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ANNA MARKOWSKA
Institute of Geodesy and Cartography
Geoinformation and Cartography
Warsaw, Poland
orcid.org/ 0000-0001-8446-6171
anna.markowska@igik.edu.pl

Cartograms – classification and terminology

Abstract. The author discusses new classifications of cartograms. Cartographic anamorphoses terminology and a multi-level classification, including not only cartograms but also anamorphical projections, have been proposed. The selected area cartograms' classes were discussed in detail and compared.

Keywords: cartograms, area cartograms, cartograms' classification

1. Introduction

A cartogram[1] is a form of cartographic presentation for which there is no unified terminology and full classification. Earlier divisions of cartograms, both Polish and foreign, were incomplete. Anamorphical projections and anamorphic pseudomaps were often omitted. The author aims to discuss the terminology referring to cartograms, present a proposed multi-level classification of cartographic anamorphoses and compare classes of area cartograms.

2. Cartograms – terminology

A cartogram is a map, on which one feature – distance (distance cartograms) or area (area cartograms) is distorted proportionately to the value of a given phenomenon (A. Faliszewska 2011). According to J.C. Muller (1982, 1983), if cartograms are a particular type of projection, all traditional maps can be treated as cartograms, and traditional equal-area maps can be treated as area cartograms. However, the literature considers cartograms mostly as maps which are the result from a purposeful modification of traditional maps, mainly choropleth and diagram maps (R. Szura 1989), i.e. "maps of visibly distorted geometry – in comparison to Euclidean geometry – but better suited to reader requirements connected to map's function" (A. Michalski, P. Tymków 2011, p. 19).

In the literature there appear various terms referring to area cartograms. The most common English terms are *cartogram* and *value-by--area map*. The term *cartogram* was originally used for graphic presentation of statistical data, in cartographic meaning it was first used in 1851 to name a series of maps *Cartogrammes a foyer diagraphiques* by C.J. Minard (H. Friis 1974). Today the English term *cartogram* refers to maps elaborated in a "scale other than a true scale" (V.S. Tikunov 1988, S. Mayhew 2004, B.D. Henning 2011).

In the sixth volume of *The history of cartography…* (M. Monmonier 2015) other terms for area cartograms are listed: *anamorphosis, diagramic maps, map-like diagrams, varivalent projections, density equalized maps, isodensity maps, mass-distributing (pycnomirastic) map projections*. Terms of cartograms often relate to the shape of basic units, e.g. *rectangular cartogram* (E. Raisz 1934) or *circular cartogram*, more often known as *Dorling cartogram* (D. Dorling

[1] In Polish terminology the term "kartogram anamorficzny" has been used for many years to describe area cartogram (R. Szura 1989). The Polish term "kartogram" means "choropleth map".

1996). Terms of many area cartograms contain the name of the algorithm used in their preparation. When W. Tobler (2004) wrote about computer elaboration of area cartograms, he mentioned J. Dougenik, N. Chrisman and D. Niemeyer (*Continuous Area Algorithm*, 1985) or Gastner-Newman (*Diffusion-based Method*, 2004) algorithms. It is possible to find other names related to algorithms used to generate cartograms (A. Markowska, J. Korycka-Skorupa 2015), e.g.:

− algorithm of W. Tobler from 1973 (*Rubber-map Method*) and from 1986 (*Pseudo-cartogram*),

− algorithm of D. Dorling from 1990 (*Cellular Automation Algorithm*),

− algorithm of D. House and C. Kocmoud from 1998 (*Continuous Area Cartogram Using the Constraint-based Method*).

Since the publication of W. Tobler's (2004) article there have appeared new algorithms, and therefore new terms referring to cartograms, e.g. *gridded cartogram* (B.D. Henning 2011), *circular-arc cartogram, rectilinear cartogram, table cartogram* or *mosaic cartogram* (S. Nusrat, S. Koborov 2016).

3. Cartograms – classification

For the purpose of map classification, seven classification criteria were set. The criteria are divided depending on which map class they can be referenced:

• cartographic anamorphoses:
A) mathematical basis,
B) transformed object,
C) transformed method;
• area cartograms:
D) graphic continuity;
E) graphic presentation;
• anamorphical projections:
F) grid transformation;
• distance cartograms:
G) the main distortion point's location.

A. Mathematical basis

Division according to mathematical accuracy of transformation was established as the main criterion of classification of cartographic anamorphoses (Z. Mudrych 1976). The term *cartographic anamorphosis* is overriding in relation to cartograms. It includes all presentations

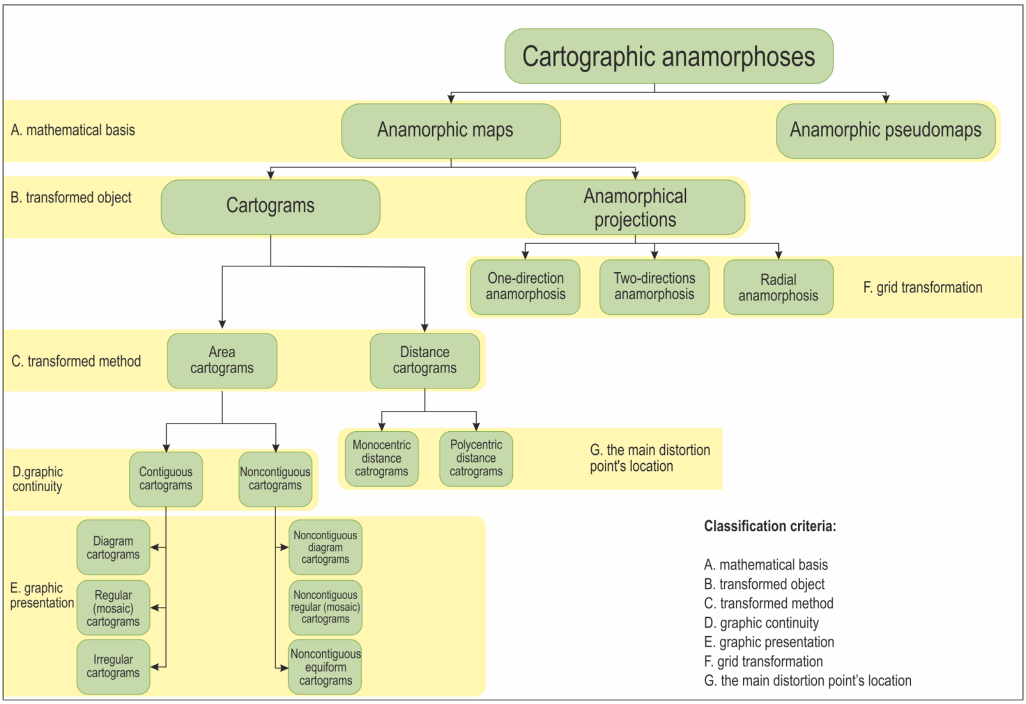

Fig. 1. Cartograms' classification

having certain anamorphic features, though they may not be drawn up according to strict mathematical rules or may not maintain spatial relations (fig. 1, criterion A):

− *anamorphic maps* (app. 1, pt 1) – presentations which have been made according to mathematical and spatial rules;

− *anamorphic pseudomaps* (app.1, pt 2) – graphically close to anamorphic maps, but elaborated in a more arbitrary way, often without clearly determined mathematical rules or rules connected to spatial relations.

B. Transformed object

The second criterion of classification considers if the anamorphic transformation is applied to the cartographical grid or to the thematic contents of the map (fig. 1, criterion B):

− *cartograms* (app. 1, pts 4a–10) – anamorphic maps in which, depending on the value of the phenomenon, the area of individual areal units is changed (area cartogram) or the distance between selected points is changed (distance cartogram); which means that changes are introduced depending on the thematic contents of the map;

− *anamorphical projections* (app. 1, pt 3) – anamorphic maps resulting from a transformation of cartographical grid (J. Korycka-Skorupa, et al. 2015).

Anamorphical projections are a type of distortion projections in which cartographical grid is distorted (S. Grabarczyk-Walus 2007). In this group of projections a local scale change is achieved by transforming linear elements of the map by specially selected transformational functions. Areas around the selected center are enlarged. Thus it is possible to observe more details in the reader's area of interest.

According to the type of distortion of the cartographical grid of the base map three classes of anamorphical projections can be indicated (fig. 1, criterion F):

− *one-direction anamorphoses* (distortion along one axis of the rectangular coordinate system),

− *two-direction anamorphoses* (distortion along two axes),

− *radial anamorphoses* (distortion in radial directions from a given central point).

C. Transformed method

In the process of editing the contents of a cartograms, the changes in the area of base units or distances between selected points can be introduced (fig. 1, criterion C). Both transformations are proportional to the value of a phenomenon (J. Olson 1976). Two classes of cartograms can be proposed:

− *area cartograms* – maps on which the area of individual spatial units is changed depending on the value of a phenomenon (app. 1, pts 5a–10);

− *distance cartograms* – maps on which the distance between selected points is changed depending on the value of a phenomenon (app. 1, pts 4a–4b).

For *distance cartograms* a general division is proposed according to the location of the main points of distortion (fig. 1, criterion G):

− *monocentric distance cartograms* – maps on which the distance between the central point and given points changes with the value of a phenomenon (e.g. distances between a selected metro station and next stations expressed in the time of journey – app. 1, pt 4a);

− *polycentric distance cartograms* – maps on which the distance between a subsequent pair of points in a given network changes with the value of a phenomenon (e.g. distances between two subsequent stations of the Warsaw metro expressed in the time of journey – app. 1, pt 4b).

D. Graphic continuity

The author of a map performs various operations during the process of cartogram elaboration (B.D. Dent 1999, B.D. Dent et al. 2009 − fig. 2). In an anamorphical transformation the area of both contiguous and noncontiguous cartograms is changed. In both classes of area cartograms the area of base units is modified proportionately to the value of a phenomenon. In contiguous and noncontiguous cartograms mutual location (orientation) of base units is preserved. Therefore changes of area or orientation cannot differentiate classes of area cartograms and should not be the criteria of their classification.

Spatial continuity of presentation differentiates contiguous and noncontiguous cartograms. Therefore this characteristic was chosen as a criterion of classification of area cartograms. Another aspect differentiating contiguous and noncontiguous equiform cartograms is the shape of base units. According to the scheme

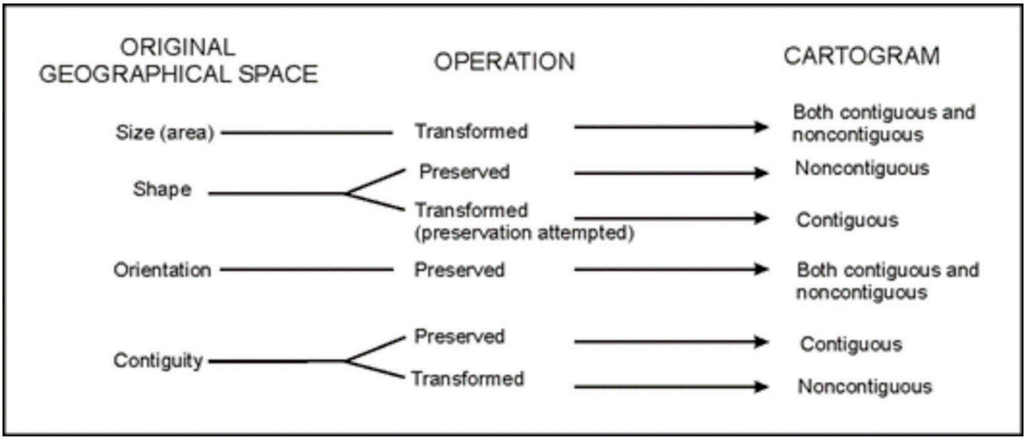

Fig. 2. The operations take to edit area cartograms (B.D. Dent 1999; B.D. Dent et al. 2009)

(fig. 2) contiguous cartograms always have a changed shape of base units, while in noncontiguous equiform cartograms the shape is preserved. However, it is possible to construct a noncontiguous cartogram, in which base units are presented in the form of geometrical figures, e.g. circles. For that reason the criterion of spatial continuity is overriding to the criterion of shape. Using spatial continuity as a criterion of classification of area cartograms the following classes can be proposed (fig. 1, criterion D):

− *contiguous cartograms* –area cartograms on which spatial continuity is preserved, which means that base units have not been separated (app. 1, pts 5a–7);

− *noncontiguous cartograms* – area cartograms on which spatial continuity is broken, which means that base units have been separated (app. 1, pts 8–10).

E. Graphic presentation

The last criterion of classification refers to the shape of base units. This aspect can be considered in both contiguous and noncontiguous cartograms. Looking at the shape of base units contiguous cartograms could be divided into (fig. 1, criterion E):

− *contiguous diagram cartograms* (app. 1, pts 5a–5c) in which units of reference have been substituted by simple geometric figures, e.g. squares, rectangles, circles (e.g. Dorling cartogram, 5c). Further in the article a simplified terms *diagram cartograms, rectangular cartograms, square cartograms* or *circle cartograms* are used;

− *contiguous regular (mosaic) cartograms* (app. 1, pt 6) in which base units correspond in shape to spatial units, but their borders are geometrized, most often build of segments. Further in the article a simplified term *regular cartograms* is used;

− *contiguous irregular cartograms (shape-like cartograms)* (app. 1, pt 7) in which base units correspond to the shape of statistical units, their borders are not geometrized, the shape of units depends on the distortion algorithm applied. Further in the article a simplified term *irregular cartograms* is used.

During the division of noncontiguous cartograms, firstly the cartograms which preserve the shape of spatial units (*noncontiguous equiform cartograms* – app. 1, pt 10) were differentiated from those which do not preserve the shape (*noncontiguous diagram cartograms* – app. 1, pt 8, and *nocontiguous regular cartograms* – app. 1, pt 9). *Noncontiguous diagram cartograms* have base units represented by geometric figures (e.g. circles, squares, hexagons), whereas in *noncontiguous regular (mosaic) cartograms* the shape of base units corresponds to spatial units, but their borders are geometrized, often built of segments.

Figure 1 presents a classification of cartographical anamorphoses. It includes the classification criteria mentioned earlier and terms describing individual anamorphic forms. Looking at area cartograms as forms of cartographic presentation it can be noted that the proposed criteria focus not only on the process of map transformation (the presentation method), but also the result of that process (the presentation form J. Korycka-Skorupa 2002). The first group of criteria (method-related) includes: mathematical basis, transformed object and transformed method. In the second group (form-related) two criteria can be noticed: graphic continuity of presentation and graphic method of presentation.

4. Classes of area cartograms

Classes of area cartograms were differentiated according to the last two criteria of the classification of cartographic anamorphoses presented earlier (fig. 1):

− graphic contiguous – information if units are connected or separated (contiguous cartogram / noncontiguous cartogram);

− graphic presentation – related to the shape of base units (geometrical figures, regular/irregular shape) and the location of centroids of spatial units.

A. Contiguous cartograms

Contiguous cartograms as a class of area cartograms preserve spatial continuity of presentation. Three subclasses of contiguous cartograms have been proposed: *diagram cartograms, regular cartograms (mosaic cartograms), irregular cartograms (shape-like cartograms)*.

A1. Diagram cartograms

In *diagram cartograms* spatial continuity of presentation is preserved, and individual units are substituted by geometric figures. *Diagram cartogram* subclasses have been distinguished by the shape of base unit, e.g. rectangular (app. 1, pt 5a), square (app. 1, pt 5b). *Diagram cartograms* using circles are an exception. In the literature they are known as *Dorling cartograms*, the term *Circular cartograms* (app. 1, pt 5c) is rarely used. These two terms can be used interchangeably (A. Faliszewska 2011).

A1a. Rectangular cartograms

In this class of diagram cartograms spatial continuity of presentation is proportional to the value of a phenomenon, and the units have the shape of a rectangle (M. van Kreveld, B. Speckmann 2004 – app. 1, pt 5a).

In the process of elaboration of *rectangular cartograms* several rules have to be followed:

− rectangle area has to be proportional to phenomenon value (e.g. size of population or value of agricultural production);

− in the case of adjoining units (e.g. voivodships) their adjoining location has to be preserved on a diagram cartogram;

− adjoining rectangles cannot border only with corners – sides always are connected.

Some of the first *rectangular cartograms* were prepared by E. Raisz (1934). The legend made by E. Raisz (fig. 3) is very useful. A square shows a certain value of a phenomenon (e.g. 1 million dollars), which makes it possible to estimate value of individual units.

An important aspect of *rectangular cartograms* is that maps can be read not only at a general level (general view of the map), or indirect level (comparing relations between base units), but also at the detail level of map reading – estimating approximate values of individual units, which is facilitated by a correctly elaborated legend.

A *rectangular cartogram* can be computer generated (M. van Kreveld, B. Speckmann 2004).

A1b. Square cartograms

Square cartograms (Demers cartograms) are a special kind of rectangular cartograms, in which base units are presented as squares (M. van Kreveld, B. Speckmann 2004 − app. 1, pt 5b).

A1c. Dorling cartograms

The name of the *Dorling cartogram* (circular cartogram – app. 1, pt 5c) came from the name of its author – Daniel Dorling, the British geographer who specializes in social-geographical issues.

In the *Dorling cartogram* the value of a phenomenon is proportional to the area of a circle (D. Dorling 1993). Initially D. Dorling did not use circles but hexagons for data presentation (A. Faliszewska, J. Korycka-Skorupa 2010). In Great Britain border on six other constituencies, and voting most constituencies were the topic of many of D. Dorling's works.

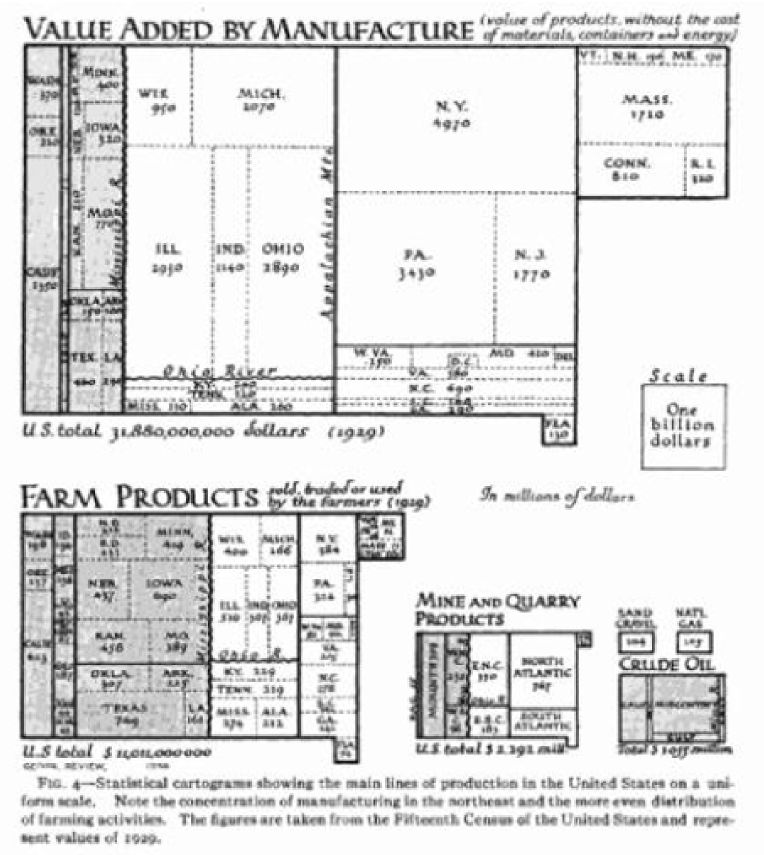

Fig. 3. Rectangular cartogram. The area of rectangles is proportional to the value of US production in 1929 (E. Raisz 1934)

A2. Regular cartograms (mosaic cartograms)

Regular cartograms attempt to maintain the general shape of a given area while geometrizing it. Individual base units are built of unit squares (segments), e.g. one square represents 150 births (app. 1, pt 6).

By using a unit square it is possible to render approximate shape of a whole country or continent. The main problem of presentations of this type is that they are time-consuming. Also, particular attention has to be paid to the data used. While presenting some statistical data it is difficult to maintain the shape of individual areas because of their high range. It may be necessary to omit the units with very low value. *Regular cartograms* can be prepared automatically, applying algorithms which enable elaboration of polygons of up to eight sides (M.J. Alam, et al. 2013) or more sides (fig. 4) called a *mosaic cartogram* (R.G. Cano et al. 2015). Despite the availability of automated methods of elaboration of regular cartograms, the popularity of this class of cartograms is still quite low. Algorithms are not available in popular GIS programs and there are no desktop or web applications with an easy interface which would enable quick preparation of regular cartograms.

A3. Irregular cartograms (shape-like cartograms)

The most important characteristic of *irregular cartograms* is that they maintain spatial continuity of a phenomenon, meaning that there is no gap between units (B.D. Dent et al. 2009). In order to generate a map of this type it is

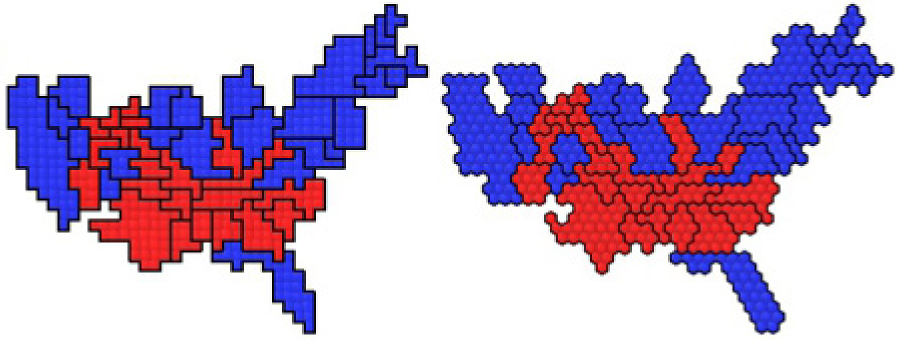

Fig. 4. Regular cartogram (squares or hexagons). Number of votes cast for candidates in the presidential election in 2012 (R.G. Cano et al. 2015)

necessary to change the shape of individual units while keeping their continuity.

In this class of contiguous cartogram the area of individual spatial units should relate to their geographical shape. For that reason they are irregular as in the illustration, app. 1, pt 7.

Elaboration of irregular cartograms is very labor-consuming and almost impossible without using computer programs. Terms of irregular cartograms often contain the name of the algorithm used for their elaboration. A comparison of the characteristics of cartograms generated with different algorithms is included in appendix 2.

B. Noncontiguous cartograms

The second class of area cartograms is *noncontiguous cartogram* in which spatial continuity of presentation is lost. Depending on the maintenance of the shape of base units three classes of noncontiguous cartograms have been proposed: *noncontiguous equiform cartograms* (maintain the shape), *noncontiguous diagram cartograms* and *noncontiguous regular cartograms* (do not maintain the shape of base units). The location of the centroids of noncontiguous cartograms' unites is an important issue (fig. 5, J. Olson 1976). It determines overlapping of base units.

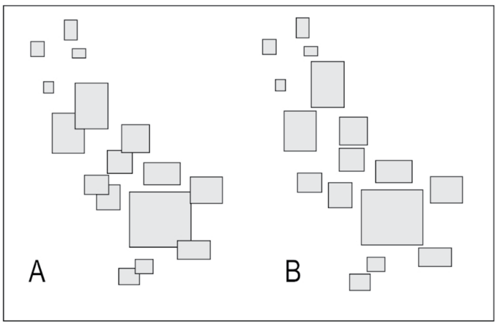

Fig. 5. Noncontiguous diagram cartogram – location of units: A – overlapping, B – non-overlapping (J. Olson 1976)

If geographical coordinates of unit centroids are maintained there is a high probability of units' overlapping (fig. 5A). This can be avoided if the location of central points is changed in the process of anamorphical transformation (fig. 5B).

B1. Noncontiguous diagram cartograms

In *noncontiguous diagram cartograms* (app. 1, pt 8) each spatial unit is represented by a single geometric figure (e.g. app. 1, pt 8 – the area of hexagons is proportional to the size of population and area in individual voivodships in Poland in 1970, J. Ostrowski 1970). Presentations of this type are difficult to interpret, because:

− they do not maintain spatial continuity, base units are separated,

− base units are replaced by geometric figures, so their shape is lost,

− the resulting cartogram is usually not drawn into the borders of the presented area – only unconnected diagrams appear in the final form.

Fig. 6. Irregular cartogram (Gastner-Newman) – the distortions mainly concern hard-to-recognize countries. Number of tankers by country (www.worldmapper.org)

B2. Noncontiguous regular (mosaic) cartograms

Noncontiguous regular (mosaic) cartogram is a subclass of noncontiguous cartograms, in which base units are presented as geometrized units. These cartograms are built of segments which can be visible on a map (app. 1, pt 9), usually borders of individual segments may be not marked on a map. *Noncontiguous regular cartograms* are easier to interpret than *noncontiguous diagram cartograms*, because they refer the shape of base units and the shape of the whole area are partially preserved.

B. Noncontiguous equiform cartograms

Noncontiguous equiform cartograms (app. 1, pt 10) are cartograms in which the shape of individual units is maintained, and only the area is modified depending on the value of a phenomenon. In order to maintain the shape of reference units it is impossible to maintain spatial continuity on such maps. This class of cartogram is considered to be the most simple to elaborate, because the only transformations of the base map are those connected to the change of the unit area and in some cases also involve moving the centroids of units (J. Olson 1976).

An interesting aspect of a *noncontiguous equiform cartogram* is that spaces between base units are significant – they can be used for interpretation of the spatial location of a phenomenon. Distances between units can show how diversified the analyzed phenomenon is. The bigger the spaces, the more diversified the intensity of a phenomenon.

5. Area cartograms – comparison

Graphic contiguity of presentation is a basic criterion of division of area cartograms, therefore the first to be compared are *contiguous cartograms* and *noncontiguous equiform cartograms*. Advantages and disadvantages of *contiguous cartograms* and *noncontiguous equiform cartograms* were listed by B.D. Dent et al. (2009, tab. 1).

A summary of selected area cartograms' characteristics was prepared by S. Nusrat and S. Kobourov (2016, app. 2). Only automatedly generated area cartograms were considered, starting with the irregular cartogram elaborated on the basis of the *Rubber map method* algorithm from 1973 (W. Tobler 1973). Some important algorithms are missing from the discussed summary, e.g. *Gridded Cartogram* (B.D. Henning 2011).

The summary in appendix 2 can be useful when choosing the cartogram which we want to elaborate. Thanks to that summary it can be noticed that the characteristic of a cartogram which is maintained by most algorithms is contiguity. The authors (S. Nusrat, S. Kobourov

Table 1. Advantages and disadvantages of contiguous and noncontiguous area cartograms (B.D. Dent et al. 2009)

Area cartogram's class	**Advantages**	**Disadvantages**
Contiguous cartogram	– in most cases it is possible to maintain the location and placement of spatial units, so they can be identified with related units on a traditional map, – due to maintained spatial contiguity of presentation the user does not have to imagine connections between separated base units, as in the case of noncontiguous equiform cartograms, – general view of the area, e.g. Poland or the world is easier to maintain than in noncontiguous equiform cartograms	– Losing the general shape of presentation and relations between units makes it difficult to read information on the map (fig. 6), – sometimes very high distortion of the shape of base units which leads to problems with identification of individual units, – there are still no proper GIS programs do elaborate all classes of contiguous cartograms, so preparation of such maps is very labor-consuming.
Noncontiguous equiform cartogram	– simplicity of construction, – the true geographical shape of individual units, – significance of gaps between base units, which can be useful for the interpretation of a phenomenon.	– spatial contiguity of presentation is broken, – separation of units makes it difficult to recreate the shape of the whole area (e.g. country or continent), – if some units are very small it may be necessary to move them to avoid overlapping.

2016) point out only several cases of not maintaining the mutual location of units (*Demer Cartogram or Dorling Cartogram*). In the summary only some classes of cartograms are marked as those in which values of phenomena are fully represented (*Statistics – accurate*). According to the authors such situation is possible if geometry is not maintained or if the shape of base units is mostly lost.

The above discussion of the characteristic of area cartograms is summarized in the *Cartogram Cube* (fig. 7, R.E. Roth et al. 2010). It is a graphic method of presenting relations between features affecting informational attributes of maps (Z.F. Johnson 2008)

− topology preservation – if placement between neighboring units is well maintained,

− shape preservation – to what extent the original shapes of units are maintained (angles and proportions of side lengths can be analyzed here),

− visual equalization – to what extent the area of a unit correctly represents the value of a phenomenon.

These three aspects of cartograms are important depending on the issues which the user reads from a map. In the process of identification of cartogram's base units their shape is more important than topology preservation. Shape preservation is important at the elementary level of map reading. At the general level of map reading topology preservation is more important. Estimation of a phenomenon value basing on a cartogram should be done only if the visual equalization is high.

The mentioned characteristics were placed on coordinate axes, i.e. edges of the Cartogram Cube. The ideal cartogram is located in the upper rear right corner of the cube, which means that it fully preserves the shape, location and placement of units and that the representation is one hundred percent (D.A. Keim, A. Herrmann 1998). Such a cartogram is impossible to reach. From the perspective of the user, shape preservation is more important at the level of detailed map reading, while topology preservation is more important at the general level of map reading. For the purpose of reading phenomenon values for individual base units, the elaborated cartogram should have as low error level as possible – visual equalization should be as high as possible (R.E. Roth et al. 2010).

In the Cartogram Cube the classes of area cartograms discussed in that article (*Dorling cartogram, regular cartogram, Gastner-Newman's cartogram, noncontiguous equiform cartogram, noncontiguous diagram cartogram),* as well as

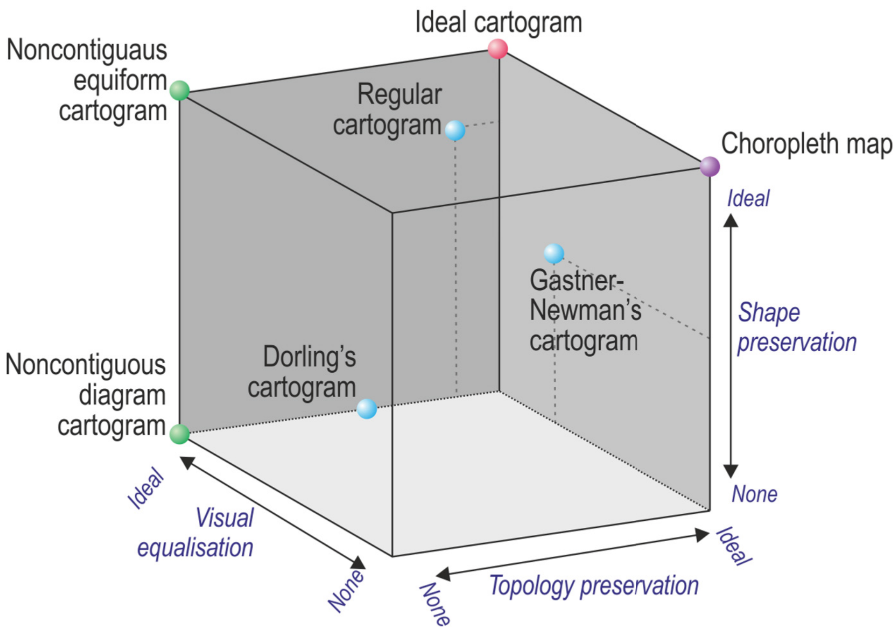

Fig. 7. Cartogram Cube – cube of preserving the characteristics of area cartograms classes. Contiguous cartograms are marked in blue, green – noncontiguous cartograms, violet – a traditional map, and red – perfect area cartogram. Map location in cube based on R.E. Roth et al. 2010 (R.E. Roth et al. 2010)

a example traditional map (fig. 7) can be placed.

From figure 7 it can be concluded that regular and irregular (Gastner-Newman) cartograms are more useful than rest area cartograms' classes. However, it should be noted that the placement of a cartogram in the scheme can be described more as a certain space rather than a particular point. Each elaborated cartogram has different distortion, and there are no indicators which would evaluate how correct a specific cartogram is. Such factors would be especially useful for irregular cartograms in which the shape of individual units and the whole area can be significantly distorted. In the case of highly distorted shapes of base fields it would be better to use a diagram cartogram or a noncontiguous equiform cartogram.

Cartogram Cube can be useful if we make different cartograms for statistical data referring to the same database and topic. Then we can set together cartograms depending on how they maintain the shape, location and placement of units and how they represent the value of a phenomenon. It is worth considering how possible it would be to use the space of a particular cartogram within the Cartogram Cube to evaluate the quality of cartograms elaborated basing on the same database.

The summary of features of various cartograms performed by the authors mentioned above makes it possible to choose the most appropriate cartogram depending on map purpose. Such elaborations are also helpful for a comparison of analyzed cartograms.

6. Conclusions

The above classification of cartograms is the most complete of all presented in the literature so far. The earlier summaries of cartograms excluded some cartogram classes, mainly focused only on area cartograms, omitted distance cartograms or anamorphical projections. They can be described as incomplete typologies rather than multi-level classifications. At the same time they did not include differentiation of classes of area cartograms. The terminology proposed in the classification, as well as the method of division of cartograms, are still open to discussion and require further analysis.

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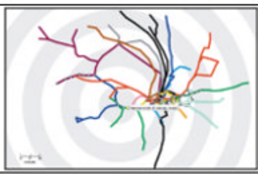
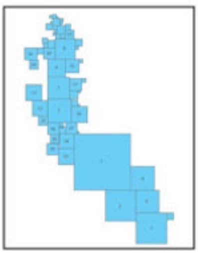
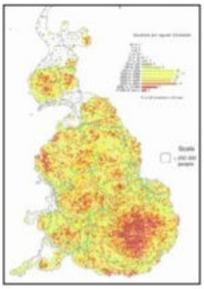
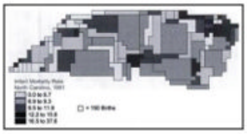
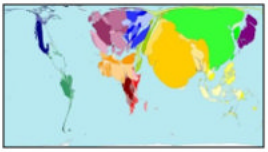
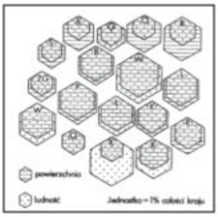
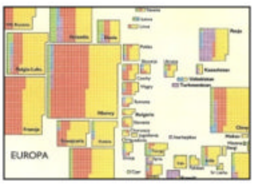
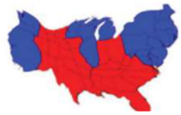
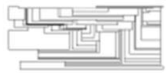
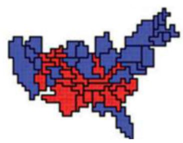
Appendix 1. Cartographic anamorphosics examples

No.	Cartographic anamorphoses	Example	Information and source
1.	*Anamorphic maps*		
2.	*Anamorphic pseudomaps*		
3.	*Anamorphical projections*		Topographic map with two-points anamorphical projection. S. Grabarczyk-Walus, 2007.
	Cartograms		
	Distance cartograms		
4a	*Monocentric distance cartograms*		London's Tube (time scale) *http://www.tom-carden.co.uk/p5/tube_map_travel_times/applet*
4b	*Policentric distance cartograms*		Fragment: Distances between Warsaw metro stations expressed in travel time. J. Korycka-Skorupa, et al. 2015
	Area cartograms		
	Contiguous cartograms		
	Diagram cartograms		
5a	*Rectangular cartograms*		Fragment: World population in 2006. https://www.win.tue.nl/~speckman/Cartograms/WorldCarto.html
5b	*Square cartograms*		Population in the counties of the state of California presented using a square cartogram. The biggest counties have been marked with the numbers poviats. http://www.ncgia.ucsb.edu/projects/Cartogram_Central/types.html

No.	Cartographic anamorphoses	Example	Information and source
5c	*Dorling cartograms*		Britain divided into counties. The circle surface presents the population inhabiting individual units. Dorling, D., 1995, *A new social atlas of Britain*, Wiley, Chichester
6.	*Regular cartograms*		Infant mortality in North Carolina in 1991. The area of counties is the number of births. Meade M.S., Earickson R.J., 2005, *Medical Geography*,, New York: The Guilford Press
7.	*Irregular cartograms*		World population in 1500 with a clear dominance of China, India, Japan and Europe. Regions of the world are distinguished by colors. www.worldmapper.org
	Noncontiguous cartograms		
8	*Noncontiguous diagram cartograms*		The area of hexagons is proportional to the population and area in individual voivodships in Poland in 1969. Ratajski L. 1989, *Metodyka kartografii społeczno-gospodarczej.*, PPWK, Warszawa – Wrocław
9	*Noncontiguous regular cartograms*		Exports of goods around the world. *Nowy atlas geograficzny. Gimnazjum*, 2001, Demart, Warszawa,
10	*Noncontiguous equiform cartograms*		Population over 65 in the United States in 1970 . Olson J., 1976, *Noncontinuous Area Cartogram*, "The Professional Geographer", Vol. 36, nr 4

Appendix 2. Automatically generated area cartograms (S. Nusrat, S. Kobourov 2016)

Type	Statistics	Contiguity	Geography	Topology	Exemple
Rubber map method	Not accurate	Contiguous	Distorted	Topology- -preserving	
Pseudo- -cartogram method	Not accurate	Contiguous	Distorted	Topology- -preserving	
Constraint based approach	Not accurate	Contiguous	Distorted	Topology- -preserving	
Cartodraw	Not accurate	Contiguous	Distorted	Topology- -preserving	
Medial- -axis-based cartograms	Not accurate	Contiguous	Distorted	Topology- -preserving	
Cellular automata method	Accurate	Contiguous	Distorted	Topology- -preserving	
Dorling cartograms	Accurate	Not contiguous	Shape not preserved (circles)	Topology not preserved	
Rectangular cartograms	Depends on the variant	Contiguous	Shape not preserved (rectangles)	Depends on the variant	

Type	Statistics	Contiguity	Geography	Topology	Exemple
Diffusion--based cartograms	Almost accurate	Contiguous	Distorted	Topology preserving	
Circular-arc -cartograms	Not accurate	Contiguous	Shape mostly preserved	Topology preserving	
Optimal rubber sheet method	Almost accurate	Contiguous	Distorted	Topology--preserving	
Fast, free-form rubber-sheet method	Almost accurate	Contiguous	Distorted	Topology--preserving	
T-shape cartograms	Accurate	Contiguous	Shape not preserved	Topology--preserving	
Non--contiguous cartograms	Accurate	Not contiguous	Shape preserved	Topology not preserved	
Demers cartograms	Accurate	Not contiguous	Shape not preserved (squares)	Topology not preserved	
Mosaic cartograms	Not accurate	Contiguous	Shape mostly preserved	Topology--preserving	
Table cartograms	Accurate	Contiguous	Shape not preserved	Topology not preserved