Mathematical model of piston ring sealing in combustion engine

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ABSTRACT

This paper presents a mathematical model of piston-rings-cylinder sealing (TPC) of a combustion engine. The developed model is an integrated model of gas flow through gaps in TPC unit, displacements and twisting motions of piston rings in ring grooves as well as generation of oil film between ring face surfaces and cylinder liner. Thermal deformations and wear of TPC unit elements as well as heat exchange between flowing gas and surrounding walls, were taken into account in the model. The paper contains descriptions of: assumptions used for developing the model, the model itself, its numerical solution as well as its computer application for carrying out simulation tests.

Keywords: exhaust gas blow-by, displacements of rings, twisting of rings, oil film, cylinder, piston, ring, mathematical model, lubrication

Introduction

Piston with rings in cylinder liner forms ring sealing which constitutes the motional closing of engine combustion chamber. Such sealing should ensure possibly highest tightness of the combustion chamber, i.e. minimization of exhaust gas blow-by to crankshaft casing. Simultaneously, piston with rings in cylinder liner serves as a slide bearing which executes to-and-fro motion and is lubricated hydrodynamically. The piston-rings-cylinder (TPC) unit should, as a bearing, show possibly low values of friction drag. Moreover it should ensure low consumption of lubricating oil and show long service life [6, 14, 19, 24].

Despite the principle of functioning the ring sealing in piston combustion engines has not been changed for several tens of years, its operation mechanisms, including impact of constructional details on effectiveness of fulfilling the above described aims, are not yet fully recognized. In view of significant importance of the unit for crucial features of engine, such as: fuel consumption, exhaust gas toxicity and service life, intensive research projects aimed at better recognition of phenomena associated with the functioning of ring sealing, as well as other projects focused on the development of more and more perfect design solutions, are under way.

The recognizing of working principles of TPC unit, including effects of its particular constructional features on its operation is not an easy task because of dynamic character of work of the sealing and impact of many interacting factors which decide on its operation. Theoretical modelling has contributed to a large extent in better recognizing various aspects of sealing operation.

In this work is presented an advanced model of ring sealing, this is an integrated model of gas flow through the sealing, piston ring dynamics and oil film forming. The model has been developed on the basis of a model described in the publications [10, 11]. However, by contrast, in the presented model twisting deformations of piston rings were taken into account and a sub-model of ring-cylinder interaction for determining oil film parameters was included, moreover it was possible to delete many simplifying assumptions concerning shape of TPC unit elements. And, a new computer software which makes it possible to conduct simulation tests, was developed.

Review of existing models of sealing

In gas flow modeling the ring sealing is considered a labyrinth sealing in which gas flows through many stages connected to each other by means of throttling gaps. In the models are described thermodynamic gas parameters in particular stages of labyrinth as well as mass fluxes flowing...
between the stages through throttling gaps. Such solution already proposed in the work \[2\] has been commonly applied till now. Successive models differ to each other with a number of factors taken into account and phenomena influencing gas flow, as well as a manner of their mathematical description.

Structure of a labyrinth, i.e. number of stages and a way of their connection by means of throttling gaps depend on a edeled engine and a degree of model advancement. In the simplest models in which gas is assumed to flow only through ring locks, labyrinth has a series structure and number of throttling channels is equal to that of sealing rings \[2, 4, 27, 17, 18, 9, 16\]. There are also models of this kind in which sealing action of piston oil rings is taken into account \[15, 25, 1, 22\]. Series structure is also attributed to the models in which gas flow through groove around ring when it has no contact with any of groove sides \[31, 20\]. However, the labyrinth scheme assumed in the models does not allow to analyze dynamic phenomena associated with an accumulating action of behind-ring spaces.

Position of ring against groove decides on that with which inter-ring space the behind-ring space is connected. Because volumes of behind-ring spaces are often greater than those of inter-ring ones, their accumulating action may significantly influence pressure runs in inter-ring spaces, consequently, also performance of the whole sealing. In order to account for the above mentioned phenomena Namazian and Heywood \[18\] integrated the model of gas flow with the model of axial displacement of rings in grooves. In the obtained model the behind-ring and inter-ring spaces were considered separately by applying a series – parallel scheme of the labyrinth. Such labyrinth scheme is commonly assumed in the integrated models \[13, 9, 26, 10, 28, 30\].

Eweis \[2\] modeled flow through ring locks by assuming perfect gas isentropic flow through orifice. Furuhama and Tada \[3\] calibrated empirically the so determined mass flux by means of a flow coefficient of constant value. This manner of the modeling of gas flow through ring lock was used in the models \[27, 18, 9, 16, 1\] where however various values of the coefficient were assumed. In the model \[26\] a flow coefficient of value depending on the ratio of pressure before and behind the orifice, was used. This approach to determining mass flux was also implemented in the models \[10, 28, 11\].

Gas flow through channel between ring side surface and groove is also modeled as an isentropic flow through orifice with taking into account a flow coefficient of constant value \[9, \]. In the model \[10\] the flow is also considered to be an isentropic flow through orifice however the flow coefficient is determined from an empirical formula in which gap geometry, ratio of pressure before and behind the gap, as well as Reynolds number, is taken into account. In the work \[18\], in view of the channel shape and character of flow (low Reynolds numbers), the flow is deemed to be a laminar isothermal flow of a compressible medium progressing through narrow channel of constant breadth. In the model \[26\] the flow through such channel is also modeled as a laminar isothermal one with taking into account a taper of gap. The similar approach was presented in the publications \[28, 22\].

In \[2\] gas flow through stage was treated as an adiabatic flow. Furuhama and Tada, taking into account results of measurements conducted on a motionless piston model stand, assumed that flow through labyrinth stages can be considered isothermal \[3\]. In prevailing majority of the models, usually referring to Furuhama, it was assumed that gas flow is isothermal and gas temperature within stage is equal to that of piston \[18, 17, 20\], or quasi-isothermal, i.e. that gas temperature changes but is determined on the basis of temperature of walls surrounding a given inter-ring space \[9, 26, 22\]. Such assumption significantly simplifies calculations because in order to determine parameters of medium within stage it is sufficient to use the equation of mass balance and gas state. However actual gas flow is of an intermediate character in between adiabatic and isothermal one. In the publications where isothermal flow, i.e. an extensive heat exchange, was assumed their authors simultaneously stress \[18, 20, 22\] that gas flow within stages is laminar at low values of Reynolds numbers. It seems to be an inconsequence as at laminar flow heat exchange intensity is relatively low. Moreover, conditions of heat exchange between gas and surrounding walls may worsen along with time of engine operation as a result of appearing sediments. Taking this all into account, in the model \[10, 11\] this author does not assume an isothermal flow, but determines gas temperature from energy balance. Wolff \[30\], in his model, made use of the manner of determining gas thermodynamic parameters, proposed in \[10\].

In the majority of models \[2, 4, 27, 17, 18, 13, 31, 20, 1, 22\] it was assumed that cross-sections of gas flow through ring lock and spaces of labyrinth stages are constant. In actual engine, volumes of inter-ring and behind-ring spaces, and especially cross-sections of gaps in locks, may change within a broad range during one cycle of engine work due to displacing motion of the piston together with rings along cylinder liner of a variable diameter. In the work \[25\] it was assumed that in order to take into account thermal deformations of liner, cross-sections of locks should change, in a logarithmic manner, along with piston actual height position changing. In case of some models, their descriptions do not allow to unambiguously state whether constant values are assumed or they change in working cycle of the engine. In the model \[10, 11\] the effect of cylinder profile on cross-section areas of gaps in locks as well as volumes of labyrinth stages was considered. In the integrated models of gas flow and ring dynamics it is commonly assumed that cross-sections of channels between ring and groove result from an instantaneous position of ring against groove \[18, 13, 9, 26, 10, 28, 11\].

Motions of ring in groove (axial, radial, twisting) have a significant effect onto sealing performance, hence many researchers have investigated the problem by developing the so called ring dynamics models. A way of description of forces acting onto rings is given, a.o., in \[5\]. And, in the work \[23\] the effect of ring twisting motions on forming oil film was estimated. However the models were not integrated with gas flow models, hence they did not allow to perform a comprehensive analysis of mutual relation between these phenomena. In the first integrated model \[18\], ring dynamics
As far as geometry is concerned, it was assumed that all elements are axially symmetrical and the piston together with rings moves coaxially in relation to cylinder. The next assumption was that rings always adhere to cylinder surface (no untightness occurs between ring face surface and cylinder surface). Temperatures and dimensions of elements were assumed unchanged during entire cycle of engine work. Nevertheless, rings may move within grooves and their cross-sections may twist dynamically. Instantaneous positions of rings in grooves and their twisting angles are determined by means of the integrated sub-model of ring dynamics. Cross-section areas of gaps between rings and groove walls result from instantaneous positions of rings in grooves. In addition, dimensions of elements may account for thermal deformations and wear, including the fact that diameter of cylinder may be different at different heights. (Fig. 2). All the above specified assumptions imply that all volumes of labyrinth stages and cross-section areas of channels which gas may flow through are functions of crankshaft rotation angle.

Model of gas flow

The model is composed of a series of stages mutually connected by means of throttling gaps (Fig. 1). It was assumed, that a semi-ideal gas whose internal energy $u$ and specific heats $c_v$ and $c_p$ are dependent on temperature only, serves as a medium flowing through the labyrinth. It was also assumed that flow of the medium through throttling gaps is isentropic, while heat exchange between gas and surrounding walls occurs within labyrinth stages. As assumed, thermodynamic parameters of gas contained within entire volume of a given stage are homogeneous and kinetic energy of the medium in the stage itself is omitted by virtue of the assumption that energy of medium flowing through the stage is entirely converted into internal energy. And, it was also assumed that pressure in the space over the first ring (stage 1) is equal to pressure inside working chamber of engine, whereas pressure behind and below the third ring (stage 6 and 7) is constant and equal to that in crankcase.
energy of medium within a single stage results from flow of stagnation enthalpy contained in substance, through control cross-sections of flow channels, heat exchange with environment, as well as work of changing volume (Fig. 3a); the change can be written as follows:

$$U = \sum_i i_{in,i}m_{in,i} - \sum_j i_{out,j}m_{out,j} + \dot{Q} - \dot{p}V,$$  

(1)

where:

- $i$ – stagnation enthalpy, $m$ – mass flux, the index $in$ stands for inflowing, index $out$ stands for outflowing, and lack of an index means that a given quantity is related to a parameter of medium within a given stage.

The presented formula is not applicable to the first labyrinth stage where pressure, in compliance with the adopted assumptions, is equal to that in the working chamber ($p_1 = p_{ind}$) and there is no throttling at inlet to this stage from the combustion chamber. By taking into account the above mentioned assumptions, mass rate of gas flowing out from the combustion chamber to the space over the first ring can be determined by using the relation as follows:

$$m_{in} = \frac{1}{i_d} \left( \frac{c_vV}{R} \dot{p} + \sum_j i_{out,j}m_{out,j} - \dot{Q} + \frac{c_p}{R} \dot{p}V \right).$$

(6)

And, the formula is formulated for the case of gas flowing from the combustion chamber to crankcase. In the presented model all possible combinations of flow directions are considered. Gas flow through throttling channels is modeled as isentropic decompression of a compressible medium. The medium always flows towards space of a lower pressure, hence in compliance with Fig. 3b, the inequality $p_{m-1} > p_m$ is satisfied. There are considered cases of sub-critical and critical flow and mass flux determined this way is corrected by means of the empirical flow factor $\psi$.

In the case of sub-critical flow taking place when the condition given below is satisfied:

$$\dot{m}_{m-1,m} = \psi_m m_{m-1,m} A_{m-1,m} \left( \frac{p_m}{p_{m-1}} \right)^{\frac{1}{\kappa}} \left[ 1 - \left( \frac{p_m}{p_{m-1}} \right)^{\frac{\kappa-1}{\kappa}} \right]^{\kappa \left( 2c_p T_{m-1} \right)^{-1}},$$

(8)

while in the case of critical flow taking place when the inequality (7) is not satisfied – from the following formula:

$$\dot{m}_{m-1,m} = \psi_m m_{m-1,m} A_{m-1,m} p_{m-1} \left\{ \frac{\kappa}{RT_{m-1}} \left( 2 \frac{c_p}{c_v} \right)^{\frac{\kappa-1}{\kappa}} \right\}^{\frac{\kappa}{\kappa-1}},$$

(9)

where $\kappa$ is the ratio of the specific heats: $c_p$ and $c_v$.

Flow factors for lock gaps are calculated from the empirical formula taking into account the ratio of pressures behind and before the lock [26]:

$$\psi_{m-1,m} = 0.85 - 0.25 \left( \frac{p_m}{p_{m-1}} \right)^2.$$  

(10)

Because shape of the gap between ring and groove much differs from an orifice, moreover its geometry changes within a very broad range (at constant length and breadth of the gap, its height changes from zero up to the value equal to axial clearance of ring in groove), in order to determine flow factor for the gap, use was made of an experimental function developed for a channel having geometry close...
to that of the considered gap; the function was determined for the gap as in Fig. 4, having the following proportion: 0.002 ≤ h/B ≤ 0.07, (where the gap breadth dimension in the direction perpendicular to the figure is very large as compared with its length B, and the surfaces K are coaxial cylinders of a very large radius as compared with B) [8]. Such function accounts for influence of gap geometry, pressure ratio before and behind the gap, as well as Reynolds number:

\[ \psi_{m-1,m} = 10^{-1.53a^2} \]  

where:

\[ Y = 0.0284X^2 - 0.459X - 0.1375 \]
\[ X = \log \left[ \frac{h}{B} \right] \text{Re}_k \frac{1 - \frac{p_m}{p_{m-1}}}{0.5} \]
\[ \text{Re}_k = \frac{p_{m-1} h}{\mu} \sqrt{\frac{\kappa}{RT_{m-1}}} \left( \frac{2}{\kappa+1} \right)^{\kappa+1} \]

\[ \mu - \text{dynamic gas viscosity determined from the relation} [13]: \mu = 3.3 \times 10^{-7} \cdot T^{0.8} \quad [\text{Pa} \cdot \text{s}] \]

The lock cross-section area A was approximated by a rectangle of the dimensions a x b, with taking into account the area A+ which may result from e.g. bevels of ring end edges (Fig. 5a).

\[ A = ab + A_+ = \pi(D_e - D_{r0}) \frac{D_e - D_p}{2} + A_+ \]  

where: \( D_e \) stands for a cylinder diameter measured at the height where the ring is situated in a given instance, \( D_{r0} \) - outer diameter of the ring, at which lock gap would be cancelled at all (\( a = 0 \)), \( D_p \) - outer diameter of shelf to which the ring adheres in a given instance (\( D_p^a \) or \( D_p^b \)). In the case when the ring does not adhere to any of the shelves, the smaller of the two diameters \( D_{r0} \) and \( D_p^a \) i.e. that for which the calculated area A is greater, should be taken into account (Fig. 5).

Cross-section area of the channel between ring and shelf depends on the axial clearance of ring in groove, l, axial position of the ring in groove \( x_r \), as well as the twist angle of ring \( \alpha \), and is calculated from the formula:

\[ A = \pi D_h \]  

where: \( D \) stands for inner or outer diameter of channel end, determined from the formulae: \( D = D_e - 2(B - e) \) or \( D = D_p \), respectively, the dimension \( e \) serves to take into account a bevel or undercut of ring inner edges (Fig. 6a), \( h \) stands for height of gap end at its inner or outer side. In the case of lower channel, \( h \) is determined from the formulae:

\[ h = \frac{x_r}{2} - \frac{B}{2} \sin \alpha - h_{od} \quad \text{or} \quad h = x_r + \frac{B}{2} \sin \alpha - h_{od} \]

where \( h_{od} \) stands for thickness of oil layer on lower shelf. The height of upper channel is calculated in an analogous way. (see Fig. 6a). Out of the two calculated cross-section areas of the channel between ring and groove (at inner and outer side) the lower value of the area A is taken for calculations of mass flux.

In the model it was assumed that sealing ring can be trapezoidal. For trapezoidal ring, by contrast to rectangular ring, the axial clearance l depends on diameter of cylinder liner (Fig. 6b). At determining the axial position of trapezoidal ring as well as cross-section of gaps between ring and groove, the following dependence of the axial clearance on varying diameter of cylinder liner was taken into account:

\[ l = l_0 + (D_e - D_{r0} - \Delta D_p) \tan \gamma \]  

where \( l_0 \) stands for the nominal axial clearance corresponding to the nominal cylinder diameter \( D_{r0}, \Delta D_p \).
is an increase in piston diameter due to temperature rise between a value at which the nominal clearance is set and its operational value, γ stands for slope angle of trapezoidal ring (Fig. 6b).

Flux of the heat exchanged between gas within a stage and walls surrounding it, in the case of the medium in behind-ring space (space 2 and 4 in Fig. 1), is equal to the sum of the heat flux which flows between gas and piston and that which flows between gas and ring:

$$\dot{Q}_{beh} = S_{p,beh} \alpha_{p,beh} (T_{p,beh} - T) + S_r \alpha_r (T_r - T), \quad (16)$$

where:

- $S$ – heat exchange surface area,
- $\alpha$ – heat-transfer coefficient.

In case of the space over the first ring or the inter-ring space (space 1, 3 and 5 in Fig. 1), it was assumed that heat exchange takes place between gas and cylinder as well as between gas and piston (but exchange between gas and ring was neglected due to its small surface area), namely:

$$\dot{Q}_{int} = S_{p, int} \alpha_{p, int} (T_p - T) + S_{p, int} \alpha_{p, int} (T_{p, int} - T). \quad (17)$$

Determination of values of heat-transfer coefficients between flowing gas and ring sealing walls constitutes a problem because it is difficult to determine gas flow velocity against particular surfaces as well as an amount of oil and deposits placed on them. For this reason it was assumed that the coefficients $\alpha_{p,beh}$ and $\alpha_r$ are constant and their values are taken on the basis of the subject-matter literature. Whereas the heat-transfer coefficients between the medium within inter-ring spaces and cylinder are calculated from the relationship:

$$\alpha_c = 0.664 \sqrt{\frac{\text{Pr}^{0.52}}{\text{Pr}_c^{0.19}}} \frac{\nu_p}{\mu RT_L}, \quad (18)$$

where:

- $L$ – height of inter-ring space,
- $\nu_p$ – piston speed,
- $\lambda$ – gas thermal conductance determined from the relation:

$$\lambda = 7.1 \times 10^{-5} \cdot T + 0.007 \quad (19)$$

obtained by applying linear approximation to the data given in [29], Pr and Pr – Prandtl numbers determined for temperature of gas and cylinder surface, respectively, from the formula:

$$\text{Pr} = \frac{C_p \mu}{\lambda}. \quad (20)$$

This relation was developed on the basis of theory by assuming laminar flow around flat plate and that gas flow velocity against cylinder is equal to the piston speed $\nu_p$, with considering an experimental factor [29] which accounts for an effect of temperature on fluid thermo-physical features.

Areas of heat exchange surfaces are determined for nominal dimensions, hence they depend neither on ring displacements against groove shelves nor on piston displacements against cylinder liner (the simplification is insignificant in view of approximated estimation of heat-transfer factors). The data used for determination of areas of the surfaces are shown in Fig. 7.

**Model of ring dynamics**

Position of ring in groove is one of the principal input quantities into gas flow model as it decides on cross-sections of channels through which gas flows and affects volumes of labyrinth stages. In the model, axial and radial displacements and angular deformations of ring in groove are considered. Like in the gas flow model, axial symmetry of piston, ring and cylinder liner was assumed, that makes it possible to consider the ring as a planar system determined by two coordinates – axial and radial.
In an analogous way are determined gas pressure forces acting upon the remaining ring surfaces.

The inertia force is derived from the relation:

\[ F_{ix} = -m_r a_p. \]  

where \( a_p \) is acceleration of piston.

The friction force between ring and cylinder liner surface, \( F_{fr} \), can be calculated in this model in a twofold way. The more exact one is based on the determination of oil film parameters as well as the use of Eq. (49). In the case when oil film sub-model is neglected in calculations (in the developed computer software such sub-model may be switched-off) the friction force can be calculated from the empirical formula [18, 10]:

\[ F_{fr} = -f \pi D_x H(p_c + p_e). \]  

where the friction factor \( f \) is derived from the relation:

\[ f = 4.8 \left( \frac{\mu_{oil}}{H(p_c + p_e)} \right)^{\frac{1}{2}}. \]  

\( \mu_{oil} \) - oil viscosity - from Vogel equation [6]:

\[ \mu_{oil}(T) = a \cdot \exp \left( \frac{b}{T - 273.2 + c} \right) \cdot 10^{-3}. \]  

and, as assumed, the oil temperature \( T \) is equal to that of cylinder at a height where in a given instance the considered ring occurs, and the coefficients \( a, b \) and \( c \) depend on oil class, \( v_p \) - piston speed, \( p_r \) - pressure acting upon rear side of the ring, \( p_e \) - ring pressure exerted onto cylinder liner, resulting from flexibility of ring itself [10]:

Axial position of ring in groove is derived from the balance equation of forces acting upon ring in axial direction, namely:

\[ m_r \frac{d^2 x_r}{dt^2} = F_{px} + F_{ix} + F_{fr} + F_s + F_a. \]  

where: \( m_r \) - ring mass, \( x_r \) - ring displacement against piston, \( F_{px} \) - gas pressure force acting axially upon ring, \( F_{ix} \) - inertia force, \( F_{fr} \) - friction force between ring and cylinder, \( F_s \) - oil squeezing-out force and \( F_a \) - adhesion force (Fig. 8 and 10).

The gas pressure force \( F_{px} \) is resultant of forces due to gas pressure acting on the upper and lower surface of ring:

\[ F_{px} = F_{px,u} + F_{px,d}. \]  

For determining the force it was assumed that if a fragment of ring surface is in a given instance connected with only one labyrinth stage, then the pressure exerted on this fragment is constant and equal to pressure within a given stage. However, if a ring surface fragment is connected with two stages, then pressure over (across) this surface changes linearly. Hence, pressure distribution across the surface depends on: position of ring in groove, its twisting angle and thickness of oil layer on shelf (Fig. 8 and 9). By taking into account the above mentioned assumptions, in the case of the example shown in Fig. 9c, the pressure force acting upon the lower surface can be determined from the relation as follows:

\[ F_{px,d} = \sum P_{x,d} = P_{x,b} + P_{x,be} + P_{x,c}. \]  

where:

\[ P_{x,b} = -p_b \frac{\pi}{4} \left( D_c - 2h_m \right)^2 - D_p^2, \]

\[ P_{x,be} = -p_b + p_c \frac{\pi}{2} \left( D_p^2 - \left( D_p - 2b_h \right)^2 \right), \]

\[ P_{x,c} = -p_c \frac{\pi}{4} \left( (D_p - 2b_h)^2 - (D_c - 2(B + h_m))^2 \right). \]
Whereas the friction force between ring and groove surface and inertia force due to ring radial acceleration were neglected as being small.

The gas pressure force $F_{py}$ is the resultant of the gas pressure forces acting upon ring rear wall, $P_{y,c}$, and non-wetted parts of ring face wall, $P_{y,a}$ and $P_{y,b}$ (Fig. 10). At determining values of the forces it was assumed that pressure is distributed as shown in Fig.10, and lengths of the segments $n_a$, $n_b$ results from the coordinates of boundaries of ring face wetted area, $x_a$ and $x_b$, which are determined by using the oil film sub-model.

The ring flexibility force $F_e$ is derived from the formula:

$$F_e = 2\pi F_T .$$

(33)

The oil film pressure force $F_0$ is in fact a reaction to the loading due to the remaining radial forces, i.e. it is derived from the formula:

$$F_0 = -F_{py} - F_e .$$

(34)

In case of switching-off the oil film sub-model, for determining radial location of ring in groove it was assumed that $h_m = 0$, i.e. that ring radial position results from cylinder diameter.

The ring twisting angle $\alpha$ is calculated from balance of the moments of forces exerted onto ring profile, determined in relation to its centre of gravity $CG$:

$$K(\alpha - \alpha_0) = M_{Fpx} + M_{Fpy} + M_{Fx} + M_{Fx} + M_{Rx} .$$

(35)

where: $\alpha_0$ – ring profile twisting angle in rest, $K$ – ring twisting rigidity derived from the relation [23, 26]:

$$K = E \cdot H^3 \cdot \ln \left( \frac{D_n}{D_n - 2B} \right) .$$

(36)

where: $E$ - Young modulus of ring material.

Moments of inertia force is equal to zero as the balance of twisting moments is related to the ring profile centre of gravity. It was also assumed that directions of action of the forces due to: ring flexibility, $F_e$, oil squeezing-out, $F_s$, and adhesion $F_a$ cross the ring profile centre of gravity, hence their moments are equal to zero too. Because values of angular accelerations are small, inertia of ring profile in rotational motion was also omitted in the balance.

Moments of gas forces are determined as the moments of particular concentrated forces (Fig. 9 and 10):

$$M_{Fpx} = \sum_{i} P_{x,i} l_i , \quad M_{Fpy} = \sum_{j} P_{y,j} l_j ,$$

(37)

and their action arms were derived from the relation:

$$r = \frac{\int px dx}{\int px dx} .$$

(38)
The moment of friction forces between ring and cylinder liner surface is expressed as follows (Fig. 8 and 9):

\[ M_{F_{fr}} = F_{fr} \cdot b_{CG} \]  

(39)

The moment of oil pressure force is determined by using the equation (Fig. 10):

\[ M_{F_o} = F_o \cdot (h_{CG} - h_u - x_{Fo}) \]  

(40)

The force \( F_{o} \) is placed in the gravity centre of oil film pressure field. Knowing distribution of the pressure \( p_{oil}(x) \), determined by using the oil film sub-model (45), one is able to determine the axial coordinate of centre of gravity of the distribution, \( x_{Fo} \), in the following way:

\[ \int_{xa}^{xFo} p_{oil}(x)dx = \frac{F_o}{2\pi(D_c - 2h_m)} \Rightarrow x_{Fo} \]  

(41)

In case the ring is not in contact with its groove, i.e. it displaces in-between the shelves, the shelf reaction moment \( M_{R_x} \) equals zero. And, when the ring starts touching a groove shelf, the shelf reaction force \( R_x \) appears. It generates the moment dependent on its value and its point of application:

\[ M_{R_x} = R_x \cdot r_{R_x} \]  

(42)

The shelf reaction which equilibrates resultant of axial forces which press ring against shelf, is derived from the formula as follows (under the assumption that \( F_{s} = 0 \), when ring is in contact with a shelf):

\[ R_x = -\sum F_x = -(F_{px} + F_{tx} + F_{fr}) \]  

(43)

In the model in question it was assumed that ring touches a shelf not in one point but this contact occurs over a surface area of \( b_k \) in breadth. Value of the breadth \( b_k \) is a function of the twisting angle \( \alpha \) as well as an assumed height of influence of the surface, \( h_k \):\[
 b_k = \frac{h_k}{\tan(\alpha)} , \quad b_k \leq B - \frac{D_c - D_p}{2} - h_m . \]

(44)

As assumed, the pressure acting along this breadth is distributed linearly. A manner of its determination, exemplified for the case of lower shelf, is presented in Fig. 11. The assumed height of influence, \( h_k \), is a simplified interpretation of surface roughness and occurrence of an oil layer on shelves.

The action arm of shelf reaction, \( r_{R_x} \), is determined by using Eq. (38), in an analogous way like in the case of action arms of gas pressure forces.

\[ \frac{\partial}{\partial x} \left( h^3 \frac{\partial p_{oil}}{\partial x} \right) = 6 \mu_{oil} \frac{\partial h}{\partial x} + 12 \mu_{oil} \frac{\partial h}{\partial t} . \]

(45)
where: $p_{oil}$ – pressure in oil wedge, $h$ – height of the gap between ring and cylinder.

Boundary conditions were assumed according to that oil pressure on wetted area is equal to that occurring in the space neighbouring to this area (Fig. 8):

$$
\begin{align*}
  p_{oil}(x_a) &= p_a \\
  p_{oil}(x_b) &= p_b
\end{align*}
$$

(46)

and, it was also assumed that pressure in the divergent part of the gap cannot drop below the saturation pressure: $p_{oil} \geq p_{satur}$ (the so called Reynolds cavitation conditions [21]).

As assumed, ring face surface is parabolic in shape, hence the gap height is described by means of the following equation (Fig. 12a):

$$
  h(x) = h_m + \frac{(x - x_{h_m})^2}{2R_r}, \quad x \in (-C, C).
$$

(47)

where: $x_{h_m}$ – coordinate corresponding to the minimum height of the gap, $h_m$, $R_r$ – radius of ring face arc, $C$ – ring face breadth (in case of some rings it may differ from the ring height $H$).

The oil pressure force $F_o$ is determined from the balance of forces (34), and simultaneously calculated by integrating oil pressure within the wetted area limits, $x_a$ and $x_b$ (Fig. 10):

$$
  F_o = \pi(D_c - 2h_m) \int_{x_a}^{x_b} p_{oil}(x) dx \Rightarrow h_m.
$$

(48)

The above mentioned condition is satisfied on assumption of an appropriate minimum gap height $h_m$, which is in fact a searched for value. It is necessary to perform calculations in successive iterations until a required conformity of a calculated $F_o$ value and a resultant value of radial forces is obtained.

The fluid friction force $F_{ff}$ is determined in compliance with the following equation [7]:

$$
  F_{ff} = \pi(D_c - 2h_m) \int_{x_a}^{x_b} \frac{(h(x) \frac{\partial p_{oil}}{\partial x} - \frac{\partial h_{oil}}{\partial x} v_{rel})}{h(x)} dx,
$$

(49)

and, its sense is opposite to that of the vector of piston speed against cylinder liner.

The wetted area limits $x_a$ and $x_b$ are determined on the basis of analysis of oil volume contained in the ring-cylinder gap. In the balance were taken into account both ring displacements along cylinder covered with an oil layer of varying thickness and its radial displacements. Change in gap height, resulting from ring radial motions generates the squeezing out of the oil gathered under ring face surface and a change in the limits of its face wetted area (Fig. 12). In the model in question the oil volume analysis was reduced to the examination of relevant surface areas.

The ring face wetted area limit $x_c$ is calculated on the basis of the oil control volume $V_k$ which is located in a free space between ring face and an oil layer of $h_{oil}$ in thickness, which adheres to cylinder liner surface (Fig. 12b):

$$
  V_k = \int_{x_{p_{max}}}^{x_{p}} \frac{\pi h_{oil}^2}{2R_r} (x - x_{p_{max}}) dx \Rightarrow x_c
$$

(50)

The coordinates of the wetted area limits $x_c$ of both parts of the profile are recalculated into the coordinates $x_a$ and $x_b$, respectively (Fig. 10), by means of an appropriate transformation of the coordinate frame.

The control volume $V_k$ may change due to oil scraping or oil supplementing in case of its lacking in under-ring space, if the oil is not squeezed out or crapped aside ring face in advance (Fig. 13b). When the oil control volume $V_k$ (Fig. 13a) is greater than that of the space possible to be filled under ring face $V_x$, some oil excess $V_z$ may be accumulated outside the profile (Fig. 13b). The oil excess comprised within the limits of the assumed maximum volume $V_{z_{max}}$ is able to flow back under ring profile and supplement the volume $V_x$, provided that such excess will happen. However if the excess $V_z$ surpasses the volume $V_{z_{max}}$, the excessive oil of the volume $V_{out}$ will be eventually squeezed out to external space. In the case when $V_k < V_x$, the space may be supplemented in an analogous way with the oil scrapped aside or squeezed out by the ring.

In the model, it was also taken into account that ring may displace in radial direction within the oil layer range ($h_m < h_{oil}$).
In this case the volume $V_k$ is additionally enlarged by sum of the squeezed out volumes $V_s$ and $V_z$, provided that the ring displacement occurs towards the cylinder liner (Fig. 14b), or lowered – when the ring departs from the liner:

$$V_{k_{-1}} = V_{k_{-0}} + \Delta V \quad \Delta V = V_s + V_b.$$  

(51)

The component volumes $V_s$ and $V_z$ of the squeezed out oil are derived from the relation:

$$V_z = (h_{m_{-0}} - h_{m_{-1}}) \cdot x_0.$$  

(52)

$$V_b = h_{ol} \left( x_1 - x_0 \right) - \int_{x_0}^{x_1} \left( \frac{x^2}{2R_e} + h_m \right) dx.$$  

(53)

and, in Eq. (53) : $h_m$ stands for the smaller of the heights $h_{m_{-0}}$ and $h_{m_{-1}}$, depending on a direction of ring radial displacement. The coordinates $x_0$ and $x_1$ of the crossing points of ring face and oil level line in the preceding instance 0 and the current one 1, respectively, are derived from the equations:

$$x_0 = \sqrt{2R_e(h_{ol} - h_{m_{-0}})}, \quad x_1 = \sqrt{2R_e(h_{ol} - h_{m_{-1}})}.$$  

(54)

Balance of oil flowing under ring face concerns the three fluxes: the inflow $q_{in}$, the flow through the gap, $q_z$, and the outflow $q_{out}$ (Fig. 14a). The fluxes per unit of ring circumference are derived from the following expressions:

$$q_{in} = h_{in} \cdot v_p, \quad q_p = \frac{1}{2} h_p \cdot v_p, \quad q_{out} = h_{out} \cdot v_p.$$  

(55)

where: $h_{in}$ and $h_{out}$ – thicknesses of oil layers before and behind ring face, respectively, $h_p$ – height of the gap under ring face in the maximum oil pressure point.

In incomplete wetting conditions, ring may slide over oil layer, owing to this the in-flowing flux $q_{in}$ becomes equal to the out-flowing flux $q_{out}$. If the flux $q_z$ is greater that the flux $q_{in}$, the scrapping out of oil occurs. The flux of scrapped-out oil, $q_z$, is equal to the difference of the fluxes $q_{in}$ and $q_{out}$. The scrapped-out oil is accumulated in the inter-ring spaces. Change in oil volume in such a space results from the difference of fluxes flowing through neighbouring rings, as well as from a volume of oil squeezed-out from under ring face into an appropriate space formed due to radial displacements of rings. Thickness of oil layer filling the space is derived on the basis of the so determined balance concerning a given space. In a similar way is derived a layer thickness of the oil which remains after ring pack passage. And, in the dead centre the scrapped-out oil volume $V_z$ is considered to be discharged away from the system either to combustion chamber or crankcase. The oil layer which remains behind the ring pack, becomes after change of direction of piston motion – a layer of oil flowing in the ring pack.

Profile of the oil layer left by ring pack along cylinder liner may vary as a result of oil evaporation on the side of combustion chamber, or oil mist deposition on the side of crankcase. The phenomena were considered in the same way by using the linear exposure time function:

$$\Delta h_{ol}(t) = I \cdot t_e.$$  

(56)

where: $\Delta h_{ol}$ – change in height due to a given phenomenon, $I$ – intensity of oil deposition or evaporation, $t_e$ – exposure time.

**Numerical model and computer software**

Numerical solution consists in transformation of the analytical differential equations describing the mathematical model, into difference equations calculated with a constant step which, in the main loop of the model, corresponds to an assumed increase of crankshaft rotation angle. Length of the step in the main loop of the model may be selected from the range of 0,1÷0,001° of crankshaft rotation (OWK). In order to increase calculation effectiveness it was made it possible to set a divider intended for extending the calculation step which concerns generation of oil film resulting from ring pack -cylinder liner interaction. In each step of this part of the model are performed several iterations consisting in matrix solving the Reynolds equation which describes oil pressure distribution in the gap between face of each ring and cylinder liner, continued until equilibrium condition of radial forces is satisfied. Determination of profile twisting angle of each ring is also conducted by calculation looping during consecutive approximations until the condition of equilibrium of moments of relevant forces is satisfied. However in view of extensive dynamics of ring angular deformations, solution is derived in each step of the main loop of the model.

Computer program was coded in C++ language by using object-orientated technique. Its structure is modular owing to this a free modification and further extension of the numerical model is possible. Introduction of input data is made by means of dialogue windows, after that it is possible to save or read all quantities. Certain data, e.g. those dealing with run of pressure in combustion chamber or profile of cylinder liner are put in the form of text files. The calculations are carried out in the runs simulating full cycles of a four-stroke engine. In order to reach continuity of results in the point in which a cycle is started and ended it is necessary to carry out a few successive calculation runs. The software is fitted with a diagram tool and animated graphic schemes which serve to visualize and analyze results both during and...
after completing calculation runs. Values of all calculated quantities are saved in text files; moreover it is possible to do a selective choice of results intended for saving, as well as to reduce number of records by data averaging.

Summary

The developed ring sealing model integrates the gas flow, ring dynamics and oil film generation models. It describes phenomena crucial for sealing process of TPC system by using physical relations. And, empirical relations were used only for description of phenomena of a minor importance or insufficiently highlighted theoretically. In the presented model, by contrast to majority of existing models, it was not assumed that gas flow through inter-ring spaces is isothermal, but that gas temperature is determined from energy balance. Such method is more realistic and it makes it possible to model isothermal and adiabatic flows as peculiar cases.

In the presented version of the model significant attention was paid to characteristic geometrical details of TPC system. It is expected that owing to this it may be possible to more precisely analyze impact of constructive and operational factors onto ring sealing performance. A comparison of preliminary results of simulation tests and the measurement results which was presented in [12], proves that the model in question is useful for predicting effects of elements wear on rate of exhaust gas blow-by. The model allows to predict rate of exhaust gas blow-by to crankcase as well gas back-flow to combustion chamber, owing to this the model may be instrumental in attempts to improve performance combustion chamber sealing and to lower emission of hydrocarbons. Knowledge of pressure distribution within inter- and beyond ring spaces as well as position and twisting deformations of rings in grooves is of a crucial importance in modeling interaction between ring and cylinder. For this reason the integrated models of gas flow and ring dynamics may be applied to modeling oil film, resistance of ring pack to motion and its wear.

Despite a large number of phenomena have been already taken into account, many other ones which could significantly affect sealing performance, have been omitted. In the future these authors plan to take into account occurrence of mixed friction between ring and cylinder as well as to model interaction between ring and groove in a more physical manner.

Bibliography


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