (2006-2007) and one of stability (2008-2016), suggests that local factors did not have a significant impact on the price dynamics. Each of the examined cities has its own specific geospatial, economic, administrative and social attributes, which affect the prices of housing; however, the almost uniform response of all housing markets makes it possible to search for network connections between them and to determine the levels of synergy, time delays and changes in the hierarchical structure between the urban areas under study (in terms of the dynamics of housing prices).

4.2. Results of conducted research

During the first stage of the study, preliminary analyses of the fitting of selected statistical models to the time series of housing prices in Warszawa, the capital of Poland, were carried out. Various numerical fittings were carried out for the collected data: using the classic linear regression model (LINEAR), polynomial regression model (POLYNOMIAL), multi-peaks fitting using the Lorentzian function (LORENTZ) and multi-peaks fitting using the Gaussian function (GAUSS). The results of the fitting procedure are shown in Figure 2.

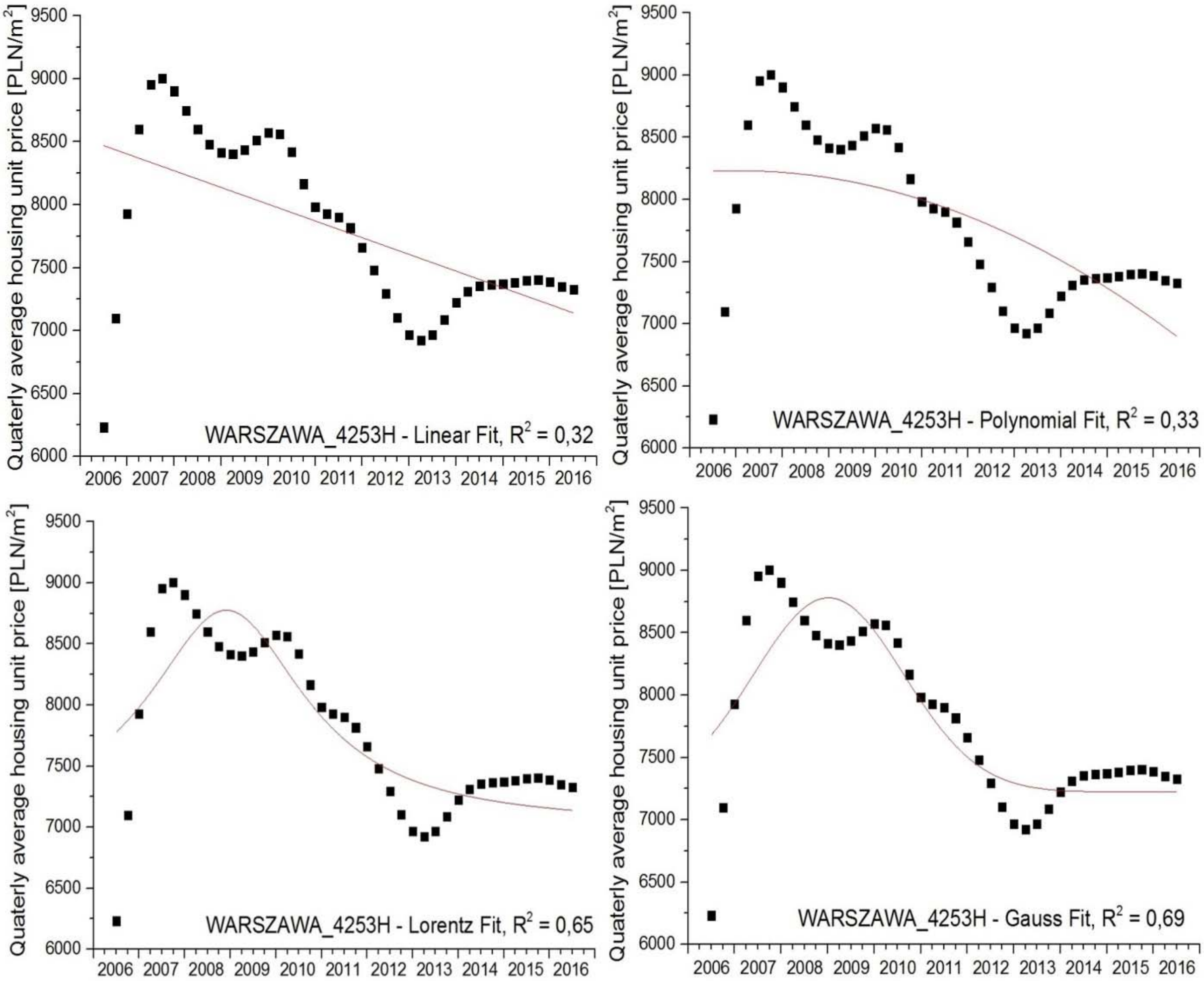

Fig. 2. Comparison of numerical fits of various statistical models to quarterly averaged unit housing prices in Warszawa. *Source:* own study.

The values of the determination index for the linear and polynomial regression models were at a similar, low level. Although the application of more complex non-linear models has resulted in a significant increase in the fitting quality, it did not exceed the R^2 level of 0.7. The results, both for linear and non-linear models, show that the adopted research concept, consisting in the search for alternative non-linear models, is fundamentally correct.

In the next stage of the study, the assumptions (presented in detail in Section 3.2) concerning the applicability of CDHO for given data series of housing prices were verified. This model is well-fitted to data consisting of at least two fundamentally different processes (rapid growth and asymptotic decrease). This directly refers to the idea of non-linear models describing the behavior of oscillators, which, after intense awakening from equilibrium (the phase of dynamics growth), return to equilibrium through gradually fading oscillations. This analogy can be applied to the time series shown in Figure 1. The calculation process was carried out in the OriginPro 9.0 statistical software, using the CDHO functions defined by the authors. The results of fitting of the CDHO model are shown in Table 3, whereas the fitting of the CDHO function to the empirical data is shown in Figure 3. The results of the fitting allow for drawing a number of interesting conclusions concerning the dynamic systems under study.

Table 3

Results of numerical fitting procedure of the CDHO model to unit housing prices (2006-2016)

	t_0 [quarters]	t_{rel} [quarters]	τ [quarters]	y_0 [PLN]	R^2
Bialystok _4253H	5.0	2.2	5.76 ± 0.43	3780 ± 50	0.89
Gdansk_4253H	4.5	1.7	4.90 ± 0.40	5080 ± 60	0.85
Krakow_4253H	4.1	1.3	2.84 ± 0.26	6030 ± 40	0.78
Olsztyn_4253H	4.4	1.6	5.28 ± 0.41	4070 ± 40	0.87
Poznan_4253H	5.1	2.3	4.20 ± 0.20	5050 ± 50	0.90
Warszawa_4253H	2.8	0.0	5.60 ± 0.40	7180 ± 80	0.89

Description: t_0 – is the absolute time delay with respect to an arbitrary time-stamp, t_{rel} - relative time delay with respect to Warszawa, τ – decay constant, and y_0 – asymptotic equilibrium level.

Source: own study.

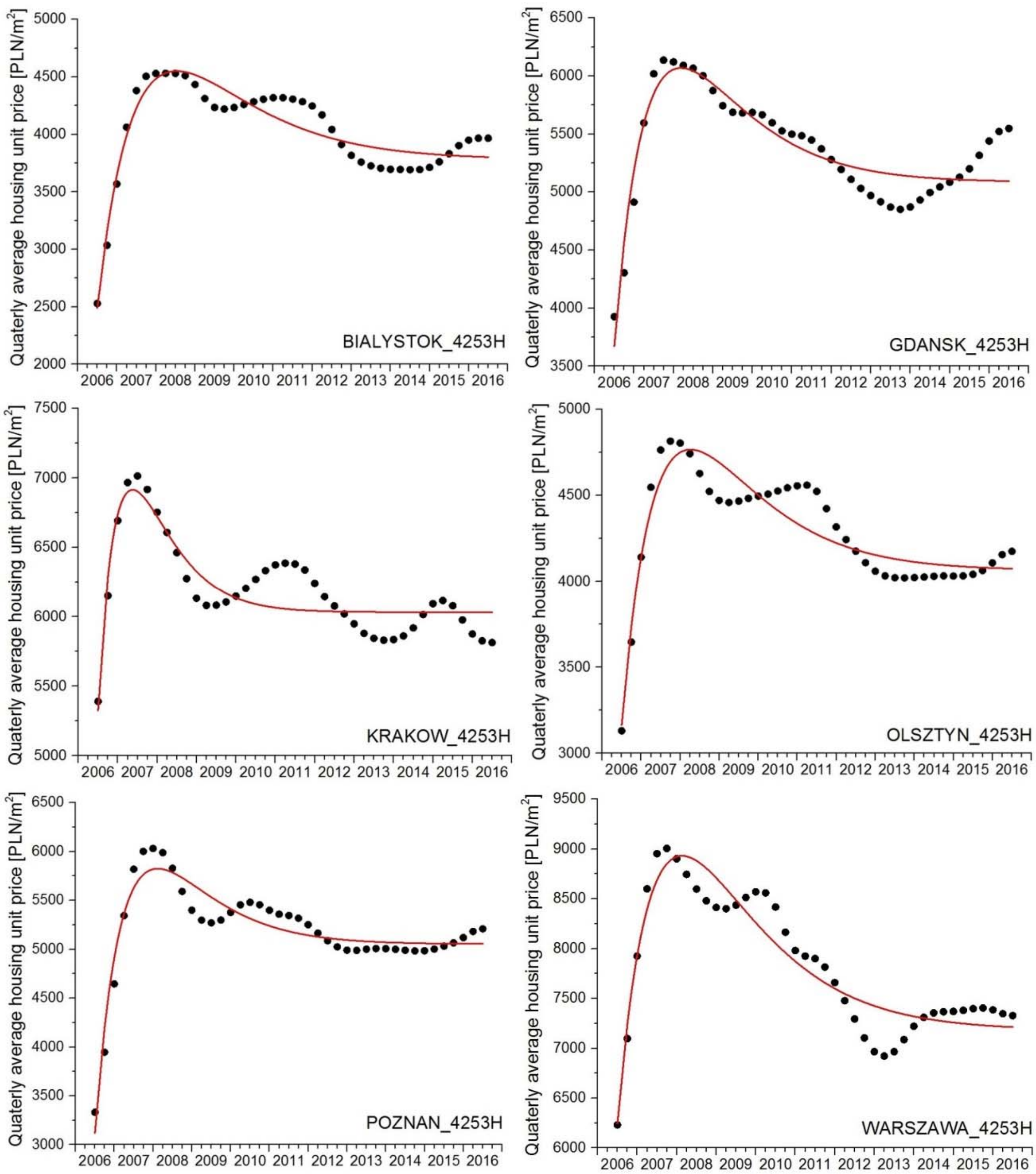

Fig. 3. Plots of housing prices smoothed with 4253H filter (closed dots) with appropriate curves from numerical fit with CDHO model (red lines). *Source:* Own study.

Firstly, owing to the use of CDHO, it was possible to significantly improve the quality of the model fitting to the empirical data compared to previous models. The determined R2 coefficient was found to range from 0.78 (Krakow) to 0.90 (Poznan). The high level of CDHO fitting in the period of rapid increases in housing prices (the period of instability 2006-2007) is in line with the high proportional growth rate for all cities. Such behavior demonstrates the high level of integration of the independent housing markets and confirms the previous assumptions about the network-like nature of the links between them. During the period of falling housing prices (the stability period 2008-2016), there are visible problems with fitting the function of CDHO, as each is reaching its own levels of equilibrium along a trajectory determined by a combination of local factors. Therefore, it may be concluded that network (synergistic) behaviors are revealed by a strong external stimulus (macroeconomic factors), while, during the periods of coming to stability, they are latent and tend to un-synchronize their trajectories of housing price dynamics under the influence of local conditions.

A common feature of all the housing markets under study is the long relaxation time τ defined as the time after which the dependent variable decreases to 1/e of its initial value (*e* –Euler's number). The longer this time, the slower the systemic changes described by the exponential decay function. The time for five cities (Bialystok, Gdansk, Olsztyn, Poznan and Warszawa) is similar – approx. 4-5 quarters. The shortest relaxation time was found for Krakow, which indicates the susceptibility of this market to rapid changes upon the influence of the surrounding macroeconomic environment.

When it comes to the next measure of dynamics, i.e. the absolute delay time t_0, it was found similar - at approx. 4-5 quarters - in all the cities except for Warszawa. The shortest time equal to 2.8 quarters for the housing market in the capital city Warszawa indicates its dominant role in influencing the trend dynamics of other housing markets. It also means that Warszawa acts as an initiator of changes (as it is the fastest to exchange impulses with the macroeconomic environment), which then spread onto the dependent markets through the network of connections. In this sense, an additional measure which shows the relative delay with respect to the dominant city has been proposed. As a result, Krakow was found to be the fastest to respond to the changes of the housing prices in Warszawa, because the relative time delay is only 1.3 quarters. Such a short response time, considering the specificity of housing markets, is in line with the previous result regarding the relaxation time τ. This means that Kraków can be considered the most unstable and dynamic system among the six Polish cities under study.

The long-term equilibrium level y_0, which is the level to which housing prices decrease asymptotically through damped oscillations, may serve as an informative measure (in the prognostic sense) and a measure of similarity. The results for Bialystok (3780 PLN ≈ 900 EUR) and Olsztyn (4070 PLN ≈ 970 EUR) indicate nearly uniform levels of long-term equilibrium levels. This result confirms the low position of these cities in the hierarchical system, determined by local factors (geospatial, economic, administrative and social). The results for Gdansk and Poznan were similar, but at a level of 5,000 PLN ≈ 1200 EUR. The highest level [y_0] was achieved by Warszawa, while Krakow was approx. 15% below this level (y_0 = 6030 PLN ≈ 1430 EUR).

Fig. 4 shows the phase diagrams of unit housing prices averaged over quarterly periods and then smoothed out with the use of the 4253H filter, plotted as a relation of price changes and prices. These graphs show that all the markets examined are, in some respects, very similar, but that sub-group-specific behavior can also be observed. In terms of similarities, the evolution of the relevant local markets starts just before a sudden jump, as evidenced by the tightening of the trajectory curve in the phase space. Moreover, the first point of equilibrium around which the trajectory runs is only a point of momentary equilibrium, as the trajectory quickly jumps to the area of attraction of the next point of equilibrium. Thirdly, finally, all paths in the phase space intersect (have nodes), which indicates that the number of degrees of freedom of each system is greater than 2. This shows that these systems are most likely linked together to form an extensive network similar to communicating vessels.

The specificity of individual housing markets is reflected in a slightly different transition between a momentary and a quasi-permanent state of equilibrium. In the case of the Warszawa market, an additional intermediate state appears, associated with the temporary halting of the downward trend in housing prices (2011), after which the system moves to a separate, long-term equilibrium state. The phase trajectories of the housing markets in Gdansk, Poznan and Krakow exhibit slightly different behavior. There are two clearly separated, but close, states of equilibrium, one of which is a state of temporary equilibrium. On the other hand, the trajectories for the markets in Bialystok and Olsztyn show a partial similarity to the market in Warszawa – both states of equilibrium are clearly separated from each other – although the intermediate state in both cases is very poorly marked. On this basis, the network may be roughly structured, with the leading markets in Warszawa, while the markets in Olsztyn and Bialystok will follow the emerging trends with some delay, and the markets in Gdansk, Poznan and Krakow, although stimulated by changes in the leading market, will be more autonomous.

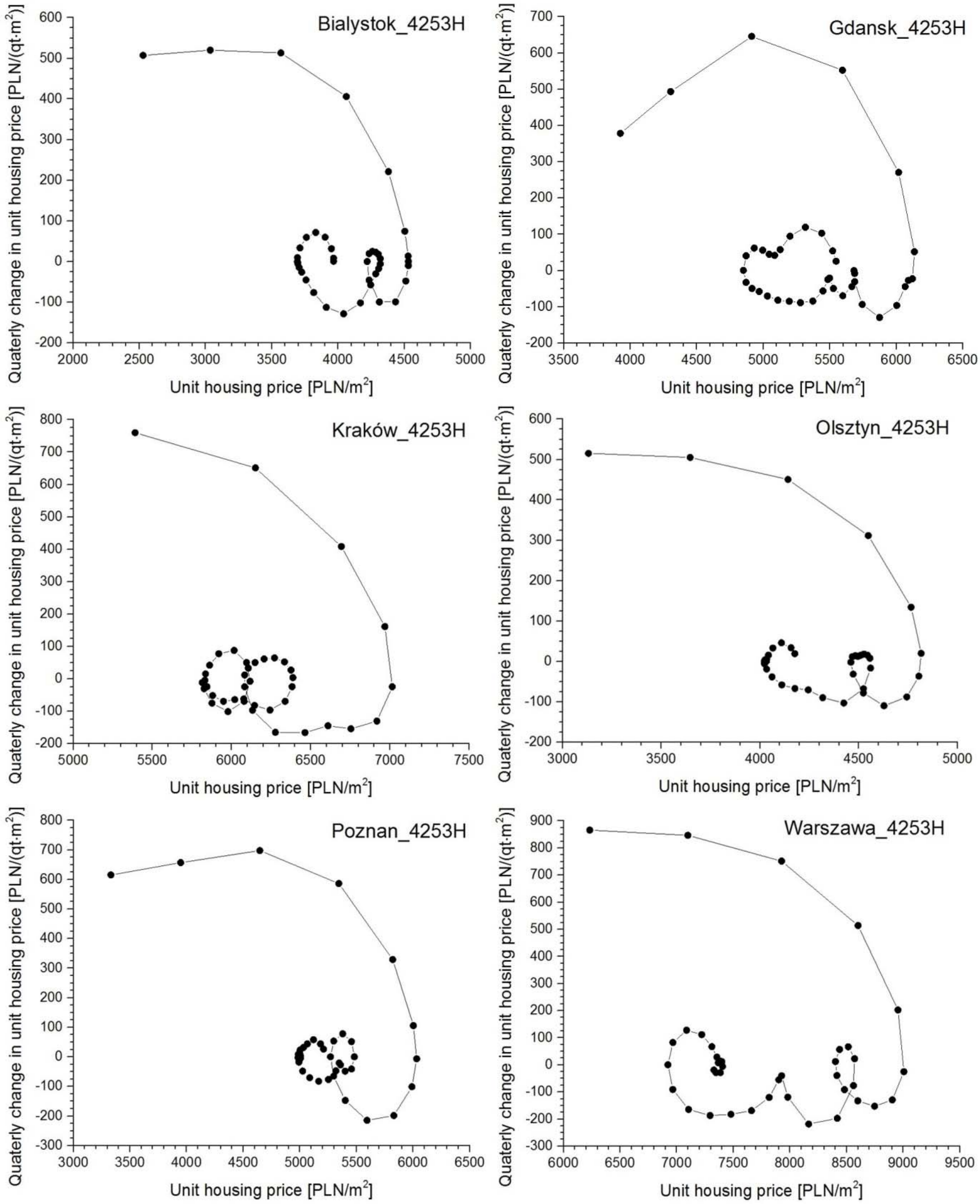

Fig. 4. Phase diagrams of transaction housing prices (per square meter) in selected towns in Poland after arithmetic averaging followed by smoothing with the 4253H filter. *Source:* own study.

5. Conclusions

This paper presents the concept of using a non-linear model of a critically damped oscillator and phase diagrams to describe housing price dynamics in selected cities in Poland in the years 2006-2016 to demonstrate the network connections of urbanized areas for the purposes of real estate management.

It was shown in the course of the study that the application of the non-linear CDHO model significantly increased the quality of fitting (measured as the determination coefficient) to the time series of housing prices, as compared to the classic models, i.e.: linear, of polynomial regression and Lorentz-Gauss model. An important aspect of using the CDHO model is the ability to draw quantitative conclusions based on non-classical measures of dynamics, i.e. absolute delay time, relaxation time and long-term equilibrium level. In terms of the absolute time of delay, the results indicate Warszawa as the center of the network of connections between the cities, which reacts the fastest to megatrends in the market environment and then initiates changes in the remaining cities. The relaxation time in the examined cities, ranging from about 6 months to almost 2 years, confirms the assumptions of the high inertia of the housing markets, which, after the rapid growth of 2006-2007, returned to the pathway leading to the original equilibrium levels. Differences in long-term equilibrium levels confirm that network (collective) behavior becomes synchronized when affected by strong external stimuli (2006-2007), while it is un-synchronized during periods of stability (2008-2016), in line with local geospatial conditions.

Using graphical methods (phase diagrams) of presenting the evolution of the systems under study, a qualitative evaluation of their dynamics was made, showing the existence of a network of links, which results from the similarity of trends and shape of particular trajectories and the existence of multiple quasi-equilibrium points.

To sum up, the article shows that the dynamics of housing prices cannot be described within the classic paradigm of independent, local housing markets because of the observed strong network connections and the emergence of synergistic phenomena.

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