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CATASTROPHE THEORY IN EXPLAINING PRICE DYNAMICS ON THE REAL ESTATE MARKET

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Abstract

The real estate market is an open system, which implies that it is able to exchange signals with other open systems and dynamic systems. The evolution of a market system over time can be described mathematically. If the system's sensitivity threshold to external stimuli is exceeded, it becomes destabilized and moves from a near-balanced state to a state that is far from equilibrium. Those dynamic processes often induce key changes in the system's trajectory of evolution. In search of equilibrium, the system becomes transformed in a process of discontinuous and discrete changes in state variables. The above statement constitutes the research hypothesis in this article.

In this study, an attempt was made to develop a mathematical model for visualizing the evolutionary path of the real estate market in the form of continuous changes interrupted by discontinuous changes. The qualitative transformation of the system will be evaluated with the use of the catastrophe theory.

Keywords: cusp catastrophe, real estate market, discontinuity of prices.

JEL Classification: C1, C51 R31, R32.

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1. Introduction

The presented interdisciplinary study verifies the usefulness of research methods applied in mathematical and nature sciences for analyses of the real estate market. Greater emphasis will be placed on the examined methods' ability to describe the events and processes on the real estate market in qualitative rather than quantitative terms.

The past decade witnessed sudden changes in real estate prices on many markets. Those changes decreased the credibility of economic decision-making and prognostication based on classical methods for investigating the variability of property prices, namely linear and multiple regression analyses. In the existing literature, the fluctuations in real estate prices which significantly diverged from long-term market trends were regarded as insignificant adjustments of current trends and a part of market noise. The importance of those events was generally minimized. This study aims to introduce into scientific discourse a specific language for describing the phenomena which are observed on unstable real estate markets.

The study was carried out in the city of Olsztyn in north-eastern Poland. The proposed method can be applied to various local markets in Poland and in other countries. The study analyzed changes in the prices of apartments in 2001-2011. The source of the analyzed data was the Register of Real Estate Prices and Values kept by the Olsztyn City Office.

2. Transformation, development and evolution of the real estate market

The real estate market can be defined at different levels of accuracy, subject to the purpose of the description, the applied criteria, the adopted reference standards and the required degree of precision. In general, a real estate market is a system comprising a set of elements and their attributes and a set of relations observed between those elements. The set of elements contains all system components which determine all processes on the real estate market. Those processes form the set of relations. Interactions between the analyzed set of elements and their attributes and the set of mutual relations take place in a strictly defined period of time, and the dynamics of those interactions are determined by the dynamics of changes in the market environment. The strength, direction and intensity of mutual interactions vary. The real estate market can receive signals from the environment, but under specific circumstances, such signals can also be transmitted by the market. The above implies that significant changes in the legal, social, economic and political environment are drivers of change in sets of market elements and their attributes, and that they contribute to the development of the real estate market.

Development may lead to changes in the system's elements in both quantitative and qualitative terms. Economic changes, such as higher unemployment or lower household incomes, can lead to a drop in property prices or the number of mortgage applications which are quantitative changes in the studied element. Qualitative changes may be driven by social factors, such as expected improvements in market infrastructure relating to the transfer of property ownership or new market trends which increase the popularity of selected types of property.

Minor changes are also observed in systems that are in a near-balanced state, and stronger impulses cause the system to grow and develop. During the development process, not only the size, but also the structure of the system undergoes change, and alternating periods of structural transformations and gradual changes are observed (DOMAŃSKI, 2008). According to SCHUMPETER (1912), the role of development analyses should be minimized in near-balanced systems, but highly accentuated in transition systems or systems that are far from equilibrium. The above approach emphasizes the importance of qualitative changes in the development process. These changes are very difficult to describe in the form of a mathematical model.

Under specific circumstances, a certain category of changes induced by mutual interactions between the analyzed system and the social, economic and political environment can shift the equilibrium point to a position where equilibrium cannot be restored through an infinite number of small movements from the previous point, but only through a qualitative reform of the entire system.

Systems which move from a near-balanced state to a state that is far from equilibrium undergo significant transformations. Dynamic processes lead to evolution. In this case, evolution is the movement towards more complex states, and it is accompanied by structural changes (DOMAŃSKI, 2008).

Based on the above assumptions, the real estate market emerges as an open and dynamic system which responds to changes in the environment and evolves with time. By sending information to the environment and responding to selected environmental factors, the market transforms its structure over time, and its evolutionary path changes in the corresponding state space.

According to the author, dynamic processes observed on the Polish real estate market in the last decade have led to instability. The market adjusted its trajectory of evolution by moving from a near-balanced state to states that were far from equilibrium in a process of discontinuous change of the system's state variable.

This study aims to visualize the evolutionary path where the state of the real estate market is conditioned by supply and demand curves which, in turn, determine the equilibrium price. It is assumed that a state of stability is maintained for longer periods of time, whereas states of instability are short-lived, and the resulting sudden changes enable the market to adjust to new conditions and pursue a stable path of evolution. The above concept can be illustrated by Poland's economic transformations. The disintegration of a centrally-planned economy led to political and economic shock therapy in the 1990s which introduced free market principles and put the Polish economy on a stable growth path.

3. The catastrophe theory in studies of the real estate market

3.1. Introduction to the catastrophe theory

The catastrophe theory, also referred to as the morphogenesis theory or the theory of jump discontinuity, examines the behavior of systems over time. According to JAKIMOWICZ (2003), the catastrophe theory describes the evolution of an entity, from its birth to death, in the form of consequences of continuous changes which are interrupted by sudden, qualitative changes. According to THOM (1991), the catastrophe theory describes possible discontinuities in system evolution. Global system evolution is a succession of continuous evolutions and sudden jumps of varied quality.

The use of the catastrophe theory requires appropriate methods and a thorough understanding of concepts such as catastrophe and discontinuity. A subjective interpretation of the above concepts can lead to superficial conclusions and methodological errors. The above words have the following colloquial meanings:

- *catastrophe* – a sudden, unexpected event with negative consequences,
- *discontinuity* – the opposite of continuity, i.e., in time and place.

The key concepts of the catastrophe theory have been defined in Table 1 to ensure that they are correctly understood in the discussed method.

Table 1

Definitions of key concepts in the catastrophe theory

Concept	Definition	Source
Catastrophe	Sudden, discontinuous qualitative change which moves the analyzed system from one set of differential equations to another.	THOM (1991)
Discontinuity	Discontinuity is observed when the set of system behaviors is divided into various qualitative types, and jumps between those types take place due to constant changes in the set of causes. Discontinuity can be attributed to the presence of boundaries in control space which increase the system's sensitivity, therefore, even minor changes in parameters can induce significant changes in the system's structure.	JAKIMOWICZ (2005)
Divergence	A system can follow a divergent evolutionary path (two trajectories), and minor changes in system parameters lead to significant (continuous) changes.	PLOEGER ET AL. (2002)
Multimodality	At certain parameter values, the potential function has more than one minimum. At least one unstable state is found between the minima.	GUTENBAUM (2003)
Alternativity	Alternativity is observed when the movement between two points in parameter space can be induced by both continuous and discontinuous changes.	JAKIMOWICZ (2005)
Catastrophe theory	The catastrophe theory is a mathematical theory of nonlinear, discontinuous phenomena. It describes discontinuities independent variables as a function of continuous changes in independent variables. It is a new mathematical method for describing the evolution of forms in nature.	THOM (1975) ZEEMAN (1976)

Source: own study.

A catastrophe is a phenomenon whereby a previously stable system loses its stability, after which it quickly moves to another stable state governed by new conditions (OKNIŃSKI 1990). In this definition, a catastrophe is not merely an adverse event, such as a construction accident or an environmental disaster, but also accounts for the positive aspects of development processes in the analyzed system, which have a high dynamic, such as heart rate, liquid-vapor transformations or chemical reactions.

Catastrophe theory models were successfully applied in various fields of research, including psychology (PLOEGER ET. AL. 2002), economic sciences (ZEEMAN, 1976; BARUNIK, VOSVRDA, 2009, JAKIMOWICZ, 2003), mathematics (THOM, 1975), biology (COBB, ZACKS, 1985), geography (CASTR, SWAIN, 1976) and chemistry (OKNIŃSKI, 1990). The above implies that the morphogenesis theory is highly versatile and that its usefulness should be tested in analyses of the real estate market.

Any attempts to apply the catastrophe theory to the real estate market should begin with the identification of catastrophes on that market. A state variable should be selected as the basis for formulating conclusions about the types of changes observed on the real estate market.

In most scientific studies and in popular science literature, the state of the market is evaluated by describing the parameters of a selected segment of the real estate market. To model the variability in property prices, the real estate market should be regarded as a dynamic system which evolves in order to achieve equilibrium. In this approach, sudden changes in property prices, both positive and negative, should be regarded as critical points in market evolution, i.e., catastrophes. At those points, the market environment has an overwhelming influence, and it prevents the real estate market from pursuing a stable growth path. For the market to develop, it has to find a completely new trajectory of evolution.

The described phenomena were observed in 2007 on the local property market in Olsztyn, north-eastern Poland, when average apartment prices increased from PLN 2,500/m² to PLN 4,800/m², as shown in Fig. 1 (*sudden changes in property prices, both positive and negative, were observed in the past decade on many local markets in Poland and in other countries, including Spain, Bulgaria, Great Britain and the United States, but due to data constraints, this study focuses on the local market in Olsztyn*).

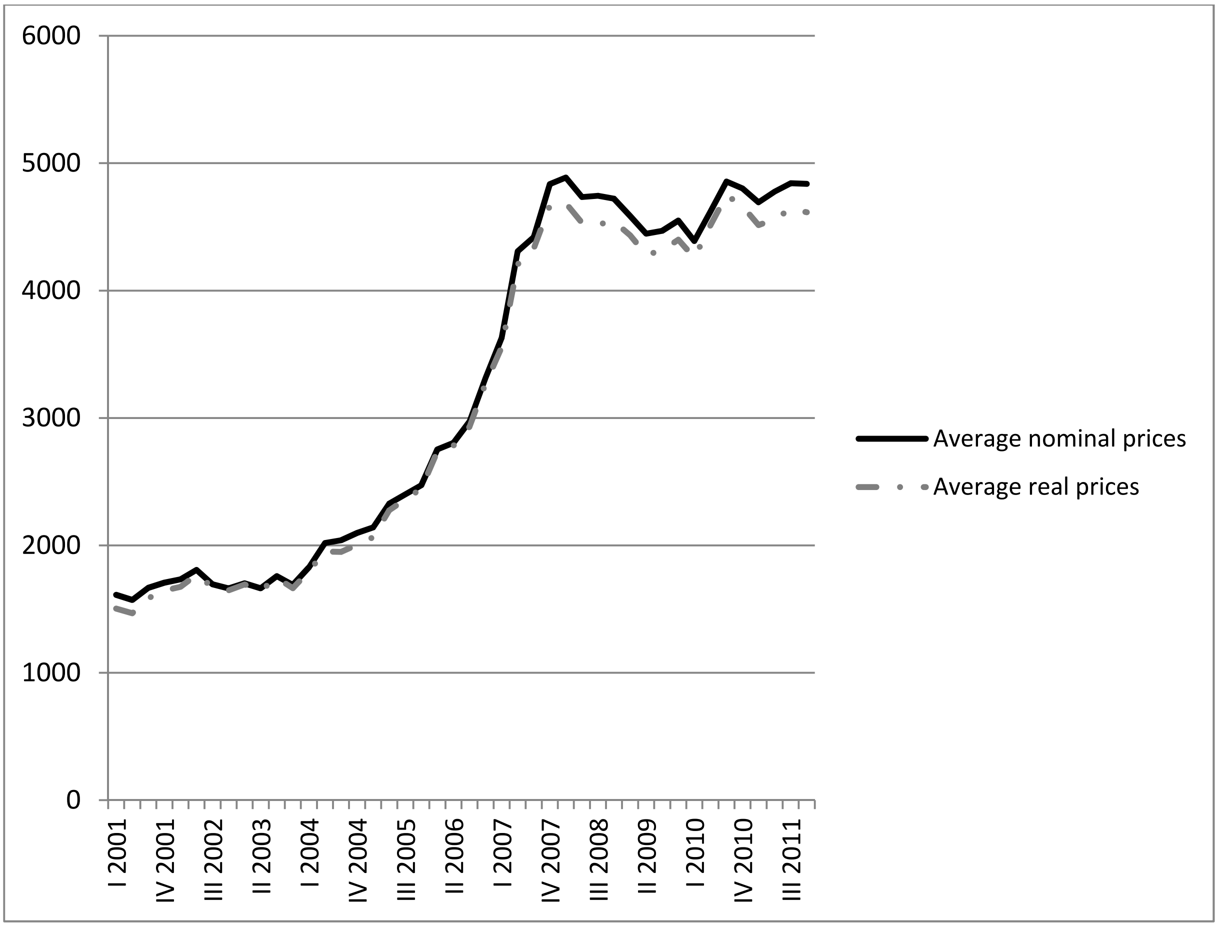

Fig. 1. Nominal and real prices [PLN] of apartments per unit of area [m²] in Olsztyn in every quarter of 2001-2011. *Source*: own study.

The next concept in our analysis is discontinuity which implies that the analyzed system is divided into segments of different quality, and discontinuous (discrete) changes between those segments follow from changes in parameters describing that system. The aim of this analysis is to determine whether price fluctuations in one market segment can be evaluated based on the example of two qualitatively different groups. The histogram of average apartment prices in Olsztyn, given in quarterly intervals (Fig. 2), points to the presence of two price distribution maxima which represent two qualitatively varied segments of the real estate market. According to the classical statistical interpretation, bimodal distribution, one of the basic concepts in the theory of catastrophe, suggests the presence of two different populations. If the set of transactions accounts only for apartment prices, the above can only be explained by sudden price fluctuations, i.e., discontinuity. A sudden drop in the number of transactions in a period characterized by rapid price fluctuations could imply that the evolutionary trajectory of the real estate market intersects inaccessibility zones, namely sets representing unstable equilibria in a system.

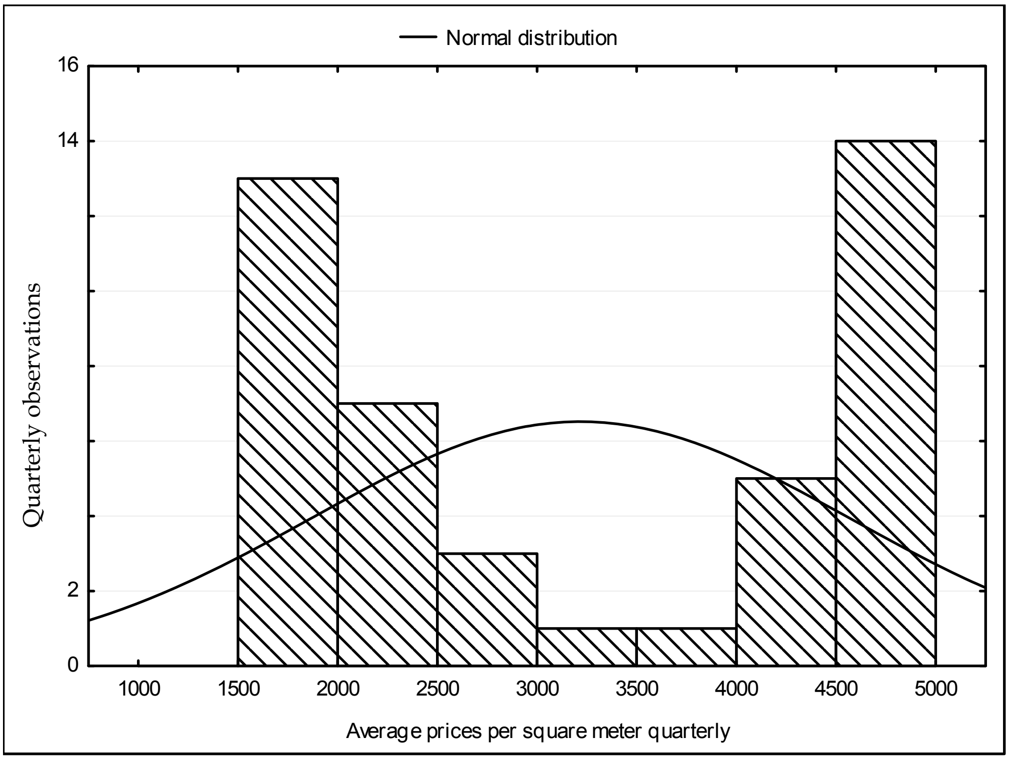

Fig. 2. Histogram of average prices of apartments per unit of area in Olsztyn in every quarter of 2001-2011. *Source*: own study.

Time remains a problematic issue in the context of discontinuity. Every model has its characteristic time which is determined by the modeled phenomenon. The division into control parameters and state variables is dictated by the fact that the former are constant values in the short-term perspective, whereas the latter are not. The system's long-term dynamics is investigated. Changes in property prices take place within a defined period of time, and the process of rapid price fluctuations is not measured in terms of seconds or hours, but rather days, weeks and months. According to Gutenbaum (2003), events which are perceived as sudden changes in the catastrophe theory have a finite duration. The theory does not analyze the dynamics of changes in the system, but investigates the evolutionary path of system equilibria.

3.2. Methodological aspects of the catastrophe theory

The catastrophe theory originated with the work of a French mathematician Rene Thom (THOM, 1975). ZEEMAN (1976) proposed several applications for the theory in economic sciences. Subject to the number of state variables and the number of control parameters, there are seven basic types of catastrophes, which exist only for codimension four and can be identified as: a fold catastrophe (1 control parameter), a cusp catastrophe (2 control parameters), a swallowtail catastrophe (3 control parameters), a butterfly catastrophe (4 control parameters), a hyperbolic umbilic catastrophe (3 control parameters), an elliptic umbilic catastrophe (3 control parameters) and a parabolic umbilic catastrophe (4 control parameters).

The names of the fundamental catastrophes reflect the shape of equilibrium surfaces in each system. The cusp model was used to analyze variations in apartment prices on the local real estate market.

The main assumption of the catastrophe theory is that systems evolve towards a state of equilibrium which is defined by local minima of the system's potential function. The arguments of that function are control variables (c) and state variables (x). Control variables determine system evolution, whereas state variables determine the system's state and describe its change dynamic. According to WAGENMAKERS ET AL. (2005), a dynamic system's evolution towards an equilibrium is best exemplified by the motion of a sphere on a curved one-dimensional surface, where the sphere represents the system's state, and gravitation represents the driving force. Those correlations are presented in Fig. 3.

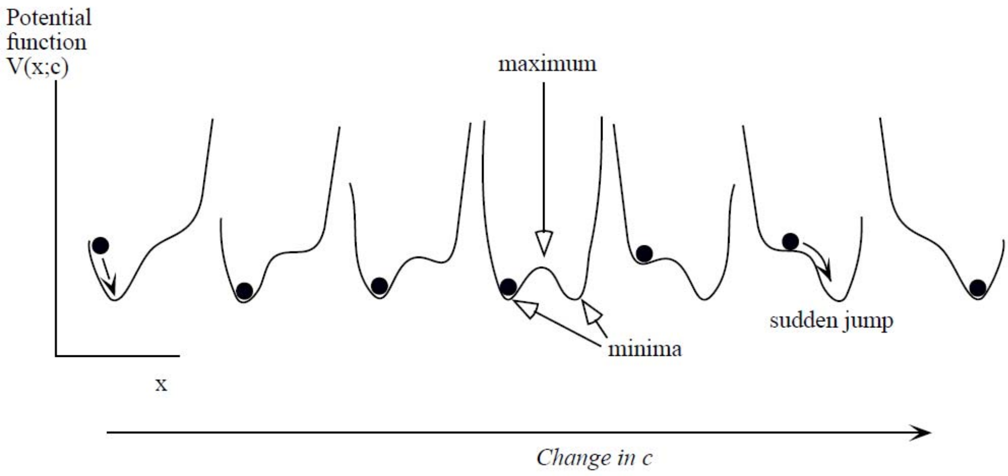

Fig. 3. Types of potential function V(x,c) minima in a system's evolution towards equilibrium. *Source:* WAGENMAKERS ET AL. (2005).

The three possible states of equilibrium proposed by WAGENMAKERS ET AL. (2005) are shown in the center of the diagram in Fig. 3. Two of them are stable equilibria (marked as minima in Fig. 3), where certain disruptions (change *c*) do not change the system's behavior. The third state is unstable equilibrium (marked as maximum in Fig. 3), where even a minor disruption causes the system to evolve towards a different state. Systems that are driven toward equilibrium values, such as the little ball in Fig. 3, may be classified according to their configuration of critical points, that is, points at which the first or, possibly, second derivative equals zero. When the configuration of critical points changes, so does the qualitative behavior of the system. For instance, Fig. 3 demonstrates how the local minimum (i.e., a critical point) that contains the little ball suddenly disappears as a result of a gradual change in the surface. As a result of this gradual change, the ball will suddenly move from its old position to a new minimum.

In a cusp model, the potential function is described by the following formula:

$$v(x;c) = -\frac{1}{4}x^4 + \frac{1}{2}\,\beta x^{\,2} + \alpha x \tag{1}$$

Parameters α and β are control variables (c), where the asymmetry indicator α is the normal variable and the bifurcation indicator β is the splitting variable. In the modeling process, those variables are defined as linear combinations of independent variables (y) with the use of the following formulas;

$$\alpha_y = \alpha_0 + \sum_{i=1}^{n} \alpha_i y_i \tag{2}$$

$$\beta_y = \beta_0 + \sum_{i=1}^{n} \beta_i y_i \tag{3}$$

The analyzed system V(x,c) evolves along an equilibrium surface, and if a potential function exists, the evolution of a given system (in this case, a real estate market) in time *t* is described by the following differential equation:

$$\frac{dx}{dt} = -\frac{dV(x;\alpha_y,\beta_y)}{dx} \tag{4}$$

If the state variable x occupies a point where dV(x;c)/dx = 0, this implies that the system is in equilibrium, and the set of all equilibrium points forming the equilibrium surface presented in Fig. 4. The points at which a catastrophe occurs can be defined precisely with the use of one of two heuristic principles: the perfect delay convention or the Maxwell convention. Analysis, in this paper, follows the principle of the perfect delay convention.

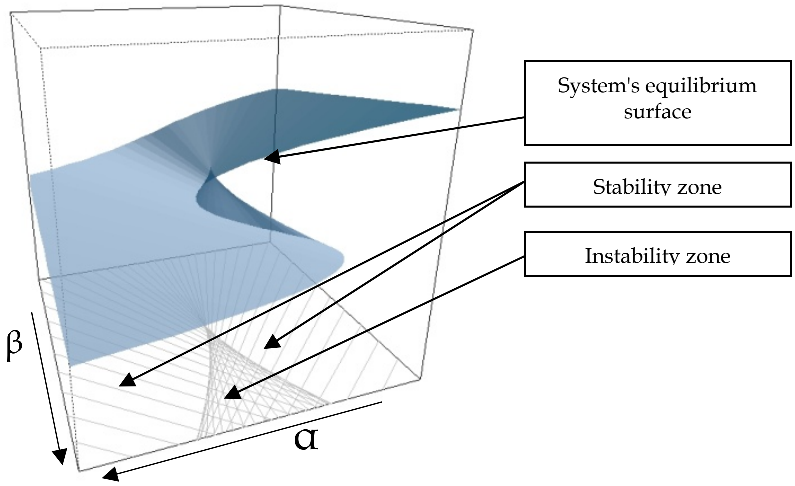

Fig. 4. The shape of a 3D equilibrium surface in the analyzed system dynamics model.
Source: own study.

A dynamic system can also evolve towards unstable equilibrium where a minor disruption can cause the system to move to another state. In the morphogenesis theory, this is known as a catastrophe. The previous trajectory of evolution undergoes sudden change on the surface of equilibrium, and the system jumps from one point on the equilibrium surface to another point in an instability zone. On a 3D equilibrium surface, this zone is represented by characteristic surface folding, and in a 2D projection, it appears in the shape of a cusp.

4. Visualization of the real estate market's trajectory of evolution

4.1. Characteristics of experimental data

A mathematical model built in line with the catastrophe theory requires the identification of a state variable (x) and independent variables which constitute the basis for the determination of the asymmetry indicator (α) and the bifurcation indicator (β). In section 3.1, the state variable was defined as the average price of 1 m^2 of an apartment (stated on a quarterly basis) in Olsztyn, the capital city of the Region of Warmia and Mazury in north-eastern Poland. The source of the analyzed data was the Register of Real Estate Prices and Values kept by the Olsztyn City Office. The initial database contained around 12,000 entries covering all types of transactions, including non-market transactions. The database had to be filtered to produce data for market transactions only. In the filtering process, non-market transactions were eliminated from the database based on the following criteria: transactions identified as non-market transactions, discount transactions, donations, real estate with easements and the sale of fractional ownership of property. The filtered database comprised 9,862 property transactions. The above data was used to calculate the average prices of 1 m^2 of property in quarterly intervals.

The state variable and independent variables, i.e., the determinants of real estate market growth, which were applied to define control variables (c) in the analyzed system are presented in Table 2.

The adopted variables were applied in detailed analyses. The sources of the above data were the Central Statistical Office (GDP, EXP, NDU, UN, ILS, WBD), the National Bank of Poland (IR) and the Register of Real Estate Prices and Values kept by the Olsztyn City Office (CTR and NT).

Table 2

State variables and independent variables – determinants of real estate market growth

Symbol	Variable	Description
CTR	Real transaction price	Average apartment price per 1 m^2, given in quarterly intervals and adjusted for inflation
GDP	Gross Domestic Product (fixed prices)	Fixed prices are average annual prices for the previous year. The corresponding period in the previous year = 100.
IR	Interest rates	Average interest rates quoted by the National Bank of Poland; rediscount rate, lombard rate, reference rate (at the end of period)
EXP	Exports of goods and services (fixed prices)	Fixed prices are average annual prices for the previous year.
NDU	Performance of the residential construction sector	Number of new dwellings, in '000
UN	Total registered unemployment	At the end of period
ILS	Economic welfare	Gross nominal monthly salary. The corresponding period in the previous year = 100.
NT	Activity on the real estate market	Number of property transactions in quarterly intervals.
WBD	Total gross value added (fixed prices)	Fixed prices are average annual prices for the previous year. The corresponding period in the previous year = 100.

Source: own study.

4.2. Development of a cusp model for the real estate market

The aim of the study was to verify the hypothesis that the real estate market evolves towards equilibrium and the trajectory of evolution intersects mainly zones of stability and, only periodically, instability zones. This theory can be verified on the grounds of the catastrophe theory by developing a mathematical model (cusp model) and visualizing the entire evolutionary path of the analyzed system (real estate market) which comprises continuous changes interrupted by discontinuous changes on the equilibrium surface described in section 3.2.

The model was built with the use of the cusp software application developed by Grasman et al. (2009) in the R-project environment. This application compares the goodness of fit between the catastrophe theory model, the linear multiple regression model and the logistic model of non-linear regression.

Table 3

Goodness-of-fit parameters.

Parameter	Asymmetry indicator – α	Bifurcation parameter – β
NDU	1.343e-02	x
IR	-1.020e+00	x
UN	-1.682e+00	x
EXP	7.272e-02	x
GDP	X	5.555e-02
WBD	X	-1.892e-01
ILS	X	1.127e-01
NT	X	1.057e-02

Source: own study.

In the process of developing the cusp model, multiple calculations were performed to identify linear combinations of independent variables and to assign them to control variables α and β or to remove those combinations which were found to be non-significant from the model. The verification process revealed that independent variables *NDU, IR, UN* and *EXP* were significant for symmetry indicator α, whereas variables *GDP, WBD, ILS* and *NT* were significant for bifurcation indicator β. The goodness of fit between independent variables and indicators α and β is presented in Table 3.

The goodness of fit of the cusp model, the linear model and the logistic model to data from the real estate market in Olsztyn in 2001-2011 is presented in Table 4 based on the relevant measures of fit. The classical coefficient of determination R^2 was adopted for linear and logistic models, and the pseudo- R^2 statistic proposed by COBB (1998) was used for the cusp model. According to HARTELMAN (1997), the cusp catastrophe is an irregular implicit regression model in the sense that the available statistical theory for implicit regression models cannot be applied to catastrophe models. In pseudo-R^2 statistic, the error variance is defined as the variance of the differences between the observed (or estimated) states and the mode of the distribution that is closest to this value. It should be noted, however, that pseudo-R^2 can become negative if many of the α's deviate from 0 in the same direction - in that case the distribution is strongly skewed, and deviation from the mode is on average larger than deviation from the mean (GRASMAN ET AL., 2009). The following goodness-of-fit indicators were additionally adopted: log-likehood ratio (logLik), where the higher the value of the indicator, the better the model's fit to empirical data, the Akaike information criterion (AIC) and the Bayesian information criterion (BIC), where the lower the value of the criterion, the better the model's fit to variable data**.**

Table 4

Goodness-of-fit between the cusp, linear and logistic models and data from the real estate market in Olsztyn in 2001-2011

Model	R^2	Cobb's pseudo R^2	logLik	AIC	BIC
Linear model	0.9581731	X	-307.32080	634.64161	652.4835
Logistic model	0.9896323	X	-276.63428	575.26856	594.8946
Cusp model	x	0.9799782	33.05514	-42.11028	-20.7000

Source: own study.

The application of the R^2 criterion and the pseudo-R^2 criterion produced the lowest goodness of fit with the linear model, but the value of R^2 (0.958) was unexpectedly high and could point to the high quality of selected independent variables (determinants of real estate market growth) relative to the adopted state variable – property price per unit area. The results reported for the logistic model (0.989) were only somewhat superior to those observed for the cusp model (0.979); it should be noted, however, that various methods were applied to determine this goodness-of-fit criterion.

In evaluations of non-linear models, the application of logLik, AIC and BIC criteria produces much more reliable results. The highest goodness of fit for the cusp model was reported for logLik (33.05), AIC (-42.11) and BIC (-20.70), and the use of those criteria produced far less satisfactory results in the linear and logistic models.

The noted results validate the presence of a "catastrophe" in the analyzed real estate market. This implies that, in certain periods of time, the system's sensitivity threshold to external stimuli is exceeded. The system becomes destabilized and moves from a near-balanced state to a state that is far from equilibrium. Those dynamic processes lead to significant changes in the system's trajectory of evolution. In search for equilibrium, the system is transformed through sudden changes in the state variable. The trajectory of evolution intersects mainly stability zones and, only periodically, zones of instability (as in the description of the equilibrium surface in Fig. 4), where the system searches for a new evolutionary path in a process of discontinuous changes. Projections of the system's trajectory in parameter space (left side – 2D) and the system's trajectory on an equilibrium surface (right side – 3D) are shown in Fig. 5.

The trajectory of evolution in the analysis of the state variable on the real estate market in Olsztyn in 2001-2011 is illustrated in Fig. 5. A geometric visualization of the system's evolutionary path reveals discontinuous changes in property prices. The state variable (average price of an apartment) is represented by circles on 2D and 3D diagrams. If we assume that no catastrophic events took place in 2001-2011, the trajectory of evolution should not intersect the zone of structural instability which is marked with a dark color and assumes the characteristic cusp shape in the 2D diagram. In the analyzed case, the points representing state variables in response to changes in control parameters α and β move from zones of stable evolution to a zone of instability. This marks a catastrophe, namely a state in which a system's sensitivity threshold to external stimuli is exceeded. The system is no longer able to pursue the existing trajectory of evolution and has to undergo sudden and discontinuous changes. The real estate market thus embarks on a new trajectory of evolution which is adjusted to the current level of external stimuli. An analysis of sudden changes in property prices in 2007 implies that, during that period, the system's trajectory of evolution intersected the instability zones shown in the 2D and 3D diagrams. Property prices, which reached new maxima within a short period of time, steadily move away from the boundaries of the instability zones marked with black lines in the 2D diagram. Discontinuous changes in the state variable are more visible in the 3D diagram which shows a characteristic folding of the system's equilibrium space. The trajectory of evolution intersects the lower section of the stability zone, and when it enters the zone of unstable behavior, which can be triggered by even minor changes in control parameters, it jumps to the upper section of the stability zone. The system has thus embarked on a new trajectory of stable evolution.

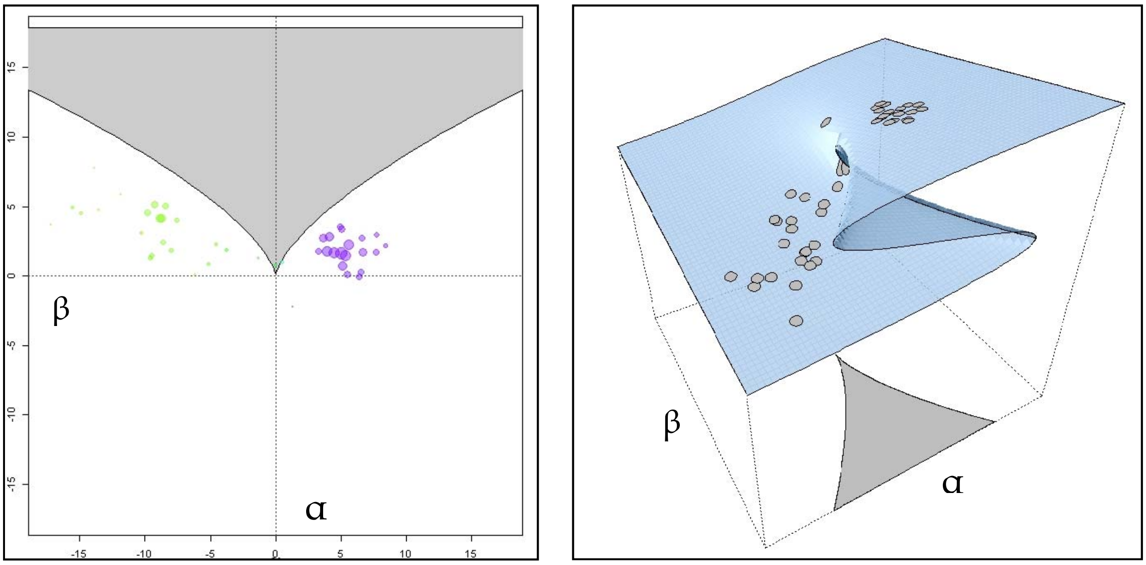

Fig. 5. Projections of the system's trajectory in parameter space (left side – 2D) and the system's trajectory on an equilibrium surface (right side – 3D) in a model describing the dynamics of changes in apartment prices per unit of area in Olsztyn in 2001-2011. *Source*: own study.

5. Conclusions

This study sheds new light on the dynamics of changes in property prices not only in a quantitative approach but, above all, from a qualitative perspective. The real estate market was analyzed as a dynamic system which always evolves towards a minimum potential function. The catastrophe theory was applied to describe discontinuities in the process of system development. From an intuitive point of view, the system's global evolution is manifested by sequences of continuous changes which are interrupted by sudden changes of varied qualitative character. The results of the study validate the theory that sudden changes in property prices significantly affect the structure of the entire real estate market, and that quasi-discrete changes should be regarded as critical points in the process of market evolution.

The visualization of results on 2D and 3D diagrams indicates that the property market remains in a stable state for a long period of time. When the market's threshold of sensitivity to external stimuli is exceeded, its rapid evolution towards unstable states will be short-lived.

The catastrophe theory expands our knowledge about processes taking place on the real estate market while graphic interpretations of the trajectory of evolution of the market's state variable illustrate the market's present state and possible avenues for future growth.

The catastrophe theory can be used to develop prognostic tools for the real estate market because performed analyses in this theory are both qualitative and quantitative, which is a major advantage of this methodology. The market's evolutionary path can be interpreted graphically to predict the possibility of changes towards stable or unstable growth, subject to variations in control parameters.

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