THE USE OF INDICATOR KRIGING FOR ANALYZING PRICES IN THE REAL ESTATE MARKET

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Abstract

The patterns and relations between real estate prices and the factors which shape them can be presented, among others, in the form of traditional statistical models, as well as by means of geostatistical methods. In the case of research involving the diagnosis and prediction of transaction prices, the key role is played by the spatial aspect, hence the particular significance of geostatistical methods using spatial information.

The main goal of the conducted research is to determine the probability of the occurrence of a price in a given location in space by means of geostatistical simulation - indicator kriging. Indicator kriging does not use the entirety of information included in a dataset, and can, therefore, be useful in situations when the assumptions involving the spatial stationarity of the examined phenomenon are not fulfilled by an entire dataset, but are fulfilled by a certain part of the set. The maps of the probability with which a regionalized variable (price) takes on particular values, limited by arbitrarily selected cutoff values, were prepared by means of indicator kriging. An alternative approach to the preparation of price probability maps is the determination of the spatial distribution of areas in which, with the assumed probability, the value of the price falls within the predetermined ranges.

The paper presents both the essence as well as a theoretical description of the geostatistical simulation of a transaction on the real estate market, as well as the results of an experiment involving the transaction prices of real properties located in the north-western part of the city of Olsztyn.

The result of the research is a set of virtual information about the places in which the transactions have occurred and about the prices of real estate, constituting a reflection of the market processes which may take place in the near future.

Key words: geostatistics, transaction price simulation, real estate market.

JEL Classification: C15, C 53, R32.

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1. Introduction

As part of the work on the future occurrence of transactions on the real estate market (the location of a transaction and its price), geostatistics has been used as the application of the formalism of random functions for the identification and estimation of natural phenomena.

In the case of research involving the diagnosis and prediction of transaction prices, the key role is played by the spatial aspect (RENIGIER-BIŁOZOR 2011), hence the particular significance of geostatistical methods which use spatial information as the crucial element describing the phenomena occurring on the real estate market. By suggesting methods for the description of spatial continuity, and by using statistical relations in order to examine spatial phenomena, geostatistics complements traditional statistical analysis.

Geostatistics constitutes a dynamically growing branch of applied (spatial) statistics, whose origin dates back to the 1950s, representing an effective tool in research on the variability of the parameters of mineral deposits, and in calculating the resources of gold ore deposits (NAMYSŁOWSKA-WILCZYŃSKA 2006, KRIGE 1951). Originally developed for the purpose of solving issues of estimation in a method used for the assessment of ore resources, it has become popular in other fields of Earth sciences. Among the most important papers in the field of geostatistics are: MATHERON (1971), DAVID (1977, 1988), JOURNEL and HUIJBREGTS (1978), CLARK (1979, 2001), ISAKS, SRIVASTAVA (1989), DAVIS (1986), ARMSTRONG (1988), CATTLE et al. (2002).

In numerous highly developed countries, geostatistical methods are widespread in various fields of science and industry. These methods, used to solve complex problems, have penetrated various areas and fields of science: geology, hydrogeology, environmental protection, geophysics, land surveying (remote sensing) and geotechnics (ENGLUND, SPARKS 1991, HOHN 1988, KOKESZ, NIEĆ 1992, LAJAUNIE 1984, MAGIERA, FORYCIARZ 2000, VERLY, DAVID 1984, WIATR 1996, ZAWADZKI 2002).

From the market research point of view, geostatistical methods are distinguished by an advantage involving the precise evaluation of the error of the performed estimates and the documentation of the spatial distribution of this error. They enable the assessment, not always available in deterministic methods, of the probability of the occurrence of values lower or higher than the introduced cutoff value in the given location, the preparation of cutoff value maps for the given probability, accounting for other variables in the interpolation process, as well as the analysis of the spatial variability of a variable or variables. By means of these methods, one can not only prepare maps of estimated average (the most probable) values, which, compared to reality, are characterized by considerable smoothing and do not reflect the existing spatial variability, but also conduct simulations, making it possible to depict this variability (URBAŃSKI 2008).

The main goal of the conducted research is to determine the probability of the occurrence of a price in a given location in space by means of a geostatistical simulation - indicator kriging (IK), which enables the preparation of maps of the probability with which a regionalized variable (price) takes particular values, limited by arbitrarily selected cutoff values.

The geostatistical simulation allows the identification of not just the most probable, but also of another possible course of the analyzed phenomenon, enabling the assessment of deviations from the average values. Conclusions resulting from analyses conducted in this manner enable the assessment of risk, and may be useful when making business decisions involving the validity and profitability of investments carried out in the real estate market.

2. Methodology and the course of study

Data from the real estate market (e.g. information about the prices and values of real estate) exhibits structural asymmetry, with visible outliers present within it, which, due to substantive reasons, cannot, however, always be rejected (BRZEZICKA, WIŚNIEWSKI 2014). One of the methods of analyzing data whose distribution of measured values considerably deviates from the normal distribution is the execution of complex, usually nonlinear, transformations of data resulting in, e.g. a distribution which is normal or close to normal. To this end, kriging methods using nonlinear data transformations are applied (HENGL 2007, ZAWADZKI 2011, DUCCI et al. 2016). One kind of nonlinear kriging, which combines the simplicity of indicator transformation with the vast possibilities of analyzing irregular datasets of complicated distributions, is coded value kriging (IK), known in Polish literature as indicator kriging (SMITH et al. 1993, ZAWADZKI 2005). This method, along with multigaussian kriging and disjunctive kriging, is regarded as one of the geostatistical methods enabling the construction of complete local models of the distribution of an estimated feature (STACH 2009, ZHANG et al. 2015). IK is one of the nonparametric geostatistical techniques which are frequently used when the data is characterized by positive skewness and the examined sample population contains outliers. In such cases, discretization is performed for the histogram of the values of the analyzed parameter, and interpolation is conducted for each class separately. The difference between ordinary and indicator

kriging involves the fact that the first interpolation is conducted based on the original data, while the second one is conducted based on transformed data (into a binary form), in accordance with the several adopted cutoff values of the analyzed parameter (NAMYSŁOWSKA-WILCZYŃSKA 2006).

In the IK method, the original values of the examined regionalized variable $Z(\mathbf{x})$ are transformed into the indicating values (indicators) – equaling zero or one (ZAWADZKI 2011, JANG et al. 2016), with the conversion of data (usually qualitative) into indicators being performed for each class separately. The places of measurement in which this class is present are assigned the value of 1, with 0 for the others. Measurement variables may also be created from continuous data by using a cutoff value which would convert this data into binary figures, assigning 0 to all values lower than the cutoff, with 1 for the others, or vice versa (URBAŃSKI 2008, PHAM 2014). The cutoff values fulfill the condition $z_{\min} \leq z_k \leq z_{\max}$, where $z_{\min}$ and $z_{\max}$ refer to the lowest and highest values in the analyzed dataset:

$$i_k(x, z) = \begin{cases} 1, & \text{when } z(x) < z_k \\ 0, & \text{when } z(x) \geq z_k \end{cases} \quad (1)$$

For the transformed indicator data (0 or 1), traditional geostatistical calculations are performed, followed by the preparation of an empirical semivariogram and its subsequent modeling. The semivariogram, being a function describing the distribution of variance depending on the distance, creates a basis for the determination of weights allowing the estimation of values in the nodes of a regular grid, assuming that the estimator is unbiased (i.e., the average value of estimate errors equals zero, and the variance of estimates is as low as possible). The parameters of the semivariogram function should be determined experimentally so that the difference between the theoretical values and the empirical values is as small as possible (ISAAKS, SRIVASTAVA 1989, ARMSTRONG 1998). The subject of the transformation of original data into indicator values, and of calculating an indicator semivariogram, is described comprehensively in NAMYSŁOWSKA-WILCZYŃSKA (2006). The indicator of a measurable feature is a binary variable, which takes the values of 0 or 1 depending on whether the value of the feature is higher than or equal to the predetermined cutoff value C . If $z(x)$ is a value of this feature in location x , then the indicator value $i(x)$ in x equals 0 if $z(x) < C$, or 1 if $z(x) \geq C$ for each adopted threshold-cutoff value.

Another possibility of determining the value of the indicator determined by the cutoff value C involves adopting a reversed indicator value $i(x)$, meaning 1 if $z(x) < C$, or 0 if $z(x) \geq C$ for each adopted threshold-cutoff value. In the case of the necessity to analyze numerous indicators, a generic model is developed, obtained for the indicator of a median.

The individual steps during the estimation of an indicator are:

- 1) the compilation of a set of proper threshold (cutoff) values via the extraction of identical classes,
- 2) the conversion of the dataset into values of zero and one, using specified threshold values,
- 3) the calculation of the value of the semivariogram function $\gamma(h)$ in each adopted class.

An indicator semivariogram can be calculated based on an internally constructed indicator variable. For the cutoff value cut_k and the value of data x_i , the indicator variable ind_i is defined as:

$$ind_i = \begin{cases} 1, & \text{when } x_i \geq cut_k \\ 0, & \text{in other cases} \end{cases} \quad (2)$$

where i means the given location.

Kriging is conducted in sequence for the transformed data based on a semivariogram. The created map shows the probability of a given class being present in a given place. These maps are made for each class separately; subsequently, the class with the highest probability of occurrence is assigned to each raster cell (URBAŃSKI 2008, PHAM 2014). The estimated value in point $\mathbf{x}_0$, determined based on the indicator values i_k , which depend on the selected cutoff value z_k , is defined by the formula:

$$\hat{I}_k(x_0) = \sum_{j=1}^n w_j i_k(x_j) \quad (3)$$

(where w_j are the weights of kriging). This is also the probability of the estimated value in point $\mathbf{x}_0$ being lower than the cutoff value z_k (CILES, DELFINER 1999). This value may be presented as the value of the cumulative distribution function of the regionalized variable $Z(\mathbf{x})$ for the cutoff value z_k in point $\mathbf{x}_0$:

$$\hat{I}_k(x_0) = P[Z(x_0) < z_k] = F(x_0, z_k) \quad (4)$$

The performance of the kriging procedure for a series of various cutoff values $z_k \in (z_{\min}, z_{\max})$, $k = 1, \dots, n$, allows the determination of the conditional cumulative distribution function of the regionalized variable $Z(\mathbf{x})$ for any location within the examined area:

$$F(x, z_k) = P[Z(x) < z_k] \text{ for } k = 1, 2, \dots, n \quad (5)$$

using its properties of a cumulative distribution function (5), we can, in a simple manner, determine the conditional probability:

$$P[z_1 \leq Z(x_0) < z_2] = F(x_0, z_2) - F(x_0, z_1) = \hat{I}_2(x_0) - \hat{I}_1(x_0) \quad (6)$$

Indicator kriging allows the preparation of maps of the probability with which the regionalized variable $Z(\mathbf{x})$ takes specific values, limited by any given cutoff values, which is extremely important in practical use, e.g., in real estate market research. Considering price as a regionalized variable, one can, therefore, prepare maps of the probability of the occurrence of a price in a given location in space within a predetermined price range.

It should be emphasized that the essence of the IK method is first of all the construction of a global model of uncertainty, based on an empirical cumulative distribution function calculated from all of the measurement data, followed by its local modification based on the vector of coded data from the measurement sites neighboring the location of the point of estimation. Based on the distributions of probability acquired by the kriging method, two general characteristics are also calculated for the coded values: the average value of the estimated cumulative distribution function, known as the expected mean value, and the conditional variance:

$$[z(u)]^*_E = \int_{-\infty}^{+\infty} z dF(u; z|(n)) \approx \sum_{k=1}^{k+1} z_k [F(u; z_k|(n)) - F(u; z_{k-1}|(n))] \quad (7)$$

where k is the number of adopted cutoff values, $z_0 = z_{\min}$, $z_{k+1} = z_{\max}$ are the minimum and maximum values of the range of the z feature, which are the parameters input by the person performing the calculations (operator). The conditional mean for each class $[z_{k-1}, z_k]$ defined as z'_k is obtained based on an interpolation procedure whose type and parameters are also specified by the operator (STACH 2009).

IK does not have to use the entire information stored in the dataset, and can therefore be useful in situations when the assumptions about the spatial stationarity of the examined phenomenon are not fulfilled for the whole set, but are fulfilled for a certain part of the set (ZAWADZKI 2011).

Properly performed indicator kriging requires adequately selected cutoff values. The ranges within which high variability of the examined regionalized variable is observed should be characterized by a dense distribution of z_k . The modeling of indicator semivariograms proceeds most frequently for each cutoff value independently. The resulting series of probability maps reflect significant changes in the distribution associated with the increasing cutoff values. In cases when semivariogram models change with an accuracy of one factor along with changing cutoff values, indicator kriging is performed using one cutoff value, which usually equals the median. This is the so-called median indicator kriging (DEUTSCH, JOURNAL 1992).

The process of using indicator kriging to analyze prices in the real estate market during research has been implemented according to the following sequence:

- the preparation of a distribution histogram for data originating from the real estate market ("price data"),
- the selection of several threshold values for the prices, with the preferable scale having equal intervals,
- the conversion of "price data" into binary values for each adopted threshold value,
- the calculation of empirical indicator semivariograms and the adjustment of theoretical models to their courses, for each threshold value separately,
- the performance of ordinary kriging, taking into account the values converted into a binary form for each threshold value,
- estimating the distribution of the probability of the occurrence of a price falling within a range between the predetermined threshold values,
- estimating the probability of the occurrence of a price falling within a specified price range, by assessing the maximum acceptable risk (determining the confidence intervals).

As a result of estimation by means of IK, taking into account the parameters of the corresponding models of indicator semivariograms, it is possible to obtain indicator maps with block evaluations, ranging between 0 and 1. They can be interpreted as the probability with which the value of a block (or a selection field) is higher or lower than the adopted threshold value, or as the share of a block above the threshold value.

The data generated from a geostatistical simulation were subsequently used to analyze the prices in the real estate market: prepare the maps of the probability with which the price takes specified values limited by the cutoff values. To this end, the statistical functions of Microsoft Excel software were used (an iterative process using a pseudo-random number generator), due to which 15 price options were generated randomly for each analyzed basic field from a selected area.

3. Results

The research was conducted based on data originating from the local real estate market in Olsztyn (a city located in north-eastern Poland). This data concerned transactions involving undeveloped real properties; during the verification of the database, the areas where the occurrence of transactions was very unlikely or impossible were excluded, e.g., lands under ditches, ponds, wooded and shrubby lands, forests, land transportation areas. As a result, 288 pieces of data were collected involving transactions which took place in the years 2010 - 2016. The real estate location points shown against a city map are presented in Fig. 1.

Fig. 1. The location of undeveloped real estate transactions in Olsztyn in the years 2010-2016.

Source: Own study.

The collected transactions involved real properties being the subject of either property rights or perpetual usufruct. The difference in property law is reflected in the prices of real estate. Therefore, in order to consolidate the database, it was necessary to introduce the correction factor W_k , which was calculated as a percentage ratio of the price of 1 m² of real estate subject to perpetual usufruct to the price of 1 m² of real estate being an object of ownership. Real properties fulfilling the criteria of being similar in terms of their features, differing only in the type of property law, were selected from the base. By means of comparison in pairs, it was estimated that the average value of the W_k factor for the city of Olsztyn was 0.74.

The prices of the examined real properties ranged between 55 PLN/m² and 764 PLN/m²; the prices remained constant in the analyzed time period, with no significant changes caused by the passage of time. As part of the indicator kriging procedure, the prices of real estate as the original values of the

examined regionalized variable were transformed into indicator values. To this end, proper threshold values were introduced, dividing the data into identical classes every 50 PLN/m², creating 15 classes. The frequency of the occurrence of real estate prices within the individual intervals is presented in Table 1 and in a histogram shown in Figure 2.

Table 1

Real estate price intervals along with their frequency of occurrence

Class number	Price range (PLN/m ²)	Frequency	Class number	Price range (PLN/m ²)	Frequency
1	50-100	16	8	400-450	9
2	100-150	40	9	450-500	5
3	150-200	64	10	500-550	6
4	200-250	52	11	550-600	2
5	250-300	37	12	600-650	2
6	300-350	28	13	650-700	0
7	350-400	26	14	700-750	0
			15	750-800	1

Source: Own study.

Fig. 2. A distribution histogram of the frequency of the occurrence of real estate prices in individual intervals. Source: Own study.

Fig. 3. A fragment of the studied area, showing the real properties being the objects of trade (parcels, the centroids of parcels). Source: Own study.

Fig. 4. Sample probability maps for the occurrence of a price in the given location in space within specified price intervals. *Source:* Own study.

In the case of the real estate market, where undeveloped real properties are the commodity, we are dealing with surface objects. For the needs of analyses using GIS software, simplified elements representing the real properties in question were used in the form of the centroids of land parcels. Figure 3 presents a fragment of the documented area.

Indicator kriging was conducted for each class separately, using version 10.0 ArcGIS software. During the process of transforming the binary data into indicator data, certain imperfections of the used method were identified. The data obtained in the process of interpolation should constitute a “set of surfaces” close to each other, but not intersecting one another, while, on rare occasions, the resulting surfaces did intersect (with considerable asymmetry of transaction prices for real properties located in close proximity). In order to enable the use of indicator kriging in the research of the real

estate market, the ArcGIS software smoothing function was applied, eliminating the intersection of surfaces and, at the same time, not affecting the fluctuations of price tendencies in the real estate market.

As a result, probability maps were prepared for the occurrence of 15 classes (15 separate maps $k_1, k_2, \dots, k_{15}$). Figure 4 presents six sample maps.

Red color in the pictures indicates a high probability of the occurrence of the specified real estate prices in the given location, with a respective palette being used through orange, yellow and green, approaching zero probability. For example, for Class 3, there is a high probability of the occurrence of prices ranging between 150 and 200 PLN/m² for a considerable part of the documented area (the districts of Brzeziny, Jaroty and the City Center), while prices ranging between 350 and 400 PLN/m² may occur only in the very center of the city. The obtained series of probability maps reflects significant changes in the distribution related to the increase in the cutoff values.

The data on the probability of the occurrence of a given price in a given location in space, originating from a geostatistical simulation, was used to create a map of probabilities with which the prices would take specified values, limited by the cutoff values. The studies involved a test area - the north-western part of Olsztyn (the Gutkowo and Redykajny districts). For the needs of the research, the area was divided into basic fields in the form of squares, with sides measuring 500 m in length, and the centroids of these fields were pinpointed. The demarcation of the studied area is presented in Figure 5.

Fig. 5. The demarcation of the test area on a city map, divided into basic fields and centroids.

Source: Own study.

Thirty-six basic fields were subjected to testing, for which 15 price options were generated randomly using the statistical functions of the Microsoft Excel software. Ordinary kriging was performed in order to depict the results in the form of the spatial distribution of the simulated prices. This step repeats for each threshold value, with a probability surface obtained as the final result, constituting a function of the price level for each block. Based on these distributions, surfaces of identical probability were drawn for the individual sections of the studied area. Figure 6 presents six selected visualizations of simulated prices.

4. Conclusions

The conducted simulation studies of the real estate market were used to determine the probability of the occurrence of a price in a given location in space by means of geostatistical simulation - indicator

kriging. As a result of the research, a series of simulated transaction prices for real properties located inside the city of Olsztyn was obtained - a set of virtual information about the places of the occurrences of transactions and real estate prices, constituting a reflection of market processes which could take place in the near future. The defects of the used data analysis method were also identified during the research. In the interpolation process, the resulting surfaces should constitute "a set of layers", close to each other but not intersecting one another, while, on rare occasions, the resulting surfaces did intersect (with considerable asymmetry of transaction prices for real properties located in close proximity).

Fig. 6. Spatial distribution of simulated prices for selected options. *Source:* Own study.

The conclusions from analyses conducted in this manner enable the assessment of risk and can be useful when making business decisions involving the validity and profitability of investments carried out in the real estate market. The use of geostatistical methods taking into account the spatial aspect may contribute to increasing the transparency of the market, including an improvement in the

efficiency of spatial management, which, in the theory of economy, is considered to be a rare commodity.

The use of indicator kriging in an analysis of the real estate market is an innovative approach. The performed research was of a pilot nature, while the obtained results and possibilities of the proposed method encourage the authors to conduct further work, in particular to verify the method with other datasets, and to use the indicator data more broadly in order to analyze the real estate market.

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