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THE ESSENCE OF EQUIVALENT MARKETS IN DETERMINING THE MARKET VALUE OF LAND PROPERTY FOR VARIABLE PLANNING FACTORS

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Abstract

The leading local legislation act defining the spatial policy is the local development plan, the financing of which is the commune's responsibility. The beneficiary of activities aimed at the transformation of the intended property use is its owner or its perpetual lessee – with lessees incurring the costs of adopting the local development plans through so-called zoning fees, the amount of which, often controversial, has become the source of numerous lawsuits. The presented problem of an outside-business and often radical change of land value corresponds to the market dilemmas in determining equivalent markets, and establishing price-setting factors and their impact on the value of real estate.

Circles of analysts and appraisers determine equivalent markets radially. This means that the analyzed or valued property is the central point of the monitored area, incorrectly "enclosed" within the geometry of a circle. Such a perception of the dispersion of transaction prices does not reflect the actual nature of the market, as evidenced by the analyses of land property values according to their planning factors, carried out as part of the present paper.

The focal point of the article is a continuous map of land value developed for properties located within the cadastral unit of Podgórze of the city of Krakow which indicates that it is not legitimate to arbitrarily take on the radial approach when designating equivalent markets, which was indirectly approved by Waldo Tobler claiming that: "*Everything is related to everything else, but near things are more related than distant things*" (TOBLER 1970).

Key words: equivalent markets, fields of value, property valuation, zoning fee.

JEL Classification: D47, L85, R14, R52, R58.

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1. Introduction

Planning factors are one of the main determinants of land property market value. The value of land in a specific location changes together with the change of its use, which is discussed by Małgorzata Krajewska (KRAJEWSKA 2008).

Estimating the value of one property in several planning arrangements poses many problems for everyone involved in the adoption or amendment of a local development plan: to the parties involved – usually due to the imaginary value of the property; to authorities – due to carrying out legislative work in the planning process, and to real estate appraisers – obliged to remain impartial and analyze the previously and newly configured properties just as they are. The number of disputable cases that often end up in court encourages everyone to seek new tools that would minimize such market

problems. The very idea of determining the value of compensation seems to be simple. The difference in land value after the establishment of a local development plan, hereinafter referred to as the local plan, and before its establishment, is part of the zoning fee. The principle is clear, but it takes account of one economic parameter, i.e. market value, in two aspects, and in three aspects in the case of adopting the local plan after 31 December 2003 (in connection with the fact that the development plan adopted before 1 January, 1995 is no longer valid: according to the provisions of the current development plan, the current use, and the provisions of the plan in force until 31 December 2003, the so-called master plan). Each aspect should be assessed separately, taking various properties into account for comparison – properties serving different purposes, and thus diversified in terms of their portfolio of market features. Consequently, the amount of the fee for the socio-economic modernization of the land is determined on two or even three independent – in terms of their factual status and spatial arrangement – real estate markets. The existing technological measures have not offered the possibility for mathematical modeling of land value subspace, which has resulted in the establishment of a trend in the art of real estate valuation in which the closest market is the appropriate one. However, in times of the dynamic development of the Geographic Information System and the overall availability of digital data on inventory items, the exploration of large areas in search of equivalent markets enters a new dimension (GOTWAY 2002).

In the present article, an attempt was taken to interpolate the subspace of continuous land value for the most important uses referred to further below. This study aims to examine the practical evidence of an unjustified or, more generally speaking, too generalized approach to determining the homogeneous real estate market, uncritically adopted by analysts as the immediate surroundings. Moreover, the significant usefulness of the process of exploring similar markets in spatial planning has been observed, from the initial phase of forecasting the financial consequences of the adoption or change of the local plan, to the zoning fees estimation phase.

2. Literature review

The study of literature in the field made it possible to distinguish a number of domestic and foreign books, articles and scientific texts analyzing the discussed topics. This paper refers to twelve publications, including the work of Radosław Cellmer entitled "Spatial Modeling in the Process of Developing Land Value Maps" (*Modelowanie przestrzenne w procesie opracowania map wartości gruntów*) (CELLMER 2014). In general, according to the concept of Radosław Cellmer, a land value map is determined based on model prices established taking into account all transactions. The diversification of real estate was minimized using the adopted price-determining factors. Then zones which are homogenous in terms of spatial planning were delimited. In the present author's studies, she suggests developing land value maps for particular uses based on which equivalent markets will be determined and, on their basis, price-determining factors, e.g. using factor analysis as described in the publication by Elżbieta Jasińska, entitled "The Application of Factor Analysis in the Estimation of Real Estate"*(Zastosowanie analizy czynnikowej w szacowaniu nieruchomości*) (JASIŃSKA 2008). Such a course of action will help to minimize the jumps in prices on borders between surveying sections or planning zones visible on the land value map developed by Barbara Prus as part of her publication entitled "Land Value Map as a Factor Supporting Commune Land Management" (*Mapa wartości gruntów jako czynnik wspomagający gospodarowanie terenami w gminie)*. In addition, determining factors on the basis of equivalent markets for variable planning factors will reduce subjectivity in property valuation, as discussed by Professor E. Kucharska-Stasiak in her work titled "Value Measurement in Economics – Consequences for Real Estate Valuation" (KUCHARSKA–STASIAK 2011).

3. Research problem

As already mentioned in the introduction, the shape of property transaction prices is determined by the intended use of a specified area, as long as its physical properties do not change (only their perception changes depending on the intended use of the land), but the reclassification of the use results in placing the property in a completely different price segment, usually x-times higher.

Changes in spatial planning imply the payment of zoning fees, which, *sui generis,* are a form of participation by the communes in profits from the sale of real estate whose value has increased as a result of the changes in or adoption of a local development plan for the area. The obligation to pay the zoning fee occurs when the following conditions are met (ŹRÓBEK, DAWIDOWICZ 2014):

- the value of a property or its part increases as a result of a change in or adoption of a local development plan,
- the property or its part is sold within a period of 5 years from the date when the local plan or change to it have entered into force.
- the sale of the property is of an equivalent nature.

The amount of the fee is defined according to the percentage share, in accordance with the Act on Spatial Planning and Development, of a no higher than 30% property value increase (ACT ... 2003). It must be noted here that attention should be paid to the essence of property valuation, understood according to the Act on real estate management as an activity related to the determination of its value (ACT ... 1997). The choice of a base of properties, similar with respect to their original intended use and that after the transformation, and thus the recognition of an equivalent market, is of primary importance. It is uncommon for a sufficient number of transactions concerning properties similar in terms of physical and functional characteristics (investment opportunities) to the valued property to have been concluded in its immediate vicinity. The approach of radial perception of the analyzed area has become popular, even though price emitters are point objects, and the direction, speed and range of price transformation is dependent on the allocation of land sold in its immediate vicinity. In fact, the geometry of an area that could be considered as being homogenous, consisting of similar properties, is not subject to any global regulations of shaping the external boundaries of the monitored market. The terms "radial approach" and "global regulations" used in the preceding sentence are understood in line with the doctrinally accepted approach to the market as an area arbitrarily similar in nature as far as the properties present in the area are concerned. Surveys comprising of representative (often in name only) samples of real estate of a given land use contain observations the price range of which is equal to the multiple of the minimum unit price observed. Therefore, it is difficult to speak of any similarity of market observations made and credibility of a certain value, as the objectification of the market is an inseparable element of property valuation (KUCHARSKA-STASIAK 2011), which in absent in such case.

4. Data

Podgórze – a cadastral unit of Krakow – was selected as the study area. This is a difficult market, as there is a large variety of intended uses noted in the area and a high number of transactions concluded within the five year period of the analysis, i.e. the years 2010-2014. However, the range of transaction prices is many times – even several dozen times – greater than the minimum price of real estate of a given use, as illustrated by the graph below (Fig. 1). Aggregation of transactions was carried out with respect to the five main uses, i.e.:

- MN – low-intensity residential development, also referred to as single-family housing,
- MW – high-intensity residential development, referred to below as multi-family housing,
- U – commercial development,
- KD - transportation,
- ZL - greenery.

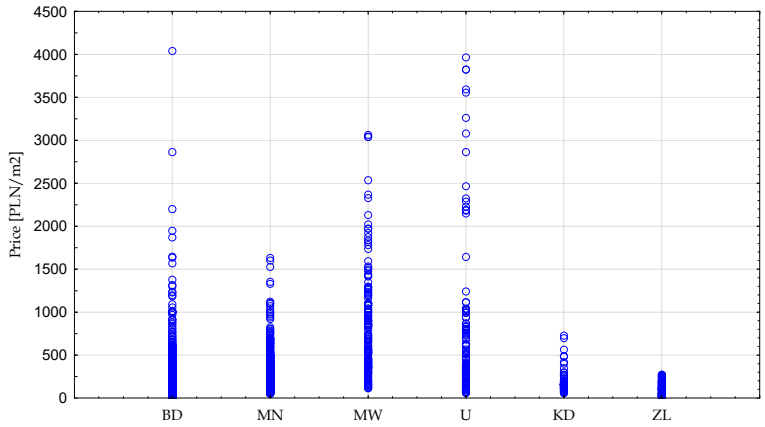

Fig. 1. Dispersal of prices of undeveloped land properties designated for different uses. *Source*: Author's own study.

The chart presents the maximum changeability of transaction prices of objects of trade in the undeveloped land property market of Podgórze by the point location of individual intended uses in azimuth columns. The collected information about the function and value of the land originate from the studies of notarial deeds, local land development plans – referred to in the present paper as local plans, and decisions regarding land development conditions.

The fraction of the transactions for which it was not possible to determine the spatial planning use was added to the chart. The test set includes 1,786 observations of the market, where the proportions of MW, U and ZL each represent 10% of all information, and KD – only 5% of the set. The most numerous group, amounting to nearly 50%, are areas intended for single-family housing development, the reason for which should be sought in the nature of the analyzed area. The fraction omitted later in the analysis amounts to 15% of the total transactions, which, in the context of 57% coverage of the surface area of Podgórze with local plans, is a small part that has no impact on the final conclusions of this study. The transaction prices adopted as a test material are devoid of the influence of time, naturally consumed by the specific nature of the local real estate market, as evidenced by the Kandell's tau correlation coefficients for specific intended uses, found to be: for KD = 0.78% per year, MN = 0.25% per year, MW = 1.12% per year, ZL = -0.29% per year, and U = 2.00% per year. The differences between the exogenous variables of individual properties are largely suppressed by the numbers within the statistical samples for each purpose. The unstable remaining part does not change the results and conclusions of this study due to the adopted further research criteria and model parameters.

5. Test method

The estimated property value is the result of many factors, the selection and evaluation of which is the responsibility of a real estate appraiser. Such a state of affairs implies the necessity of maximizing market information by determining price equivalence fields that may be allocated in any area of the analysis, as well as having any size and shape. The collection of all price equivalence fields is a map of single use land value.

In this paper, the above mentioned map of land value has been drawn up, presenting the methodology and tools used for its creation, as well as the assessment of its suitability for spatial planning processes. The study was conducted in a GIS environment, more and more widely used in fields where it had used to be neglected (GUNES, KOVEL 2000). Data from the Municipal Spatial Information System and market information on transactions of undeveloped land properties with designated intended uses were used. The transactions were located on the plane of the city arrangement, according to the location of the properties. The altitude coordinate (C) represented by the unit transactional price and appropriate land use markings were applied to plane coordinates (X, Y), locating the objects of transactions and retaining the actual distance relationships. For each intended use, the subspace of land value was interpolated. One or more price emitters are based on such subspace, understood as the extremes of the geospace approximating function starting from which the predictors of property value decrease in non-linear progression (BRAUNBRÜCK, RAVASOO 2016). This paper documents the results of the study of the three interpolation methods.

5.1. Inverse distance method

Mathematical modeling of land value continuous subspace (V) according to the inverse distance method consists of estimating unit values of land property for each point of a polygon stretched on the transactions located on the extremes. It is carried out according to the formula presented below:

$$z(u)_j = \left(\sum_i w_i c_i\right) \times \left(\sum_i w_i\right)^{-1} \tag{1}$$

where:

$z(u)_j$ – estimated land value in the j location, where $z(u)_j \in V$,
i – transactional point ordinal number,
c_i – unit price achieved in the ith transactional point,
w_i – unit price weight.

As shown in the formula above, the inverse distance method involves appropriate weighing of average unit prices of the closest transactions, whereas the weight is determined in inverse proportion to the d_i (def. $\overline{x(x_0)_j, c_i}$), in accordance with the formula:

$$w_i = \frac{1}{d_i^n} \tag{2}$$

In the study, the modeling of subspace for the leading intended land uses in the Podgórze district of Krakow was performed, and the 12 closest observations ($i \in [1 \div 12]$) with weights built on the cubic inverse of the Euclidean distance ($n = 3$) of the observations from the interpolated point were adopted. The inverse distance method does not distort transactional points, i.e. those which, by virtue of having a third coordinate, are the correct points of the modeled subspace. The C coordinate is attributed to other points as a result of estimating the hypothetical result of a transaction in the interpolation process.

5.2. Natural neighbor method

The natural neighbor method differs from the previous method in terms of its approach to the issue of weighing adjacent observations. After all, distance plays an indirect role here, but is necessary in order to establish the appropriate surface relationships. Each point in the land value subspace is interpolated in three successive operations (MITKA, PIECH 2012). The first one consists of the triangulation of transactional points according to the Delaunay's criterion, rationalizing the system of triangles stretched on an analyzed market. Then (stage two), the bisectors of the sides of each generated triangle are created, and the vertices of the Thiessen polygons are formed at their intersections. Combined, they form closed areas within which the points are characterized by the shortest distance to the transactional point of each of the polygons (MITKA, PIECH 2012). In the final - third stage, for each interpolated point, an autonomous Thiessen polygon is created, built on the background of the polygons of the second stage, as shown in the figure below (Fig. 2).

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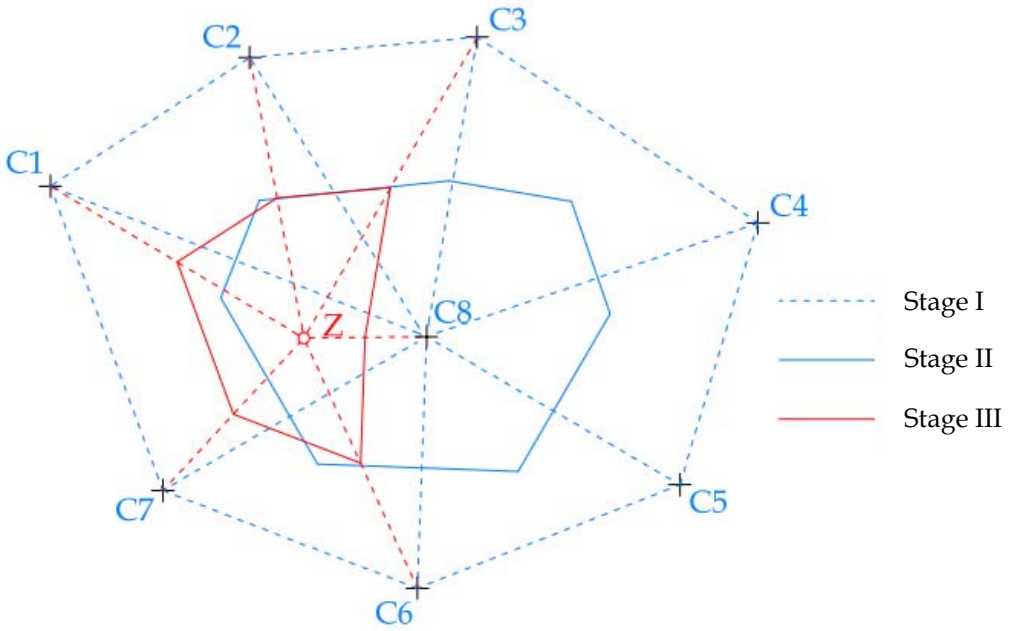

Fig. 2. Three stage interpolation of the Z point according to the natural neighborhood method. *Source*: Author's own study.

The $\bar{C}$ coordinate of the *j* point is created as the sum of the products of transactional points and their weights corresponding to the relationship between the participating surface of the portion belonging to the ith Thiessen polygon ($P_{t(part\_t)}$) and the total surface area of the polygon (P_t). The approximation of the j-th $\bar{C}_j$, point, where $\bar{C}_j \in V$, with *V* meaning the subspace of land value for the analyzed intended use, has been carried out in accordance with the following model:

$$\bar{C}_j = c_1 \times \left(\frac{P_{1/part\ 1}}{P_1}\right) + c_2 \times \left(\frac{P_{2/part\ 2}}{P_2}\right) + \cdots + c_i \times \left(\frac{P_{i/part\ i}}{P_i}\right) \quad (3)$$

where:

$\bar{C}_j$ – j-th interpolated point of land value subspace (V),

i – transactional point ordinal number,

c_i – unit price of the i-th neighboring transactional point,

P_i – surface area of the i-th Thiessen polygon,

$P_{i/part\_j}$ – participating part of the Thiessen polygon for the j-th interpolated point in the entire surface area of the i-th Thiessen polygon (P_i).

The interpolation of land value subspace for individual intended uses was carried out considering all the points constituting the V subspace divided into Thiessen polygons in the Voronoi diagram. The exemplification of geospatial data prepared for the approximation (the first and second stage completed) for the MN use has been provided below (Fig. 3).

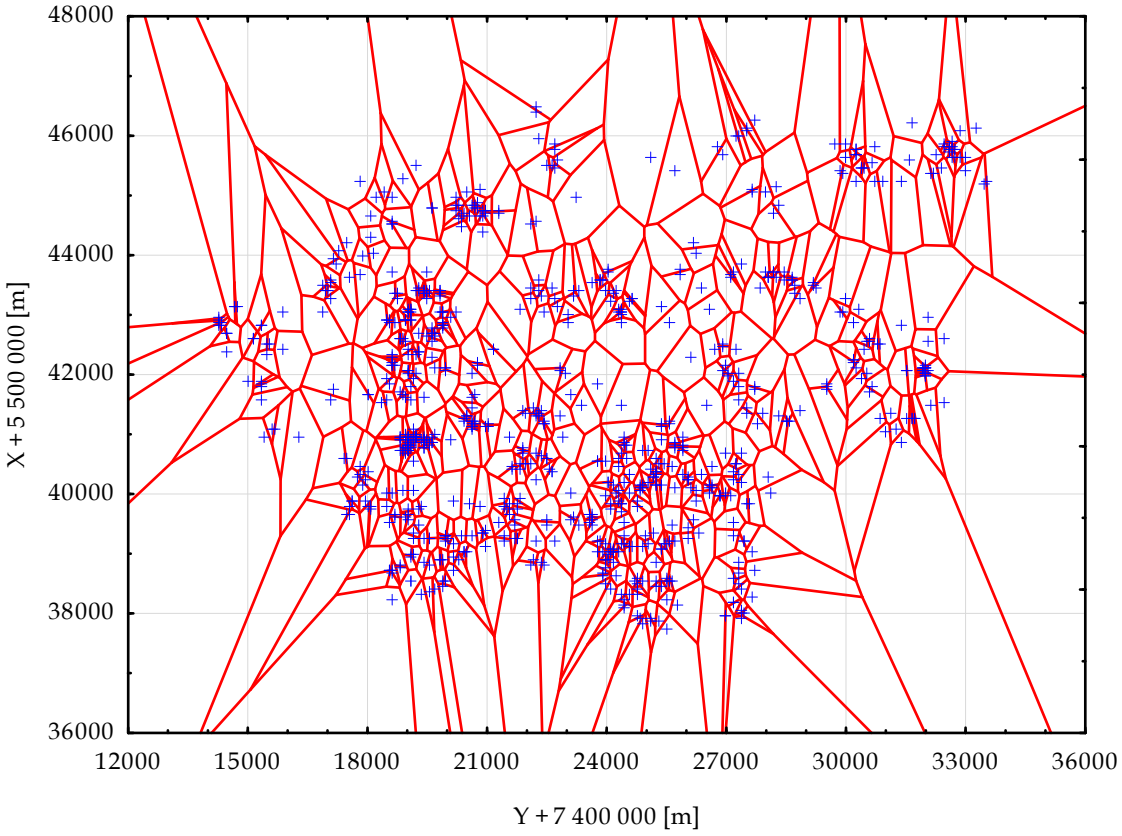

Fig. 3. Voronoi diagram for MN real estate. *Source*: Author's own study.

5.3. Ordinary kriging method

Due to the very strong geostatistical basis for ordinary kriging, studies were undertaken on its usefulness in modeling subspace (W). Such studies had already been conducted both in Poland and abroad, also with certain modifications, e.g. using the regression-kriging model (LIGAS 2009) and implementations in other fields of study (LIU et al. 2009; HUANG et al. 2010). However, the conclusions were not entirely clear, and no answer was found to the question of whether the method is universal in economically diverse environments, where price fluctuations are high even inside rotary focuses, i.e. places where a large number of transactions of varied amounts have been carried out, localized in a small area. Kriging assumes the existence of isotropic autocorrelation between the location of transactional points (X, Y) and the price – C coordinate (CELLMER 2011). The estimates of the value of successive V subspace points were based on the following formula:

$$C_j = \sum_{i=1}^{n(g)} w_i \times C_{T,i} \tag{4}$$

where:
C_j – j-th interpolated land value subspace point (V)),
i – ordinal number of the transactional point,
$n(g)$ – number of transactional points in the interpolation group,
w_i – weight of the i-th transactional point determined based on the semivariogram function,
$C_{T,i}$ – unit price of the i-th adjacent transactional point (MUCHA, WASILEWSKA 2007).

Just as in the inverted distance method, in this case it is also assumed that the closer to the interpolated point the transactional points are located, the more reliable the C_j value. This also arises from the stochasticity of the method itself, in which IDW is an interpolation that truly belongs to the group of deterministic methods. In the study, two forms of the semivariogram weighing the geospatial market observations were used:

– exponential semivariogram (NIEDOBA 2011):

$$\gamma(d) = C \exp\left(\frac{d}{a}\right) \tag{5}$$

– exponential semivariogram (NIEDOBA 2011):

$$\gamma(d) = C\left(1 - \exp\left(-\frac{d^2}{a^2}\right)\right) \tag{6}$$

where:
$\gamma(d)$ – value of the semivariogram function at point d,
C – semivariogram threshold value,
d – distance between two similar transactional points,
a – coefficient reducing the value of coordinates to a unit square.

The stochastic surface area of land value (W) was modeled assuming that the number of transactional points in interpolation groups is equal to 12 market observations, which is identical to the IDW method, and which will allow for a clear assessment of model assumptions without the impact of selectable parameters.

6. Study results

Generally, equivalent markets are determined radially, which means that the analyzed or valued property represents the center of gravity of the monitored area. This is not reflected in the factual nature of the market, which is proven by the land value maps presented below – at this stage, equivalent to the modeled subspaces of land value – categorized according to their spatial planning conditions and determined using the ordinary kriging method with the application of the exponential semivariogram (Fig. 4, 5 and 6). The example of real estate in the Podgórze district indicates the adoption, without prior analysis, of neighboring markets as equivalent ones, since the level of land value is the result of a number of factors distracting the allocation of similar micro-markets. Similarities between properties should be sought in areas of a homogeneous nature in terms of price and spatial planning.

As correctly pointed out by R. Cellmer *(…) "on the real estate market there is a high spatial autocorrelation, which means that property prices are heavily dependent on the prices of real estate in the neighbourhood"* (CELLMER 2014). It is worth mentioning here the price centroids, also referred to as price emitters, the importance of which and allocation in the analyzed market space is variable depending on the use. In the following visualizations, price dissonance concerning surface price emitters, the interaction of which depends on the analyzed market segment, is noticeable.

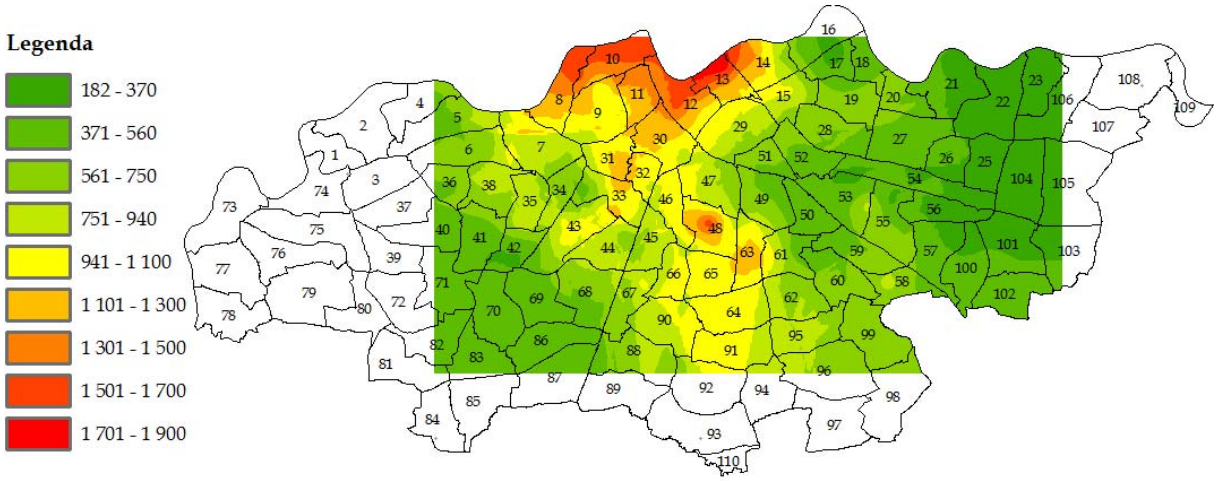

Fig. 4. Value map of real estate intended for high intensity development. *Source*: Author's own study.

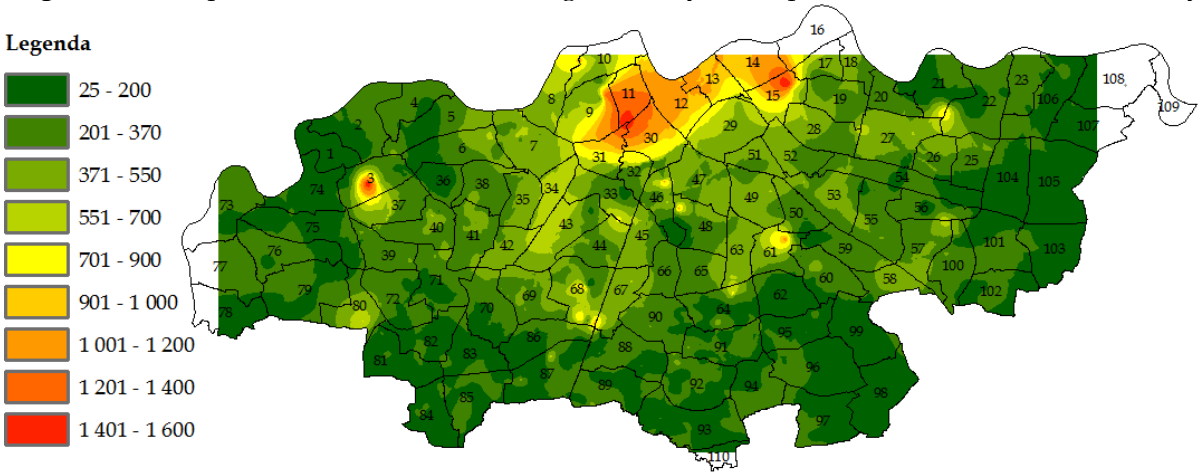

Fig. 5. Value map of real estate intended for low intensity development. *Source*: Author's own study.

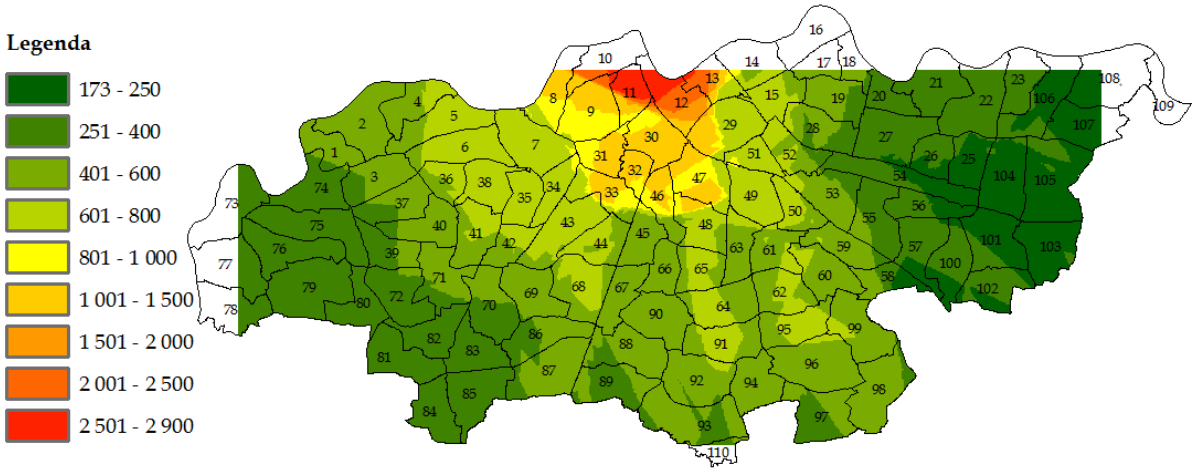

Fig. 6. Value map of real estate intended for service development. *Source*: Author's own study.

7. Discussion and conclusions

The problems of processing information mainly concern the quantity and quality of the available data. The accumulation of data can be observed in the undeveloped land property sector, which is characterized by a significant price dissonance being the consequence of the allocation of real estate in the analyzed area, as well as the diversity of spatial planning directions. The equalization of price determinants is suggested to be performed by determining a continuous subspace of land value for the main spatial planning trends, which will be the cornerstone of the concretization of price emitters.

The interpolation of transaction prices was done using three methods, namely:

1. the inverse distance method (Fig. 7),
2. the natural neighborhood method (Fig. 8),
3. the ordinary kriging method, using the Gaussian semivariogram model (Fig. 9).

The results of each method are similar, and possible differences are a consequence of the assumptions made during the interpolation of the grid. In the case of the uniform density of large amounts of information, the selection of the method of grid interpolation is of secondary importance, which is evidenced by the images of modeling the transactional price subspace of land intended for low intensity housing development (Fig. 7, Fig. 8, Fig. 9). The above is not justified in markets with a limited amount of data and of varied density. The exploration of results generated using the compared methods indicates kriging to be the method that the most accurately reflects the value of land property for other uses (MW, U, KD, ZL).

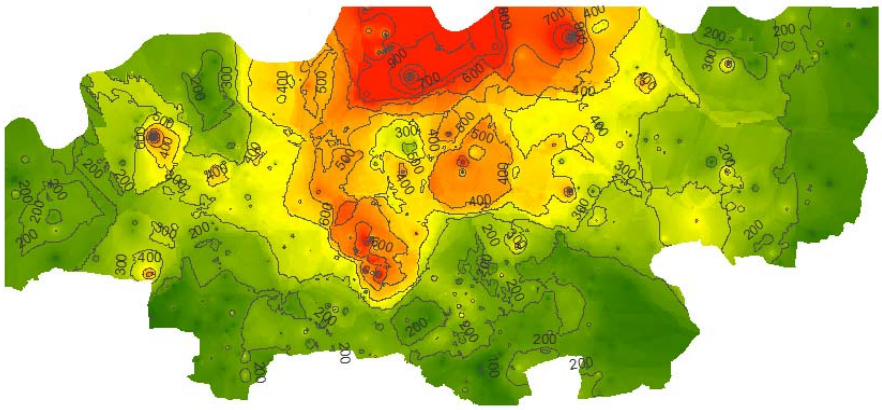

Fig. 7. Map of land value for low intensity development, generated using the inverse distance method. *Source*: Author's own study.

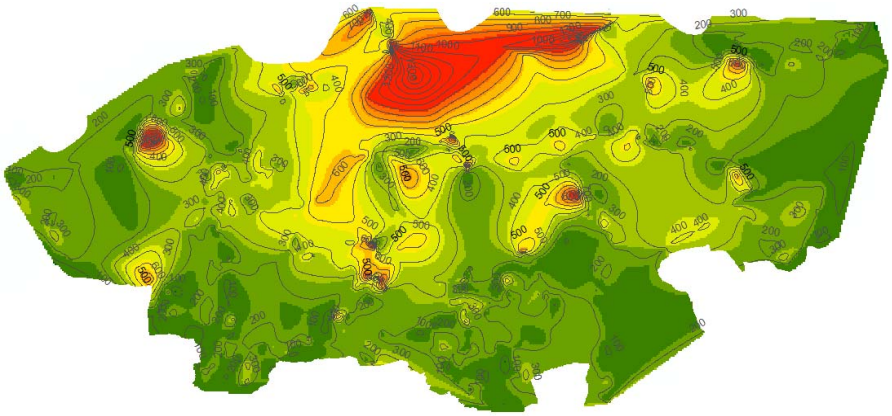

Fig. 8. Map of land value for low intensity housing development, generated using the natural neighbor method. *Source*: Author's own study.

The visualizations presented above depict a slight shift in the allocation of regressive surface price emitters and their range, which in no way precludes the possibility of their prediction. Regardless of the methodology applied, the main emitter creating the value of real estate in the Podgórze district intended for MN use is the historic part of Podgórze, local shopping centers and the neighborhood of green areas. The open catalogue of factors interacting with transaction prices has been summarized in Table 1 below.

The scope of the impact of the above-mentioned price-determining centroids is varied and largely correlated with the use of the surrounding areas. The scope of the effect of two intensive emitters results in local price shocks.

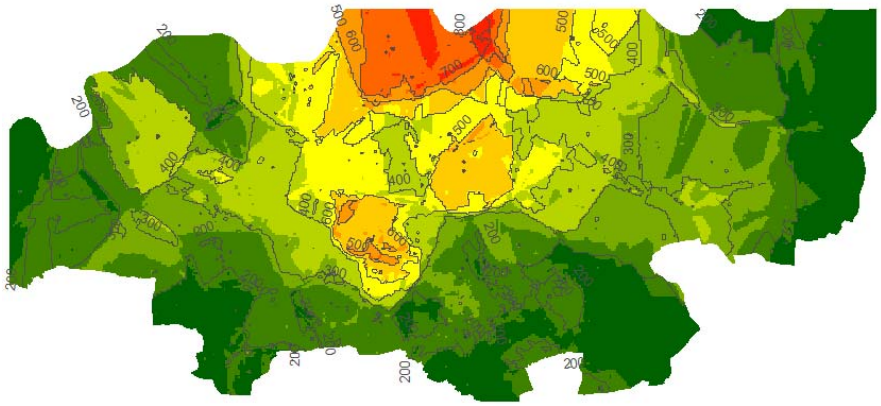

Fig. 9. Map of land value designated for low intensity housing development, generated using the kriging method. *Source*: Author's own study.

Table 1

Regressive price emitters

Property purpose	Surface and point price emitters
MN – single-family housing development	Historic part of Podgórze, shopping centers, green areas.
MW – multi-family housing development	Historic part of Podgórze, shopping centers.
U – service development	Historic part of Podgórze, office complexes.
KD - transportation	Surrounding areas.
ZL – green areas	Recreational areas, communication of strategic nature.

Source: Author's own study.

The recognition of major spatial centroids affecting the value and use of the analyzed area allows for the determination of value areas defined by the border extremes of a given price range. The modelled subspaces of land value should be identified with equivalent markets, the selection of which is crucial in the process of estimating property value. For example, for real estate intended for low-intensity residential development, a continuous area of land value with prices extremes ranging from PLN 371/m2 to PLN 700/m2 (Fig. 10).

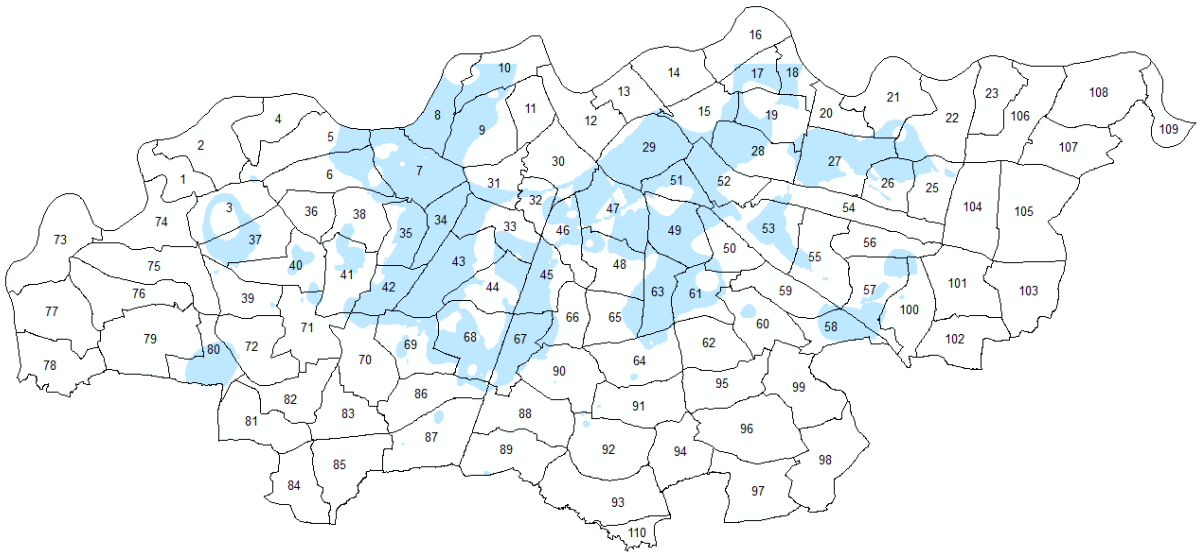

Fig. 10. Map of equivalent markets generated for low-intensity land property with the price extremes of PLN 371/m2 and PLN 700/m2. *Source*: Author's own study.

The final conclusion from the conducted studies refers to the determined problem of the field of real estate management, presented in the introduction to this paper and concerning the arbitrary

treatment of market similarity understood as a continuous (uninterrupted) area around the appraised property. As demonstrated by deterministically and stochastically interpolated land value maps, the behaviorism of the market is significantly non-linear and dependent on the frequency and length of the price wave, retaining some features of an acoustic wave (CIESZKO et al. 2016) and water wave (DINGEMANTS, KLOPMAN 2010). This means that market values are not distributed regularly around the surface price emitters. Therefore, the inclusion of real estate located around the epicentre, i.e. the valued property, in the valuation of real estate or located within the borders of one or several neighboring areas, results in the similarity of the real estate of the so-called representative samples being retained in physical terms only. The even more important economic aspect, including the vital attribute of real estate, i.e. its price, is entirely omitted. In this study, the total elimination of the economic inconsistency of property has been shown; it is suggested to stray from the radial perception of the market and explore the equivalent markets within specific uses. The deterministic methods of the interpolation of land value maps (IDW and neutral neighborhood) and the stochastic method of ordinary kriging (in two versions of a theoretical semivariogram: exponential and Gaussian) presented in the paper provide geospatial information about the location of equivalent markets. In addition, the author points to the superiority of strictly distance-weighted methods (kriging, IDW) over the indirectly-weighted ones determined by surfaces. The presented methods of the interpolation of land value maps put the approach to determining the market value of real estate with changed spatial planning factors in a new light. The value should be established in three configurations, where each of them is attributed a different regionalization (concentration) of properties similar in physical and economic terms. Geospatial pre-orientation concerning the location and size of equivalent markets in the era of GIS development should become an element of the set of appropriate skills possessed by a real estate appraiser dealing with the valuation of real estate accounting for various spatial planning factors. As a result, the beneficiaries of zoning fees will pay for the actual increase in value resulting from the adoption or change of a local plan and not the value which was attributed as a result of an incorrectly recognized market.

Finally, attention is drawn to the evolutionary nature of the study, especially in terms of the nomenclature adapted in the planned research. For the purposes of this paper it was assumed that land value maps are modelled subspaces of land value. However, the meaning of the term "subspace" is slightly narrower, as it is the price-randomized implementation of the interpolation model that minimizes the dissonance of value by the action of self-functioning free market mechanisms. The subspaces of randomized land values will be the basis for determining the n-dimensional model space of the real estate market (Full Space Real Estate Market), where n is the number of vectors of price-determining factors.

8. References

BRAUNBRÜCK A., RAVASOO A., 2016, *Nonlinear Wave Propagation and Reflection – Comparing the Numerics with the Analytics*, Wave Motion, Vol. 60, pp. 108-120.

CELLMER R., 2014, *Modelowanie przestrzenne w procesie opracowania map wartości gruntów (Spatial Modeling in the Process of Developing Land Value Maps)* (in Polish), UWM, Olsztyn.

CELLMER R., KURYJ J., 2011, *Defining the Land Value Zones with Use of Geostatistical Methods*, Journal of the Polish Real Estate Scientific Society, Vol. 19, No. 3, pp.. 7-19.

CIESZYŃSKI M., DRELICH R, PAKUŁA M., 2016, *Wave Dispersion in Randomly Layered Materials*, Wave Motion, Vol. 64, pp. 52-67.

DINGEMANTS M. W., KLOPMAN G., 2010, *Reflection in Variational Models for Linear Water Waves*, Wave Motion, Vol. 47, pp. 469-489.

GOTWAY C.A., YOUNG L.J., 2002, *Combining Incompatible Spatial Data*, Journal of the American Statistical Association, Vol. 97, pp. 632 – 647.

GUNES A., KOVEL J., 2000, *Using GIS in Emergency Management Operations*, Journal of Urban Planning and Development, Vol. 126(3), pp. 136-149.

Huang D., Qian Z., Shen G., Xu J., 2010, *Spatial Analysis and Assessment of Soil Fertility by Using GIS and Kriging Method*, World Automation Congress (WAC), pp. 19-23.

JASIŃSKA E., 2008, *Zastosowanie Analizy Czynnikowej w Szacowaniu Nieruchomości (The Application of Factor Analysis in the Estimation of Real Estate)* (in Polish), Zastosowanie metod statystycznych w badaniach naukowych III, StatSoft Polska.

KRAJEWSKA M., 2008, *Value of Built-up Eeal Estate in Different States of Spatial Planning*, Journal of the Polish Real Estate Scientific Society, Vol. 16, No. 3, pp. 47-54.

KUCHARSKA – Stasiak E., 2011, *Value Measurement in Economics – Consequences for Real Estate Valuation*, Journal of the Polish Real Estate Scientific Society, Vol. 19, No. 1, pp. 7-16.

LIGAS M., 2009, *Application of Regression – Kriging Model to Real Estate Value Prediction*, Journal of the Polish Real Estate Scientific Society, Vol. 17, No. 1, pp. 7-15.

LIU C., QIAN Z., SARRIS A., SHEN G., XU J., 2009, *Applications of Regression Kriging and GIS in Detecting the Variation in Leaf Nitrogen and Phosphorus of Spruce in Europe*, Computer Science and Information Engineering, Vol. 7, pp. 336-340.

MITKA B., PIECH I., 2012, *Modelowanie powierzchni terenu (Land Surface Modelling)*, Kraków, Infrastruktura i Ochrona Terenów Rolnych, No. 3/I/2012, pp. 167-180.

MUCHA J., WASILEWSKA M., 2007, *Geostatystyka nieparametryczna w dokumentowaniu złóż (Non-Parametrical Geostatics in Deposit Documenting)* (in Polish), Kraków, Zagrożenia naturalne w górnictwie – materiał warsztatowy, pp. 339-352.

NIEDOBA T., 2011, *Zastosowanie krigingu zwyczajnego dla oszacowania zawartości popiołu i siarki w węglu w zależności od gęstości i rozmiaru ziarna (Using Ordinary Krining to Estimate Ash and Sulphur Content in Coal Depending on Grain Density and Size)*, Górnictwo i Geologia, Vol. 6, No. 2, pp. 159-166.

ŹRÓBEK R., DAWIDOWICZ A., 2014, Scientic Editors, *Spatial Data in Wide Geospace*, Croatian Information Technology Society, GIS Forum 10 000 Zagreb, Croatia, University of Warmia and Mazury in Olsztyn, University of Silesia, Poland, pp. 27-34.

PRUS B., 2010, *Mapa wartości gruntów jako czynnik wspomagający gospodarowanie terenami w gminie (Land Value Map as a Factor Supporting Commune Land Management)*, Infrastruktura i Ekologia Terenów Wiejskich, Polska Akademia Nauk, No. 12, pp. 169-177.

TOBLER R. W., 1970, *A Computer Model Simulating Urban Growth in the Detroit Region*, Economic Geography, Vol. 46 (2), pp. 234-240.

Act of 21 August 1997 *on Real Estate Management*, Journal of Laws of 2015, Item 1774, as amended.

Act of 27 March 2003 *on Spatial Planning and Land Utilization*, Journal of Laws of 2015, Item 199, as amended.