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SPECULATIVE BUBBLES AND THEIR COMPONENTS ON THE REAL ESTATE MARKET– A PRELIMINARY ANALYSIS

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Abstract

Various speculative phenomena arise on the real estate market, and the speculative bubble (SB) is one of the best known events of the type. Speculative bubbles still have many unidentified components, and are characterized by high research potential due to the multiple factors responsible for bubble creation, as well as considerable practical implications on account of the multivariate results describing the real estate market (REM) and its surroundings.

Speculative price bubbles are associated mainly with changes in price trends on the real estate market. A thorough analysis of a speculative bubble over time demonstrates trend changes also in other research categories which constitute bubble components and elements of the real estate market and its surroundings. The above criteria were used to identify a new research category termed speculative bubble components (SBC).

The research hypothesis states that speculative bubbles should be analyzed based not only on prices, but also on bubble components. The objectives of this study were to: 1) classify speculative phenomena on the REM, 2) describe a speculative bubble based on market prices and SBC, and 3) present the results of a study evaluating speculative bubble components in relation to market prices, and discuss the trajectories of the analyzed research categories over time.

This study attempts to determine whether a speculative bubble can be analyzed in view of its components, and which elements of the real estate market and its surroundings can be classified as SBC. Attempts were also made to identify a research method that supports the identification of SBC variables and classification of variables into groups, and explains market prices in view of the identified variables and groups. The research relies on a review of literature in the theoretical part and statistical analyses in the experimental part. The results will broaden our knowledge of the mechanisms behind speculative phenomena on the real estate market.

Keywords: speculative bubble, speculative bubble components, real estate market, speculation

JEL Classification: D84, R21.

Citation: Brzezicka J., 2016, *Speculative Bubbles and their Components on the Real Estate Market – a Preliminary Analysis*, Real Estate Management and Valuation, Vol. 24, No. 1, pp. 87-99.

DOI: 10.1515/remav-2016-0008

1. Introduction

Speculative bubbles (SB) on the real estate market (REM) have a multi-dimensional and complex character. A one-dimensional study of real estate markets does not generate sufficient data for analyzing speculative bubbles. The complexity of the investigated phenomenon requires a detailed examination of its constituent elements. This paper discusses the concept of speculative bubble components (SBC). Speculative bubble components represent a broader approach to the classification,

description and measurement of speculative phenomena on the REM. In the first part of the study, an attempt will be made to classify speculative phenomena on the REM, and SBC will be analyzed in successive chapters.

2. Classification of speculative phenomena on the REM

2.1. Definition of speculative phenomena

Speculation is defined as the purchase or sale of assets in the hope of reselling or repurchasing them on the same market at a higher or lower price (CARTER 1998, p. 4), or as a "rat race of trying to get the highest possible return in the shortest period" (HAGSTORM 1994, p. 43). Speculative behaviors apply to a series of market phenomena that can be collectively referred to as speculative phenomena. Speculative phenomena are an inherent part of the real estate market. According to BRYX (2006, p. 38), speculation is a function of the real estate market. Speculative behavior on real estate markets leads to the rise of a speculative bubble, which appears to be a nomological phenomenon (TYC 2013, p. 340) but, in reality, is a highly complex event.

In this study, a speculative bubble is defined as an above-normal increase in property prices which is not justified by intrinsic factors, which contributes to the attainment of a price at which the bubble bursts and, consequently, causes market prices to freefall. This definition is consistent with the existing nomenclature relating to speculative bubbles on the real estate market and the key assumptions regarding the creation and life cycle of speculative bubbles on the REM (SHILLER 1990; STIGLITZ 1990 as cited in HIMMELBERG, MAYER and SINAI 2008, p. 1; CASE, SHILLER 2003; KINDLEBERGER and ALIBER 2005 as cited in LAI and VAN ORDER 2010, p. 753; HUI, YUE S 2006; SMITH and SMITH 2006, p. 2; ŻELAZOWSKI 2007, pp. 139-140; 2008, p. 99; GOODMAN, THIBODEAU 2008; KUCHARSKA-STASIAK, SCHNEIDER and ZAŁĘCZNA 2009, pp. 25-30; ŁASZEK, AUGUSTYNIAK and WIDŁAK 2009, p. 3; GARBER 2000; TYC 2013, p. 341; BRZEZICKA and WIŚNIEWSKI 2014).

In view of the complexity of the analyzed phenomenon, speculative events were examined in horizontal and vertical dimensions for the needs of a detailed analysis. In this approach, speculative phenomena are expressed by a speculative bubble, whereas the speculative bubble is expressed by its full life cycle. A life cycle is defined as the overall set of processes which take place over time and support a holistic description of the bubble, from its creation until the end of its life cycle. Those processes can be collectively referred to as a **speculative process**. A speculative process is not linked with any single aspect of speculative phenomena, but it applies to both the horizontal and the vertical dimension.

2.2. Speculative phenomena in the horizontal dimension

In the horizontal dimension, a speculative bubble is observed (a shift in the trajectory of prices and other components) in reference to the time variable based on the bubble's life cycle. The following research categories linked with speculative bubbles on the real estate market can be identified in the horizontal dimension: a speculative bubble (as the primary concept for speculative phenomena), a negative speculative bubble, and a speculative anti-bubble. The above research categories are presented in Figure 1.

ROUBINI and MIHM (2011) proposed the concept of a negative speculative bubble (NSB) which involves a rapid drop in prices after the bubble has burst. This is a minority view because the speculative cycle is determined by an increase in prices until the burst, and the ensuing freefall of prices (see SHILLER 2000). In this approach, NSB is part of the definition of a speculative bubble, whereas in the structural approach, it is a part of the SBC concept.

The speculative anti-bubble (SAB) is defined as an above-normal drop in property prices which is not justified by intrinsic factors, leads to the attainment of a *depressed price* and a continued rise in prices above the long-term trend on the market[1]. This approach is characterized by a specific "symmetry of definitions" which is derived from the "symmetry of prices" in the life cycle of a speculative bubble on the assumption that the price trend describing the anti-bubble in a simplified

[1] The concept of a speculative anti-bubble was defined during a scientific debate in November 2014. The author was unable to find any references to the anti-bubble concept in the international literature predating that event.

model represents the prices before the burst[2].

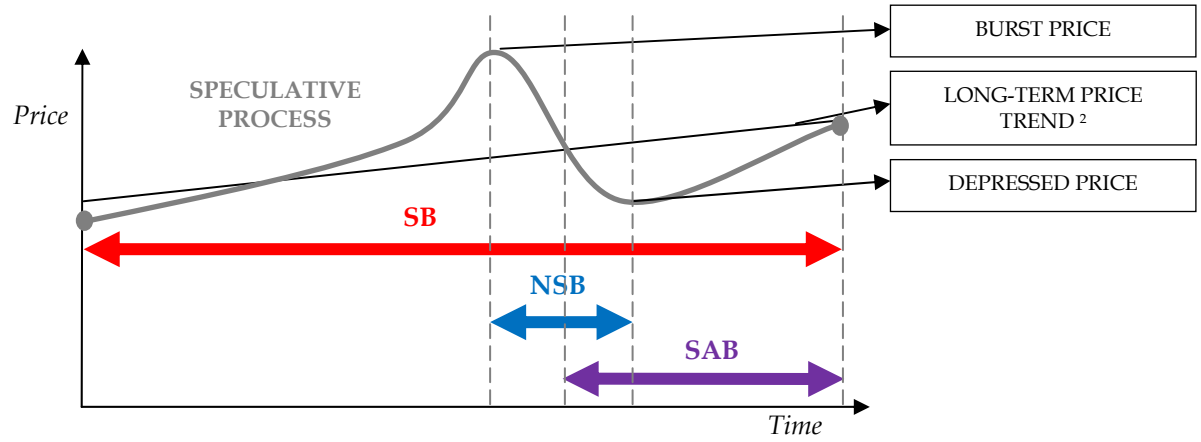

Fig. 1. Speculative phenomena on the REM in the horizontal dimension. *Source:* own elaboration.

2.3. Speculative phenomena in the vertical dimension

In the vertical dimension, a speculative bubble is observed (shift in the trajectory of prices and other components) in reference to the prices and bubble components in the vertical dimension, including with the use of the time variable. According to Wiśniewski (WISNIEWSKI 2007, pp. 22-23) and Belej (BELEJ 2013, p. 52), a systemic approach to the real estate market supports simultaneous observations and analyses of the elements of the real estate market and its surroundings. The observations of changes in property prices enable simultaneous evaluations of changes in other research categories which are elements of the real estate market and its surroundings. A new research category termed speculative bubble components (SBC) was identified based on the above considerations. Speculative bubble components are used to analyze bubbles in the vertical dimension, over a given time interval that corresponds to a bubble's life cycle (red line in Fig. 1) or a shorter period that ends with the attainment of a depressed price.

Speculative bubble components are defined as variables in the real estate market or its surroundings whose trajectories are bound by a unique relationship with the market prices of property. Those variables are classified as SBC because, due to the complex character of speculative processes, their trajectories accompanies price changes. The above holds true on the assumption that the place and significance of the variables in the life cycle of a speculative bubble cannot be reliably described if those variables are merely regarded as the cause or outcome of a speculative bubble. In principle, SBC and market principles are bound by multi-dimensional causal relationships (Fig. 2).

3. Data and Methods

3.1. Objective

Speculative bubble components were analyzed. The key objective of the study was to present speculative bubble components, and was achieved through the following intermediate objectives: 1) to present changes in selected variables from the real estate market and its surroundings which are bound by a relationship with property prices over time, 2) to classify variables into similarity groups, and 3) to verify whether the selected variables can be regarded as SBC.

3.2. Methods

Speculative bubble components were identified and analyzed in two stages. In the first stage, variables were selected, described and classified into similarity groups based on their trajectories. The second stage involved an analysis of the mutual interactions between variables and prices. The deployed analytical methods included graphic data analysis, linear correlation analysis, multiple regression analysis and cluster analysis.

[2] The simplified approach may seem doubtful because the price trend describing the anti-bubble could be shifted due to changes in the absolute "level" of a linear trend function over time. Those analyses are not an object of interest in this study, which is why the simplified approach was adopted.

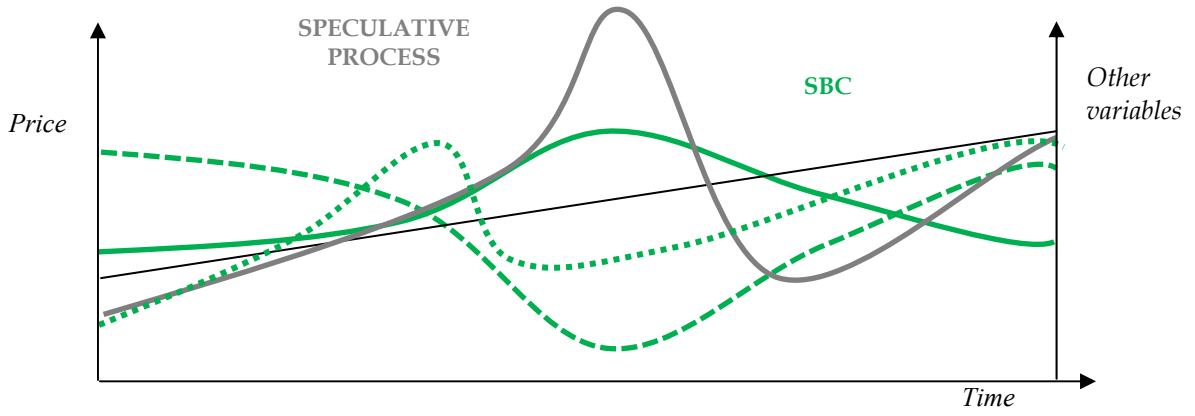

Fig. 2. Speculative phenomena on the REM in the vertical dimension. *Source:* own elaboration.

3.3. Database

The study was performed on data relating to the property market in Olsztyn. The analyses were carried out based on average annual prices of apartments in 2004-2013, calculated as average prices for all transactions recorded in the Register of Real Estate Prices and Values kept by the Olsztyn City Office. More than ten variables were selected for the SBC analysis. The analyzed data did not produce a closed set. The selected variables described various social and economic categories, including demographics, construction, investments, and macroeconomic factors that are directly or indirectly linked with the real estate market. The below database of variables was developed based on the information provided by the Central Statistical Office, unless indicated otherwise. Local data describe the city of Olsztyn, whereas some data relate to another administrative unit (region), as indicated in the description. The following variables were selected for the study (values are given in Table 1):

- Variable 0 (C_N) – property price [PLN/m²],
- Variable 1 (TRN) – number of concluded transactions [numb.],
- Variable 2 (L_LUD) – population [pers.],
- Variable 3 (P_N) – rate of natural increase per 1000 population [pers.],
- Variable 4 (DZIET) – total fertility rate, defined as the number of children born per woman in her child-bearing years [pers.], data for the Olsztyn area,
- Variable 5 (L_MIESZ) – number of apartments [numb.],
- Variable 6 (POW_U) – apartment floor area [m²],
- Variable 7 (POW_U1) – average floor area per apartment [m²],
- Variable 8 (MIESZ_O) – total number of apartments commissioned for use [numb.],
- Variable 9 (GR_KOM) – total municipal land [ha], data for the Olsztyn area (city with county rights),
- Variable 10 (W0) – population per year *t*/number of apartments per year *t* (W0) [pers./apart.],
- Variable 11 (D_LUD) – population growth rate per year *t*, defined as the rate of population growth in year *t*, where population in year *t*-1 = 100 [dimensionless quantity],
- Variable 12 (D_MIESZ) – percentage rise in the number of apartments, defined as an increase in the number of apartments in year *t*, where the population inhabiting apartments in year *t*-1 = 100 [dimensionless quantity],
- Variable 13 (W1) – population growth /apartment growth (W1) [dimensionless quantity],
- Variable 14 (W2) – relationship between inter-regional migration flows in each region and the population inhabiting apartments in those regions (W2), where W2 = inter-regional migration flows in year *t* / number of apartments in year *t* * number of people per apartment (W0) * 100%[3] [dimensionless quantity],
- Variable 15 (WYN) – average gross monthly income [PLN],
- Variable 16 (DOCH) – budget revenue in Olsztyn (city with county rights) per inhabitant [PLN],

[3] Detailed methodology and justification for calculating variables W0, W1 and W2 were described by BRZEZICKA and WISNIEWSKI (2015; 2016).

- Variable 17 (WYD) – budget expenditure in Olsztyn (city with county rights) per inhabitant [PLN],
- Variable 18 (BIL) – current balance in Olsztyn, revenue - expenditure, per inhabitant [PLN],
- Variable 19 (N_INW) – investments per inhabitant [PLN/pers.],
- Variable 20 (N_BR) – internal R&D spending [PLN], data for the Region of Warmia and Mazury,
- Variable 21 (STUD) – percentage of foreign university students [%], data for the Region of Warmia and Mazury,
- Variable 22 (INFL) – annual inflation (previous year =100) [dimensionless quantity], data for Poland,
- Variable 23 (S_REF) – reference interest rate, calculated as a weighted average of interest rates in a given year, where the number of months with a given interest rate, rounded off to 30 days, was the weight [%], data for Poland, developed based on data provided by the National Bank of Poland,
- Variable 24 (S_BEZ) – registered unemployment rate [%].

Table 1

Data used in the study

Year	C_N	TRN	L_ LUD	P_N	DZIET	L_ MIESZ	POW_U	POW _U1	MIESZ _O	GR_ KOM	W0	D_ LUD	D_ MIESZ
2004	**1998**	463	173850	2.0	1.27	64153	3689764	77.2	1213	4169.0	2.7099	100.00	100.0
2005	**2327**	631	174473	1.2	1.29	65230	3759157	68.5	1119	4137.0	2.6747	100.36	101.7
2006	**2933**	739	174941	2.1	1.32	66506	3843878	66.9	1299	4092.0	2.6305	100.27	102.0
2007	**3995**	713	175710	2.4	1.35	68133	3942368	61.5	1653	8833.0	2.5789	100.44	102.4
2008	**4704**	642	176142	2.8	1.41	69704	4057260	73.1	1573	8832.9	2.5270	100.25	102.3
2009	**4460**	642	176457	3.2	1.41	70375	4104666	70.5	676	8832.9	2.5074	100.18	101.0
2010	**4604**	1236	175388	2.4	1.33	71363	4123273	65.9	994	8833.0	2.4577	99.39	101.4
2011	**4749**	2187	175420	1.8	1.25	72190	4178663	66.1	957	8833.0	2.4300	100.02	101.2
2012	**4209**	3941	174641	1.0	1.21	73407	4255205	63.5	1224	8833.0	2.3791	99.56	101.7
2013	**4044**	4440	174675	0.8	1.177	74581	4325341	60.7	1187	4119.0	2.3421	100.02	101.6

Year	C_N	W1	W2	WYN	DOCH	WYD	BIL	N_ INW	N_ BR	STUD	INFL	S_ REF	S_ BEZ
2004	**1998**	1		2477	2483	2545	-62	2165	39	0.2	103.5	5.8	10.8
2005	**2327**	0.99	0.046	2563	2637	2607	30	2735	46	0.2	102.1	5.3	9.2
2006	**2933**	0.98	0.050	2742	2916	2912	4	3312	39	0.2	101	4.1	6.8
2007	**3995**	0.98	0.050	2931	3348	3204	144	3769	68	0.2	102.5	4.4	4.4
2008	**4704**	0.98	0.041	3133	3578	3539	39	4140	57	0.2	104.2	5.3	4.3
2009	**4460**	0.99	0.039	3323	3573	4042	-469	4101	81	0.2	103.5	3.8	7.3
2010	**4604**	0.98	0.041	3443	3823	4457	-633	4503	120	0.3	102.6	3.5	6.9
2011	**4749**	0.99	0.041	3547	4493	4706	-213	5334	138	0.4	104.3	4.2	7.2
2012	**4209**	0.98	0.036	3649	4743	4609	135	5190	146	0.4	103.7	5.0	8.4
2013	**4044**	0.98		3795	5080	4916	164	4275	112	0.6	100.9	2.9	8.4

Source: own elaboration based on the Register of Property Prices and Values (Price), Central Statistical Office (Variables 1-22 and 24) and the National Bank of Poland (Variable 23).

4. Empirical Results

4.1. Stage 1 – data clustering

In the first stage of the study, variables whose trajectories changed in response to a speculative bubble were identified. The results are presented graphically in the figures below. Due to significant differences in the values of the analyzed variables and their units, all variables, excluding the price, were standardized. Price was the dependent variable and, therefore, was not standardized. Price was also not standardized to preserve real prices and provide a real reference point. The remaining variables were independent variables, and they were standardized with the use of Formula (1):

$$z_{ij} = \frac{x_{ij} - x_{j\,śr}}{s_j} \quad (1)$$

where: z_{ij} – standardized value of the j[th] attribute in the i[th] object; x_{ij} – initial value of the j[th] attribute in the i[th] object; $x_{j\,śr}$ – arithmetic mean of the j[th] attribute; S_j – standard deviation of the j[th] attribute.

Standardization supported simultaneous observations of all variables (Fig. 3). The relationship between every variable and price was plotted individually, but not all relationships were presented in this paper due to space constraints.

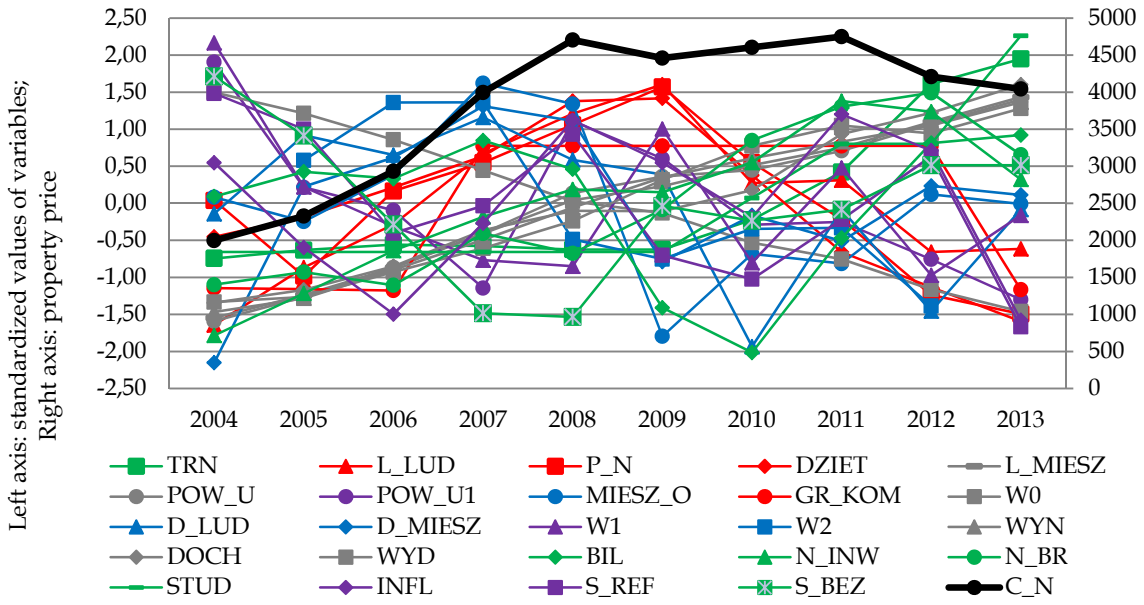

Fig. 3. Standardized variables used in the study. *Source:* own elaboration.

Five similarity groups were identified based on similarities in variable trajectories. Similarities were defined graphically by observing the relationships between each variable and price (refer to the "Limitations" section). Every group was presented in a separate figure (Figs. 4, 5, 6, 7 and 8). In each group, the arithmetic mean of standardized variables belonging to that group was calculated for each year. The arithmetic mean was used to simulate the trajectory of each group, and the most representative variable identified in each one. The selected variable was characterized by the highest correlation with the group mean. Correlation coefficients are presented in Table 2. The type of variables that influence property prices (stimulating and inhibiting factors) was taken into account. The main emphasis was placed on the classification of variables into groups based on similarities in their trajectories.

Table 2

Correlation coefficient

Group 0	Variable	L_MIESZ	POW_U	W0	WYN	DOCH	WYD	Mean
	Mean	0.9977	0.994	-0.997	0.9985	0.981	0.9883	1.000
Group 1	Variable	TRN	BIL	N_INW	N_BR	STUD	S_BEZ	Mean
	Mean	0.982	0.286	0.640	0.810	0.934	0.268	1.000
Group 2	Variable		L_LUD	P_N	DZIET	GR_KOM	Mean	
	Mean		0.928	0.907	0.895	0.760	1.000	
Group 3	Variable		MIESZ_O	D_LUD	D_MIESZ	W2	Mean	
	Mean		0.797	0.750	0.748	0.780	1.000	
Group 4	Variable		POW_U1	W1	INFL	S_REF	Mean	
	Mean		0.880	0.610	0.741	0.802	1.000	

Source: own elaboration.

Description of groups:

1) Group 0 – variables whose trajectory does not change significantly relative to property prices when a speculative bubble arises. The following variables were classified into group 0: Variable 5 (L_MIESZ), Variable 6 (POW_U), Variable 10 (W0), Variable 15 (WYN), Variable 16 (DOCH) and Variable 17 (WYD). Interestingly, one of the variables has a negative slope (decreasing), whereas the remaining variables have a positive slope (increasing). Those variables are marked

in grey in Figure 1. Variable 15 (WYN) (average gross monthly income) is the representative variable.

2) Group 1 – variables whose trajectories are symmetrical to the price trajectory. The following variables were classified into group 1: Variable 1 (TRN), Variable 18 (BIL), Variable 19 (N_INW), Variable 20 (N_BR), Variable 21 (STUD) and Variable 24 (S_BEZ). Those variables are marked in green in Figure 1. Variable 1 (TRN) (number of concluded transactions) is the representative variable.
3) Group 2 – variables whose trajectories are similar to the price trajectory. The following variables were classified into Group 2: Variable 2 (L_LUD), Variable 3 (P_N), Variable 4 (DZIET) and Variable 9 (GR_KOM). Those variables are marked in red in figure 1. Variable 2 (population) is the representative variable.
4) Group 3 – variables whose trajectory changes significantly relative to property prices when a speculative bubble arises and are characterized by a rising trend. The following variables were classified into Group 3: Variable 8 (MIESZ_O), Variable 11 (D_LUD), Variable 12 (D_MIESZ) and Variable 14 (W2). Those variables are marked in blue in Figure 1. Variable 8 (total number of apartments commissioned for use, *t*) is the representative variable.
5) Group 4 – variables whose trajectories change significantly relative to property prices when a speculative bubble arises and are characterized by a falling trend. The following variables were classified into Group 4: Variable 7 (POW_U1), Variable 13 (W1), Variable 22 (INFL) and Variable 23 (S_REF). Those variables are marked in violet in Figure 1. Variable 7 (average floor area per apartment) is the representative variable.

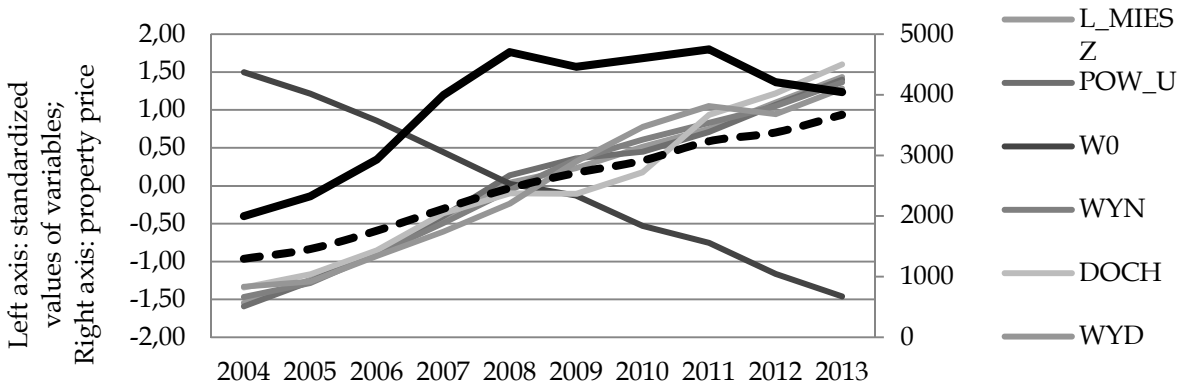

Fig. 4. Group 0. *Source:* own elaboration.

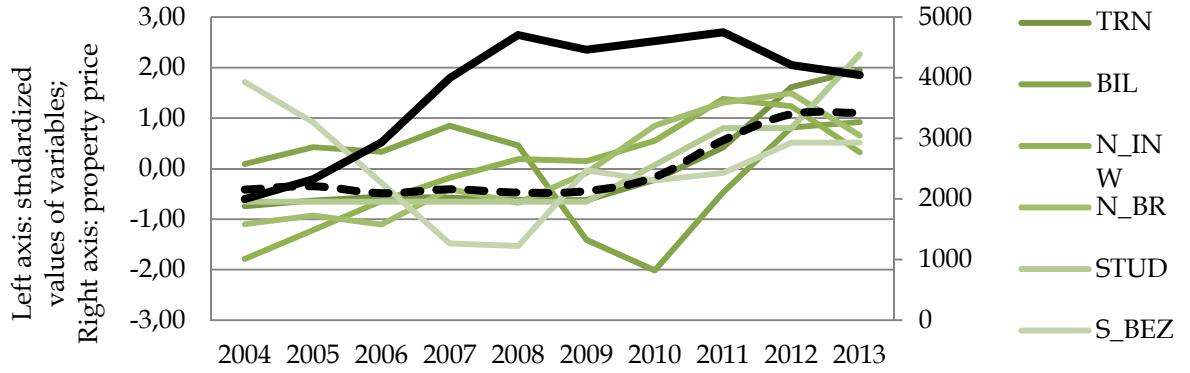

Fig. 5. Group 1. *Source:* own elaboration.

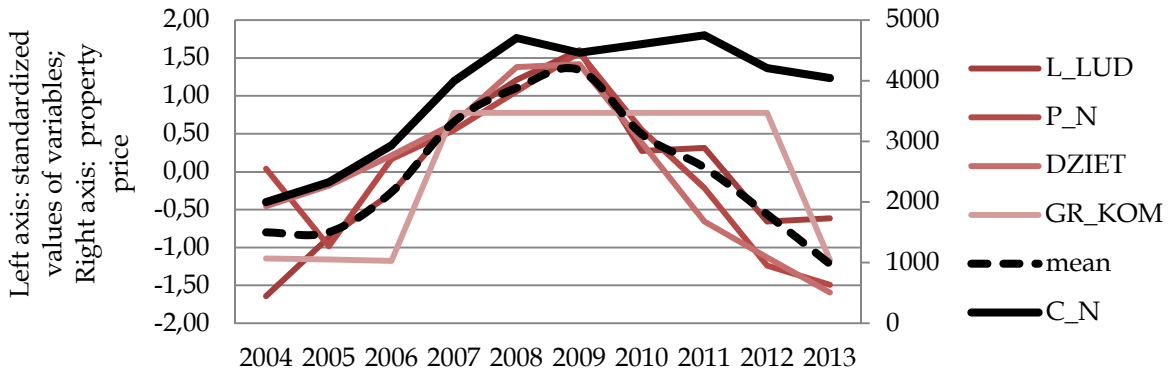

Fig. 6. Group 2. *Source:* own elaboration.

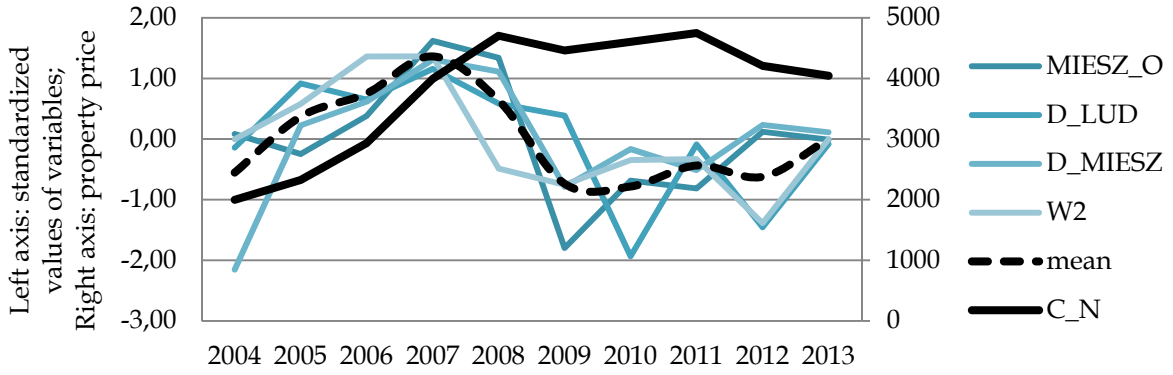

Fig. 7. Group 3. *Source:* own elaboration.

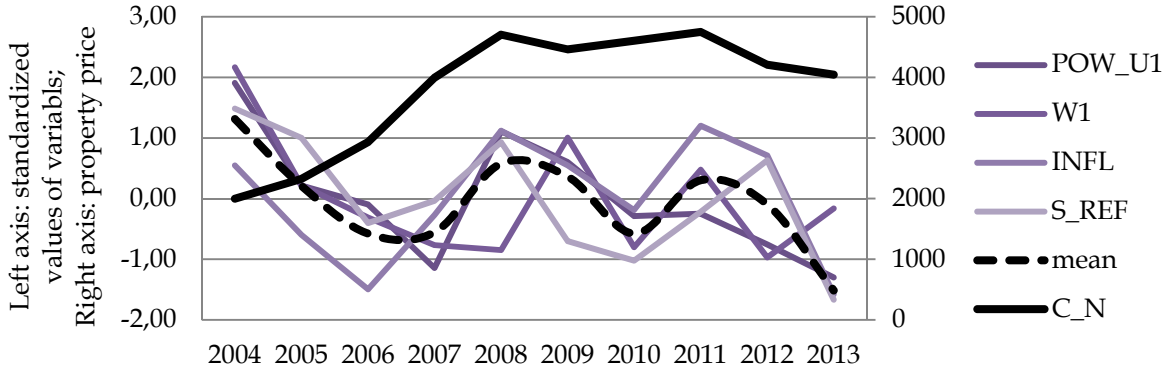

Fig. 8. Group 4. *Source:* own elaboration.

4.2. Stage 2 - practical implications

In the first stage of the study, Group 0 was eliminated because its variables did not change trajectories in response to a speculative bubble. In the first stage, variables were classified into groups, and values describing that group with the use of two criteria: 1) **representative variable in the group (Criterion 1)** and 2) **mean value in the group (Criterion 2)**, were identified. Those values are presented in Table 3 and Figures 9 and 10.

The correlations between the variables and the price were verified before further analyses, and strong correlations between variables for the adopted criteria were not determined. A multiple regression model was developed. In successive stages of the study, multiple regression models were developed for both criteria and their goodness of fit was checked. The results revealed that both models were characterized by good fit to real data. The significance level for values in Groups 3 and 4 was > 0.05 for both criteria.

Table 3

Values describing groups of variables based on two criteria

Year	Price	Representative variable				Mean value			
		Group 1	Group 2	Group 3	Group 4	Group 1	Group 2	Group 3	Group 4
2004	**1998**	-0.75	-1.64	0.08	1.91	-0.41	-0.80	-0.55	1.31
2005	**2327**	-0.63	-0.87	-0.25	0.22	-0.35	-0.80	0.37	0.21
2006	**2933**	-0.56	-0.28	0.38	-0.10	-0.49	-0.27	0.75	-0.58
2007	**3995**	-0.58	0.67	1.62	-1.14	-0.41	0.66	1.36	-0.55
2008	**4704**	-0.62	1.21	1.34	1.11	-0.47	1.11	0.64	0.58
2009	**4460**	-0.62	1.60	-1.79	0.60	-0.44	1.34	-0.74	0.36
2010	**4604**	-0.22	0.27	-0.68	-0.29	-0.17	0.49	-0.78	-0.58
2011	**4749**	0.42	0.31	-0.81	-0.25	0.56	0.05	-0.44	0.30
2012	**4209**	1.61	-0.66	0.12	-0.76	1.08	-0.56	-0.62	-0.10
2013	**4044**	1.95	-0.62	-0.01	-1.30	1.10	-1.22	0.00	-1.52

Source: own elaboration.

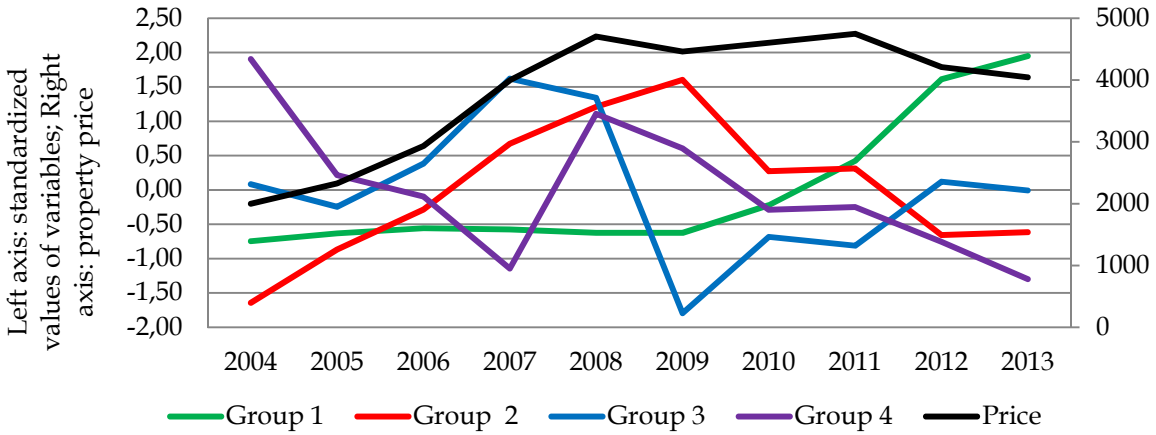

Fig. 9. Representative variables in groups. *Source:* own elaboration.

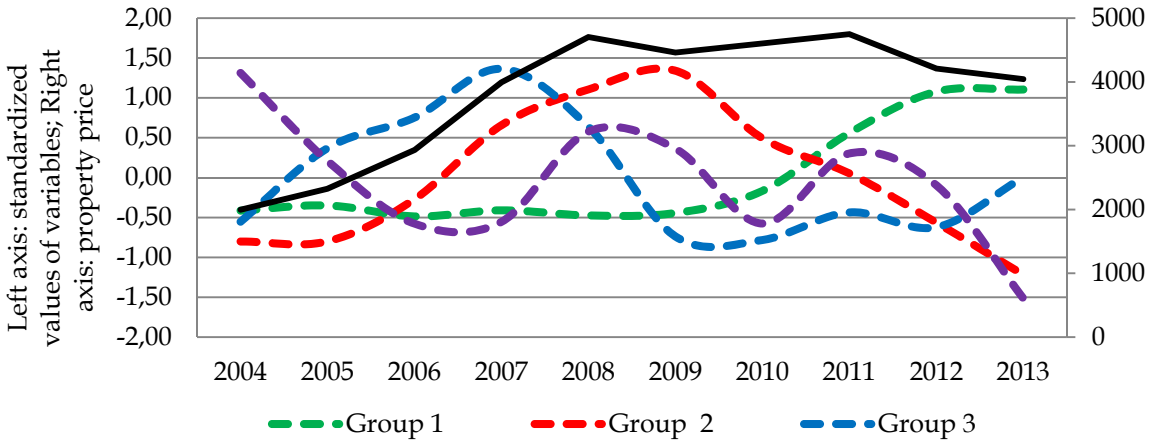

Fig. 10. Mean values in groups. *Source:* own elaboration.

Results modeled based on criterion 1 (representative variable):
- R^2: 0.87,
- significance of F: 0.01778 (critical value of F: 0.05),
- F-statistic: 8.71088 (critical value of F-statistic: 3.3258),
- *t*-statistic for Group 3: 0.1178 and Group 4: 0.2919 (critical value of *t*: ±2.5706).

Results modeled based on criterion 2 (mean values):
- R^2: 0.95,

- significance of F: 0.00185 (critical value of F: 0.05),
- F-statistic: 23.94933 (critical value of F-statistic: 3.3258),
- *t*-statistic for Group 3: -0.0358 and Group 4: -1.2020 (critical value of *t:* ±2.5706).

The model based on criterion 2 is characterized by better fit to real data.

Another series of models was developed to check the goodness of fit of models containing only values for Group 1 and Group 2. The results are presented in Tables 4 and 5. Both models were adopted, but the model based on mean values was characterized by better fit to the data due to the significance of F and the coefficient of determination. Critical values of F and *t* are not shown in the tables, and they were determined at: critical value of F for both models - 3.1355, and critical value of *t* for both models - ± 3.1824.

Table 4

Values describing groups of variables based on the representative variable criterion for Groups 1 and 2

Coefficients		*Standard error*	*t -statistic*	*p-value*	*Regression statistics*	
Intersection	3802.261	129.772	29.300	0.0000	Multiple R	0.93
Group 1	580.935	141.935	4.093	0.0046	R-squared	0.87
Group 2	917.541	141.935	6.465	0.0003	Adjusted R-squared	0.84
F			23.91489		Standard error	410.37
Significance of F			0.00074		Observations	10

Source: own elaboration.

Table 5

Values describing groups of variables based on the mean value criterion for Groups 1 and 2

Coefficients		*Standard error*	*t-statistic*	*p-value*	*Regression statistics*	
Intersection	3802.261	96.593	39.364	0.0000	Multiple R	0.96
Group 1	1364.13	182.089	7.492	0.0001	R-squared	0.93
Group 2	1226.400	136.305	8.997	0.0000	Adjusted R-squared	0.91
F			45.98329489		Standard error	305.45
Significance of F			0.00009411		Observations	10

Source: own elaboration.

The above implies that property prices can be explained by a regression model that takes into account SBC based on the adopted criteria. The elements of a model that accounts for mean values are presented in Figure 9, where the area characterized by intense speculative behavior is marked with an oval line. A model that accounts for the adopted methodology was described with the use of Formulas (2) and (3):

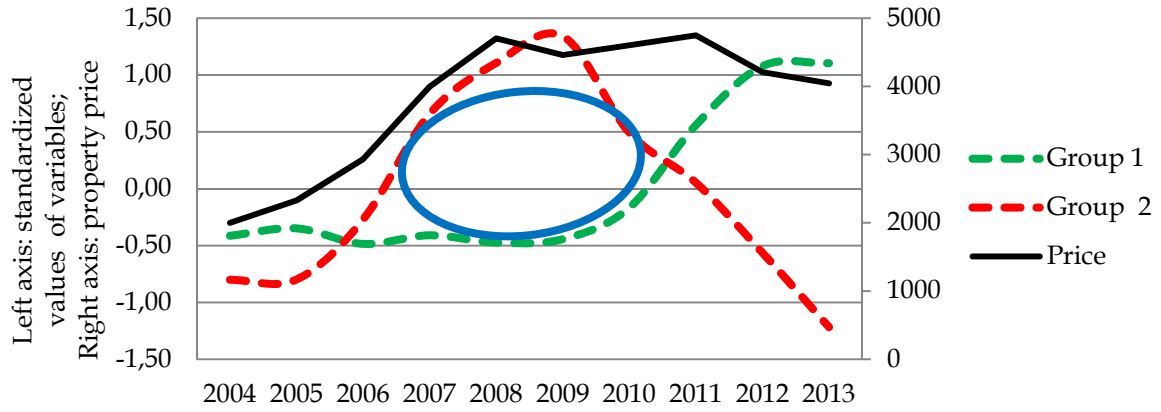

Fig. 9. Mean values in groups. *Source:* own elaboration.

$$Price = Group\,1 * X1 + Group\,2 * X2 + Intersection \quad (2)$$

$$Price = 1364.13 * X1 + 1226.40 * X2 + 3802.26 \quad (3)$$

where the value for Group 1 is the arithmetic mean of the standardized values classified into Group 1, and the value for Group 2 is the arithmetic mean of the standardized values classified into Group 2.

5. Limitations

The presented study has numerous methodological limitations resulting from the structure of the analyzed problem, the pioneering character of the research and the type of analyzed data. The key research limitations are presented below.

1) The analyzed data differed in their thematic reference, units expressing variable values and statistics describing samples for different variables. Variables were standardized to create a more homogeneous database.
2) Standardized data were subjected to cluster analysis to classify variables into groups, but homogeneous similarity groups could not be identified due to significant differences in the analyzed data. k-means clustering was used to determine the number of similarity groups. When 5 groups were selected, satisfactory similarity groups were not obtained after 20 iterations. An increase in the number of iterations did not improve the results.
3) The variables were correlated. The correlation matrix was calculated in the initial stage of the study, but ultimately, all variables were included in the analyses. The presence of correlations was dictated by the adopted research assumptions. It should also be noted that the classification of highly correlated variables into similarity groups could make the classification process somewhat "artificial". For example, the value of the correlation coefficient between Variable 5 (number of apartments) and Variable 6 (apartment floor area) was determined at 0.998, and between Variable 5 and Variable 7 (average floor area per apartment) – at 0.592. This example confirms that SBC is a complex problem. Non-clustered data cannot be subjected to regression analysis due to strong correlations between variables.
4) Both correlation and regression analyses were linear. A linear correlation coefficient has the following limitations: linear relationship between variables, sensitivity to terminal points, and sensitivity to an absence of normality (STANISZ 2006, p. 295). A linear regression model is built on the key assumption that independent variables are not correlated and that all independent variables are correlated with the dependent variable. Due to a high number of independent variables in the model and similarities in the thematic reference of selected variables, the analyzed variables were collinear (non-zero linear correlations) (ACZEL 2000, pp. 590-592), which could significantly influence the modeled parameters. The assumptions had to be relaxed due to the complexity of the problem. As a result, non-linear variables were described with a multiple linear regression model.
5) The results could be compromised by apparent correlations (TYSZKA 2010, p. 71), i.e. the presence of seeming correlations between two phenomena which, in reality, are not related. The above could be attributed to the presence of a third phenomenon which influenced the remaining two. Variable trajectories were not determined by property prices, but by the overall economic performance because prices of real estate were influenced by the economic crisis. At the same time, the extent to which social and economic factors are influenced by a speculative bubble and an economic crisis is difficult to estimate, and the two causative factors are difficult to separate. According to many authors, the main cause of the economic crisis was a speculative bubble on the US real estate market (BARBERIS 2013; BOOTLE 2009, p. 8; BLANCHARD 2009, p. 7). The structure of the research problem points to its complexity and the need for multiple-aspect analyses.
6) The last limitation is the apparent inconsistency of the research methodology. A speculative bubble arises on a real estate market when prices significantly exceed the intrinsic value of property (BRZEZICKA and WIŚNIEWSKI 2014; ŁASZEK, AUGUSTYNIAK and WIDŁAK 2009). Does this imply that this study undermines the above assumption? It should be noted that the results of this study explain property prices in complex manner, not through simple dependencies. Many of the analyzed variables are not intrinsic values on the real estate market. At this point, the author is unable to provide clear answers to the above questions, but research into the analyzed problems on the real estate market will be continued in the future.

6. Conclusions

The following conclusions can be formulated based on the results of this study:

1) A speculative bubble is a complex phenomenon, and its creation is accompanied by changes in the trajectories of variables from the real estate market and its surroundings. Many variables change their trajectories in response to a speculative bubble, but the values describing groups of variables are non-linear and are bound by mutual relationships.
2) In view of the proposed methodology, the variables classified into Groups 1, 2, 3 and 4 can be regarded as SBC, and the variables included in Groups 1 and 2, when clustered according to the described criteria, can explain market prices. In Figure 9, the system of two functions which are clusters of variables classified into Group 1 and Group 2 describes property prices. Those functions delineate an area characterized by intense speculative behavior. This study supported the development of a model describing this critical area. Unfortunately, the resulting conclusions have to be adjusted in view of the research limitations. The functions describing the clusters of values in Groups 1 and 2 are non-linear, whereas the applied multiple regression model is linear.
3) Calculations performed on composite measures produce similar results to simple solutions, i.e. mean value in a group and the choice of a representative variable in a group.
4) A detailed analysis of the trajectory of Variable 9 (total municipal land) produced interesting results which could indicate that Olsztyn also fell prey to the illusory increase in property prices.
5) Another question that arises in connection with this study is whether a speculative bubble can be reliably analyzed based on many variables that constitute SBC. A speculative bubble is a massive phenomenon which changes many elements both within and outside the market. The trajectories of variables are analyzed to determine the probability of a speculative bubble, make inferences about the stages of the speculative process and evaluate the condition of the REM. Further research is needed to overcome the existing limitations (non-linear models, elimination of collinearity of independent variables, other variable clustering methods) and offer new practical implications. The concepts presented in this paper have practical implications and can be used to develop new projects.

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