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Dimensional and Geometrical Errors in Vacuum Thermoforming Products: An Approach to Modeling and Optimization by Multiple Response Optimization

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In the vacuum thermoforming process, the product deviations depend on several parameters of the system, which make the analysis, the computational modeling, and the optimization of errors a multi-variable process with conflicting objectives. In this sense, the aim of this work was to study the dimensional and geometrical errors as well as the optimization (minimization) of these errors in one typical vacuum thermoforming product made of polystyrene (PS). In particular, it was intended to predict and minimize errors in a range of ideal tolerances using Multiple Response Optimization (*MRO*) Models. Thus, through the fractional factorial design (2^{k-p}), initial experimental tests were performed using proposed measurement procedures, and Analysis of Variance being the data analysis is discussed. Following that, the *MRO* models were implemented which were also validated to represent the sample data. Through this analysis of the results, it can be concluded that the regression models of errors are not linear functions, hence, the developed models are valid for the studied process, and finally that the validation results proved the efficiency of *MOR* models developed, but these models will not be able to generalize to new situations in a range far from the values studied.

Keywords: Dimensional and geometrical errors, vacuum thermoforming process, multiple response optimization, plastics processing.

1. INTRODUCTION

Thermoforming is a generic term for a set of thermoplastic manufacturing processes which allow the production of thin wall plastic parts from flat sheets or plastic films, such as vacuum thermoforming, also known as vacuum forming, drape forming technique, thermoforming with the use of air-slip forming, and other little-used techniques such as billow or free bubble forming, mechanical bending, matched-mold forming, and twin-sheet forming, which are the earliest and simplest methods of thermoforming [1], [2].

In this context, the vacuum forming technique is defined by [2]-[4] as the process where the vacuum force obtained by the negative atmospheric pressure is used to force a preheated sheet against the "cold" surface of the mold, which takes on its shape. Specifically, this is the forming technique and/or stretching where a sheet of thermoplastic material is preheated by a heating system (Fig.1.a), Fig.1.b)) and forced against the mold surface (positive or negative) by means of the negative vacuum pressure produced in the space between the mold and sheet (Fig.1.d1)), by mold suction holes and a vacuum pump which "sucks" the air from the space and "pulls" the sheet against the surface of the mold (Fig.1.d1)), transferring it, after cooling (Fig.1.e)) and removing excess material to shape it (Fig.1.f)), [4], [5]. The typical sequence of this technique is presented in Fig.1. [6].

However, according to [7], [8], there are still a number of challenges to be overcome in this process, caused by the conflict of objectives between the quality aspects and the adjustments of the process control variables. The evaluation of the performance of the system is usually dependent on many processing variables such as environmental manufacturing characteristics, equipment characteristics,

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stretch speed, plug characteristics, temperature of heating, and cooling system [1], [9], [10]. Therefore, for [7], [8], [11] it is necessary to understand the complex and multi-variable process, with non-linear characteristics and conflicting objectives, in order to optimize the product quality characteristics and reduce errors before molding the part.

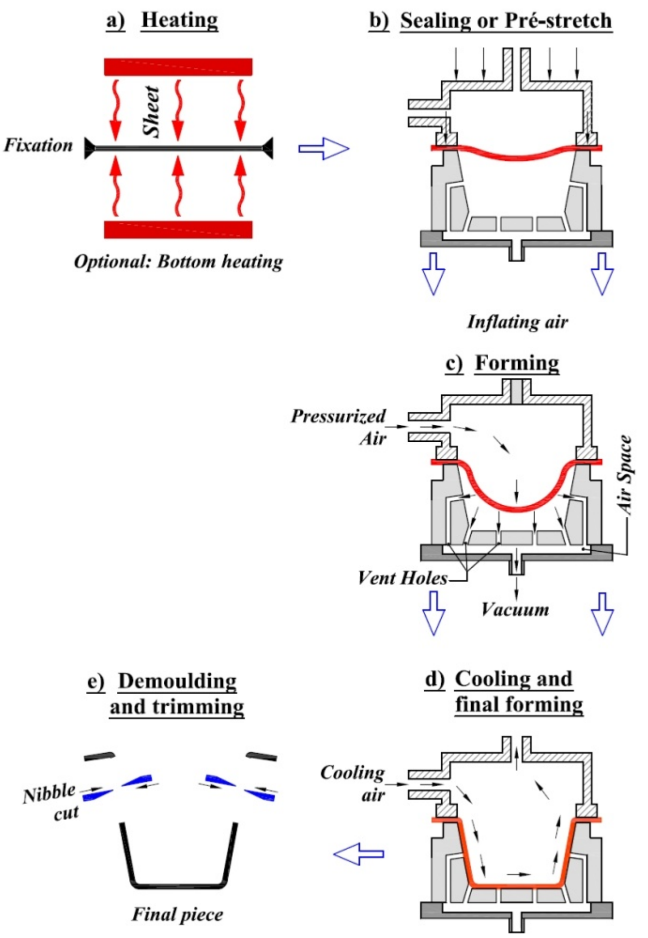

Fig.1. Schematic of basic vacuum thermoforming.

Several authors have developed work with the objective of modelling and predicting the quality of the final product of the vacuum thermoforming process, [12] using computational optimization techniques, Finite Element Method (FEM), Artificial Neural Network (ANN), and [7], [8] statistical models, aiming to predict and to optimize the quality characteristics. We still have the works of [10], and others focusing on the development of an elastic-plastic model for thickness analysis [2], [13]. Leite *et al.* describe the application of a methodology based on Artificial Neural Network models with objective function [11], the model has processing parameters as inputs and the product errors as outputs. References [6], [14]-[17] concentrated their studies on aspects of mold geometry and process parameters to verify their influence on the distribution of product thickness, [4], [18] have developed a methodology for optimization of production technologies with the product design. Martin *et al.* have studied the instrumentation and control of thermoforming equipment in real-time analysis with the control of multiple variables [19]. Other researchers have focused on modeling, simulation and prediction of sheet temperature and optimization of the heating system by different methods and techniques [20]-[22].

However, in complex manufacturing processes such as this, [23]-[25] suggest that the traditional approaches to process control fail to understand all aspects of process control or existing subsystems. Thus, researchers are using Multiple Response Optimizations (*MRO*) to model systems with multiple input and response variables, in order to minimize or maximize all responses based on an objective function. With a multi-criteria optimization problem involving more than one objective function to be optimized simultaneously, usually, the objective functions are in conflict with, or compete with each other, thus, the possible optimal solution functions do not allow the minimization of all objectives simultaneously. Researchers present several approaches and methods to optimize problems of multiple objectives [26], some of them reported by [27], [28].

One of the widely adopted techniques for *MOR* models uses Multiple Linear Regression Models (*MLR* models) to describe the relationship between a response and its regressor variables (process parameters), and also to estimate the response [29]-[31]. These models described by Montgomery [32] are linear regression equations that contain more than one independent variable or regressor and a dependent or response variable, that are related to *k* regressors or variables of input. Thus, these models need to be developed for each of the response variables for the modeling of an *MLR* algorithm [33]. The developed models are converted into a system that combines the n individual equations and an objective function through the programming of a multiple response optimization algorithm [18], [32]. Thus, the objective of this algorithm is to find a satisfactory solution or several possible configurations of the input variables that simultaneously offer the best performance for the multiple objectives of the n models [33], using solution space of input variables [29]. These equations can be solved by several mathematical methods of solving systems of linear equations or software [29].

First, the objective of this work was to study the dimensional and geometrical errors and the optimization (minimization) of these errors in one typical vacuum thermoforming product. For this purpose, the manufacturing parameters (factors) were studied statistically to determine their influence on the deviations of the product (response variables), and then, the Multiple Response Optimization Method that uses Multiple Linear Regression models to describe the relationships of the variables studied, (simultaneously) was used to simultaneously minimize the partial errors. A validation test was performed to evaluate the predictive capacity of the models and efficiency of the methodology studied. Finally, this study allowed us to identify the main significant factors, and also to develop models and algorithms that estimate and minimize errors of vacuum thermoforming parts.

2. EXPERIMENTAL WORK

2.1. Material and equipment

In this work, 2.0 x 2.5 m of white laminated polystyrene (PS) sheets with a thickness of 1.0 mm were used to manufacture the parts. The plates were cut into 300 x 360 mm sheets, cleaned with water and neutral liquid soap (pH), and then dried and wrapped in plastic film packages previously heated at 50°C and maintained for two hours.

For the manufacturing of the mold, considering the inherent aspects of the manufacturing process and the volume contraction of the product of 0.5 % [1], [34], Medium Density Fiberboard (MDF) plates were used as the raw material. The three-dimensional (3D) design of the model was developed using Computer-Aided Design (CAD, SolidWorks® 2008) software, which was integrated with Computer-Aided Manufacturing (CAM, Edge CAM® 2010) software. The mold was machined in a Computer Numeric Control machine (CNC, Discovery 560 ROMITM Machining Center), and subsequently, the vacuum holes and the final finish were performed. Finally, we performed the Computer-Aided Inspection (CAI) of the mold in a Coordinate Measuring Machine 3D (CMM 3D, Micro-Hite 3D TESATM with Reflex Software) to determine the dimensional and geometric deviations present in the mold.

A semi-automated vacuum thermoforming machine was developed and automated by the researchers. This equipment has the capacity to work with plates of thickness of 0.1 to 3.0 mm, a useful area of 280 x 340 mm, displacement of the mold (*z* axis) of up to 150 mm, vacuum pumps of 160 mbar with motors of 1.0 CV, infrared heating systems composed of two resistors of 750 W and 1,000 W, movement by pneumatic systems and acquisition of temperature data by "K" thermocouples and non-contact infrared. The system is programmable through a commercial Personal Computer (PC) integrated with microcontroller board (Arduíno UNO Revision 3).

2.2. Parameters and measurement procedure

There is no consensus among authors [4], [5], [7], [8] about the measurement parameters of control and quality in the vacuum thermoforming process and still, [1], [4], [9] there is no specific measurement procedure or equipment to be used. As a result, they were defined and developed to measure the errors of the piece. The procedures, scales, measurement process and tolerances are described in the following paragraphs.

For measurement errors, 3D MMC was used carrying a 4 mm diameter solid probe, calibrated with an error of ± 0.004 mm and CAI software. The reference values for dimensions were calculated, based on the final dimensions of the mold. Also, according to [3], [9], a deviation of ± 1 % for linear dimension and ± 50 % for flatness on surfaces are acceptable, and as a reference, the values calculated for dimensions were adopted as the general criteria for acceptance of sample dimensions. Fig.2. presents the geometry of the standard product, where dimensions and parameters to be measured in the samples are represented.

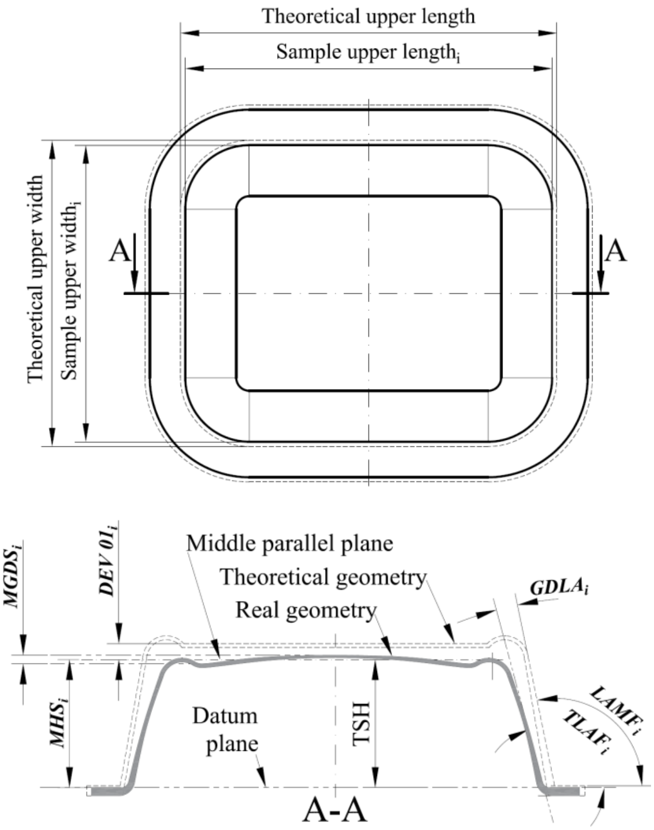

Fig.2. Product standard: dimensions on piece or dimensional deviations parameters.

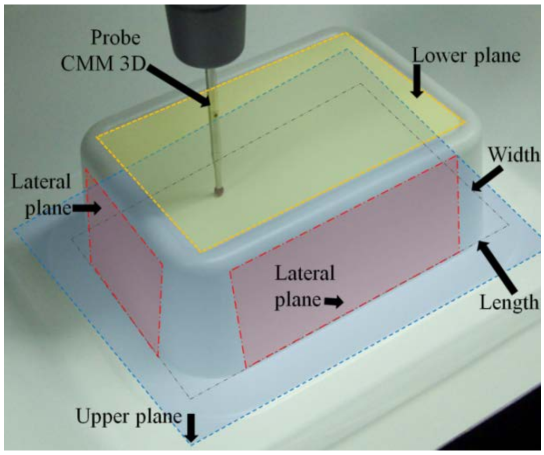

Fig.3. Measurement: Planes and references in the sample.

So, the Dimensional Deviation Height (*DDH$_i$*) or *DEV 01* was defined thus:

$$DDH_i = (MHS_i - TSH\ ) == DEV01_i = (MHS_i - 57.92) \quad (1)$$

where *i* is the index of the analyzed sample and *TSH* is the Theoretical Sample Height (57.92 mm). The *MHS$_i$* is the Measured Height in the Sample, calculated by the distance between two parallel planes formed by the upper and lower parts of the sample (Fig.2. and Fig.3.).

To determine the planes, 8 points were collected in each region using the 3D MMC (Fig.3.) and via CAI software the perpendicular distance between the planes was calculated. A negative (-) mean value indicates that the height is less than the ideal and a positive mean value (+) that it is greater than the ideal.

The Deviation of the Diagonal Length (DDL_i) or *DEV 02* is calculated by the difference between the values of the $MLDS_i$ and the value of the *TDL*, being:

$$DDL_i = (MLDS_i - TDL\ ) \tag{2}$$

where, $MLDS_i$ is the Measured Length of the Diagonal in the Sample or *DEV 02*, which in this work was defined as the quadratic relation of the lateral distances of the upper end of the sample (length and width) (Fig.2. and Fig.3.) and *TDL* is Theoretical Diagonal Length of the Sample = 207.97 mm, so:

$$DDL_i = DEV02_i(\sqrt{(width_i)^2 + (length_i)^2 - 207.97}) \tag{3}$$

To determine lateral distances, 5 points were collected along each side of the samples (Fig.3.), and later, via CAI, the distances between sides were calculated. A negative (-) mean value indicates that the length is smaller than the ideal and a positive mean value that it is greater than the ideal. Also, *TDL* is the Theoretical Diagonal Length of the Sample (208.0 mm).

The *DEV 03* or Geometric Deviation of Side Angles ($GDSA_i$), in this study, is expressed as:

$$\begin{gathered} GDSA_i = \tfrac{1}{z}\sum_{J-1}^{z} GDLA_i == ... \\ DEV04_i = \tfrac{1}{4}\sum_{J-1}^{4} (LAMF_S - TLAF_S\ ) \end{gathered} \tag{4}$$

where *z* is the number of sides and *s* the evaluated face. The *GDLA* is the difference between the Lateral Angle Measured on the Face of sample *i* ($LAMF_i$) and the Theoretic Lateral Angle of the Face (*TLAF*), for *s* = 1 ... 4, respectively, 95.93º, 95.93º, 96.02º and 96.06º. To determine each *ALMF*s, 09 points were collected on the surface to design the plan of control (lateral planes, Fig.2. and Fig.3.). The *GDLA*i was calculated, using CAI software, by the difference between the planes of the angles.

The Geometric Deviation of Flatness (GD_i) or *DEV 04,* that will have a zero value (0) for an ideal surface or positive values, was calculated as:

$$GD_i = (MGDS_i - TGDS\ ) == DEV03_i = (MGDS_i - 0.11) \tag{5}$$

where $MGDS_i$ is the Measurement Geometric Deviation of the *i-th* Sample, calculated by measuring 09 points on the surface of sample bottom (lower plane, Fig.2. and Fig.3.). Later, using CAI software, the distance between the two boundary planes of measured surface was calculated. Also, this procedure was used to calculate the Theoretical Geometric Deviation of the Sample (*TGDS*), which is 0.11 mm.

2.3. Analysis method

The Analysis of Variance (ANOVA) method has been performed to determine the importance of the process input parameters. The ANOVA is a set of statistical methods used to analyze data and to investigate implication of the main effects and interactions in the response variable. Moreover, it provides enough data to compare the parameter levels and the significances. [35].

Also, to estimate the response variable and evaluate the first-order models in A, C, ..., E, along with the AC, ..., AE interaction, the Fractional Factorial Designs [35] technique was used. For this, the coefficients of multiple linear regression models (*MLR*) were calculated. The *MLR* is the regression model that contains more than one independent variable *x*, that is, the response variable *Y*, is related to *k* input variables [29], so:

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_j x_k + \ldots + \beta_j x_1 x_2 + \ldots + \beta_j x_k x_{k+1} + \varepsilon \tag{6}$$

Finally, Multiple Response Optimization Models (*MRO* models) were developed [29], [32]. For this, the coefficients of *MLR* and statistical analysis data are used for computational modeling of the *MLR* models for each type of response variable. Afterwards, the *MRO* algorithms are developed with the *MLR* models, in order to generalize and estimate minimum error values and generate a list of possible optimal solutions. The script codes are programmed in the MATLAB® numerical analysis and programming software, with the techniques of analysis described.

2.4. Experimental study and analysis of data

In this research, we used the parameters (factors) described by [3] and compatible with the geometry of sample and equipment, namely: A. Heating Time (in seconds - s), B. Electric heating power (in percentage - %), C. Mold actuator power (in Bar and cm/s), D. Vacuum time (s), and E. Vacuum Pressure (in millibar - mbar). Table 1. shows the levels - high and low and values of parameters. For these selected values, test trials were performed to determine the operating value (center points) and limits with which the samples could be manufactured [7].

Table 1. Factors and levels selected for the main experiments.

Level	**Factors**				
	A [s]	***B [%]***	***C [bar and cm/s]***	***D [s]***	***E [mbar]***
1 (-1)	80	3.4 (100%)	18.4 (100%)	7.2	10
2 (+1)	90	4.0 (85%)	21.6 (85%)	9.0	15

The experiment consisted of 17 treatment combinations according to the planning Fractional Factorial Design $2_V^{5-1}2$ with one center point [35]. For each treatment, two (2) runs were performed in a random sequence being that, 01 sample and 01 repetitions were manufactured in the same run, totaling 68 runs (4 samples per treatment combinations). The 68 samples of PS were produced and then cooled completely in an air-conditioned room at 22ºC with 60 % humidity. Then, to quantify the linear and geometric errors of the parts, the inspection methods described in the previous chapter were applied using CMM 3D and the results found were tabulated. Table 2. shows the types of deviations and respective mean values of 34 samples. It is observed that the data *vs.* type of deviation are well distributed, except for only two (02) points for *DEV 03*, respectively, samples 26 and 31 (outliers in data: values below $Q1 - 1.5 \times IQR$ or above $Q3 + 1.5 \times IQR$, respectively for *DEV* 03 the interval from -0.415 to 1.345. Where: *IQR* is the Interquartile Range, *Q1* first quartile and *Q2* is third quartile.).

Table 2. Experimental main results.

Order test	Responses (values)			
	DEV 01[a] ***[mm]***	***DEV 02***[a] ***[mm]***	***DEV 03***[a] ***[º]***	***DEV 04***[a] ***[mm]***
1	-0.122	-0.238	0.358	0.269
2	-0.628	-0.217	1.050	0.476
3	-0.464	-0.305	0.238	0.113
4	-0.463	-0.248	0.213	0.204
5	-0.467	-0.231	0.294	0.082
6	-1.565	-0.160	0.910	0.539
7	-0.229	-0.294	0.270	0.218
8	-0.490	-0.288	0.204	0.302
9	-0.323	-0.198	0.418	0.410
10	-0.943	-0.277	0.281	0.398
11	-0.463	-0.383	0.288	0.107
12	-1.000	-0.241	0.451	0.416
13	-0.989	-0.217	0.301	0.128
14	-0.328	-0.377	0.265	0.239
15	-0.597	-0.423	0.110	0.235
16	-0.492	-0.473	0.279	0.292
17	-0.563	-0.227	1.150	0.476
18	-1.431	-0.254	0.955	0.462
19	-0.645	-0.328	0.502	0.433
20	-0.576	-0.460	0.213	0.232
21	-1.234	-0.245	0.805	0.442
22	-0.794	-0.301	0.457	0.322
23	-1.022	-0.310	1.106	0.442
24	-0.639	-0.366	0.531	0.242
25	-0.757	-0.297	0.230	0.043
26	-1.306	-0.248	1.551[b]	0.628
27	-0.785	-0.317	0.505	0.285
28	-0.419	-0.358	0.265	0.223
29	-0.692	-0.407	0.238	0.181
30	-0.792	-0.466	0.213	0.164
31	-1.294	-0.279	1.532[b]	0.642
32	-0.824	-0.455	0.062	0.221
33	-1.096	-0.288	0.320	0.477
34	-0.832	-0.430	0.736	0.231

[a] Mean average value for 02 pieces; [b] Outlier

The ANOVA assumptions were verified and validated using analysis of normality assumption (Anderson-Darling), assumption of homogeneity of variances (plot of residuals versus fitted values) and independence assumption (plot of residuals in time sequence) processed by MiniTab 16® software, none showed abnormal values. The ANOVA results for deviations versus the factors studied are summarized in Table 3., or *F*-test table, with a confidence level of 95 % ($\alpha = 0.05$), and the critical test value for the *F* distribution $f_{0,05;1;17} = 4.45$.

Table 3. ANOVA summary table, results for the deviation analysis vs. factors in main experiments.

Factor	**Responses**							
	DEV 01		***DEV 02***		***DEV 03***		***DEV 04***	
	$F_{(0)}$	*p-valor*	$F_{(0)}$	*p-valor*	$F_{(0)}$	*p-valor*	$F_{(0)}$	*p-valor*
A	10.2	0.005	89.7	0.000	77.72	0.000	0.42	0.542
B	37.0	0.000	82.6	0.000	86.23	0.000	22.5	0.000
C	0.30	0.592	4.6	0.046	8.93	0.008	1.44	0.246
D	0.98	0.336	6.43	0.021	56.03	0.000	0.02	0.899
E	0.08	0.776	4.50	0.049	1.36	0.259	0.34	0.567
A*B	1.92	0.184	52.1	0.000	43.81	0.000	3.91	0.065
A*C	4.86	0.042	2.73	0.117	6.24	0.023	0.27	0.612
A*D	6.13	0.024	1.29	0.271	5.58	0.030	2.27	0.150
A*E	1.87	0.189	2.63	0.123	2.04	0.171	0.29	0.596
B*C	5.66	0.029	0.01	0.943	0.42	0.525	5.04	0.038
B*D	0.05	0.833	6.98	0.017	30.14	0.000	0.12	0.739
B*E	0.63	0.438	0.08	0.783	2.45	0.136	0.89	0.359
C*D	0.03	0.867	1.81	0.196	1.54	0.232	0.14	0.709
C*E	3.02	0.100	2.23	0.154	29.55	0.000	1.12	0.305
D*E	4.89	0.041	0.37	0.550	0.25	0.817	1.38	0.257

All: $S = 0.0648608$; $R^2 = 70.26\%$ and; $R^2_{(adj)} = 42.28\%$.

P-Value by Anderson-Darling test: *DEV 01* = 0.235, *DEV 02* = 0.100, *DEV 03* = 0.057 and *DEV 04* = 0.123.

From Table 3., it is concluded that the critical manufacturing parameters are B and A, and also for *DEV 01*. For *DEV 02*, the factor B stands out as significant; for *DEV 03*, all factors are significant; and in *DEV 04*, in sequence, the most significant factors are B, A, and D. Also, at least 01 factor, or its interaction effect, is significant for one error type analyzed (except the factor E for *Dev 4*).

It is graphically presented in Fig.4.: the interactions of the factors vs. the errors using ANOVA. The analysis of the graphs confirms that the critical process factors are A and B, and that correlation between them is predominantly inverse and not proportional. Also, for all deviations, there is evidence of interaction between all the factors, and we see that there is no direct relationship between the levels (-1 and +1) of the factors and lower value of deviations.

Finally, it can be concluded, by this data analysis, that the modification of factor levels cannot be studied in isolation for each type of deviation because the optimal levels are different for each deviation, for example: for *DEV 01* are the +1 levels of factors A, B and D combined with the -1 levels of factors C and E (+A, +B, -C, +D and -E); for *DEV* 02, the optimal selection would be + A, + B, - C, + D and + E; for DEV 03: +A, +B, +C, +D and + E; and to minimize DEV 04 are +A, -B, -C, -D, and 0 (center point level).

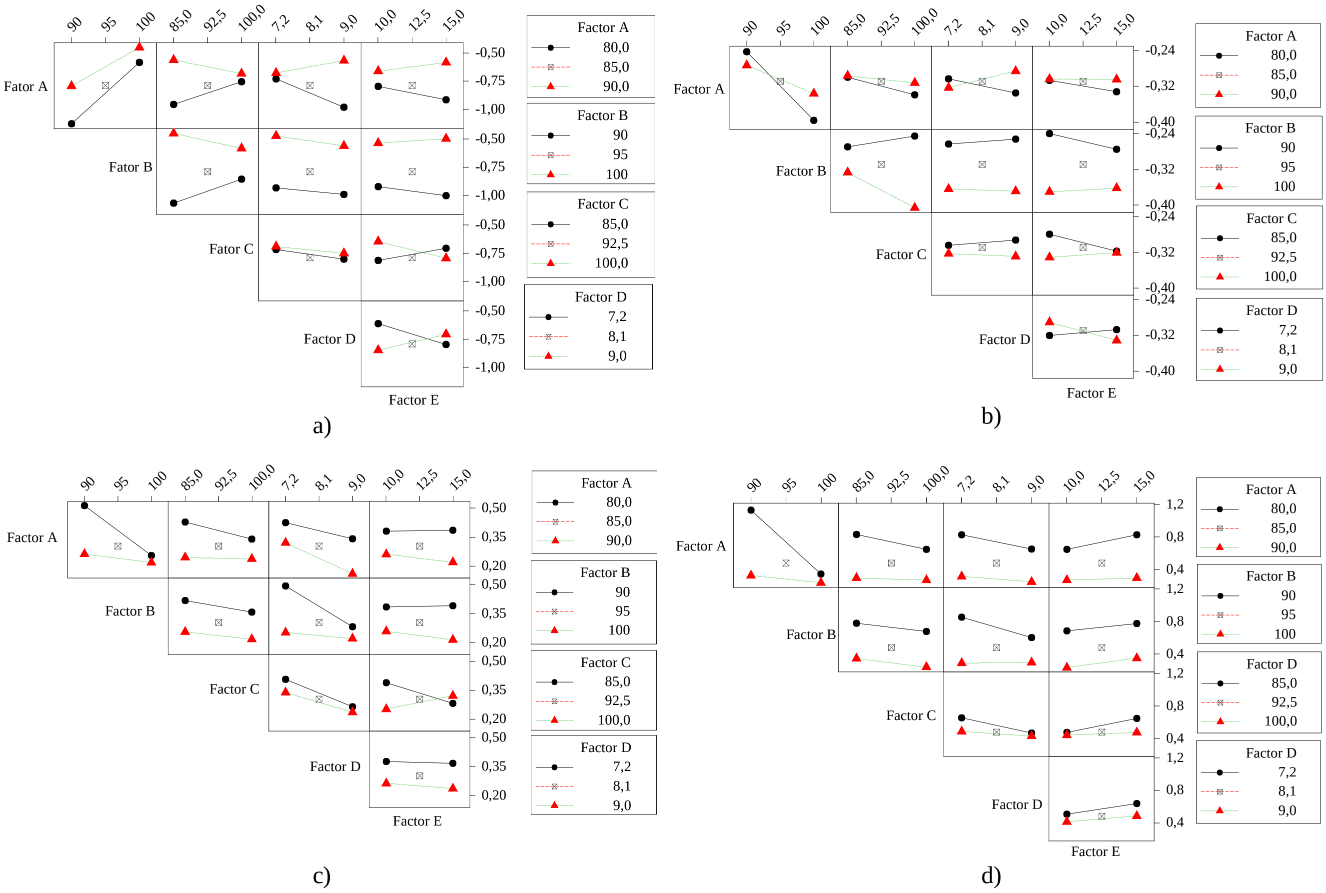

Fig.4. Interactions plot of factors: a) Interactions for *DEV 01*; b) Interactions for *DEV 02*; c) Interactions for *DEV 03*;d) Interactions for *DEV 04.*

3. Development of multiple response optimization models by multiple linear regression

First, with the data analysis developed, Multiple Linear Regression Models were developed for each type of response variable (error). So, for each one it was calculated: the constants β_0, the regression coefficients β_j, the cross-product terms of the input variables taken two to two $\beta_j x_k x_{k+1}$, and the random error term ε. Table 4. presents the calculated coefficients for each type of error being *MLR* model 01 for DEV 01, *MLR* model 02 for *DEV 02*, and so on, respectively. To evaluate the adequacy of these models to the data, we calculated the Coefficient of Determination (R^2), the Pearson Correlation Coefficient (*r*), the Mean Squared Error (*MSE*), and Mean Absolute Error (*MAE*), which are presented in the table. It is evident by the R2 value that capabilities of the models to predict the data are higher that 94 %, the r values above 0.97 indicate a very strong correlation between the response variables of regression models and *MSE* and *MAE*, which prove that mean values of prediction errors are less than 6 %.

The Multiple Response Optimization Models algorithms were developed and implemented using the *MLR* set as sub-models. For this purpose, the script sub-codes were developed in numerical analysis and programming software (MATLAB®), which uses *MLR*s to develop part of the *MRO* algorithm. The regression models developed for each type of error were programmed and the coefficients and constants of the estimates were coded and converted into external data files. The correlation of outputs of multiple response models were tested and evaluated, being that the *r* values found were lower than 0.76.

Also, to select the set of optimal values of factors by the *MRO* algorithms, a general objective function was developed. Equation (7) presents this O_j estimator value used to quantify a solution given a set of input factors.

$$O_j = \frac{1}{8}\sum_{i=1}^{4}\{(D_i)x\ Weight_i\} \qquad (7)$$

$$\text{for } D_i = \frac{MLR_{i,j}}{admissible\ error_i}$$

where *j* represents the *j-th* coefficient of performance for a (01) solution vector and i the deviation type, where i = 1, 2, 3, and 4 for the deviations *DEV* 01, *DEV* 02 *DEV* 03, and *DEV* 04. Consequently, they are pretested and restrict the desired ranges for all deviation of solution for: $0 \leq MLR_i < admissible\ error_i$. For each test the function O_j assigns numbers between 0 and 1, where $O_j = 1$ is a value completely undesirable and $O_j = 0$ is the optimal value and so, the function's internal search minimizes D values for the target values of $D_i = 0$. The values of the "admissible errors" for i = 1, 2,…4 were defined as | 0.6 mm |, | 2.1 mm |, | 1 mm |, | 0.72º | and the *i-th* weights adopted are: 2, 2, 3 and 1, respectively.

Table 4. Multiple linear regression models: Coefficients of the model parameters.

Parameter	**Model Type**			
	MLR for DEV 01	***MLR for DEV 01***	***MLR for DEV 01***	***MLR for DEV 01***
	Coefficient value			
Constant	-27. 949777290	9. 868354195	69. 065370830	27. 416482210
A	0.147198177	-0.127872101	-0.710060972	-0.208447796
B	0.309674115	-0.064336080	-0.713180139	-0.221828149
C	-0.166175729	-0.047886613	0.131432500	0.007012008
D	4. 219830150	0.274325867	-0.459804784	-1.188922512
E	-0.992314896	-0.005321494	-0.739571944	-0.250265384
A*B	-0.002028656	0.000906562	0.006966083	0.002162254
A*D	-0.017926562	0.001316982	0.008850926	0.004532616
A*E	0.007246438	0.001383035	0.002194667	-0.001543027
A*C	0.001335354	0.000165253	-0.001043778	-0.000311275
B*D	-0.019336285	-0.005722334	-0.000390741	0.001178064
B*E	-0.000626063	-0.000311248	0.005097667	0.003586834
B*C	0.000775354	0.000287991	0.000180222	-0.000340673
D* E	0.002764931	-0.001937292	0.014426852	0.004500225
D*C	-0.009421644	0.001797519	-0.005333642	0.006576914
E*C	0.004313208	-0.000717682	-0.000784556	-0.000218731
Center point adjustment constant	-0.049108594	0.003270649	-0.029714583	-0.007910518

R^2: *MLR* for *DEV 01* = 94,29%; *MLR* for *DEV 02* = 99,99%; *MLR* for *DEV 03* = 99,97%; *MLR* for *DEV 04* = 99,98% and; All models = 98.56%;
r: *MLR* for *DEV 02* = 0.971; *MLR* for *DEV 02* = 1.000; *MLR* for *DEV 03* = 1.000; *MLR* for *DEV 04* = 1.000 and; All models = 1.000;
MSE: *MLR* for *DEV 01* = 0,1950; *MLR* for *DEV 02* = 0,0; *MLR* for *DEV 03* = 0,0002; *MLR* for *DEV 04* = 0,0; all models = 0,0488;
MAE: *MLR* for *DEV 01* = 0,0484; *MLR* for *DEV 02* = 0,0016; *MLR* for *DEV 03* = 0,0149; *MLR* for *DEV 04* = 0,0040; all models = 0,0172.

Based on this pre-development, the first code of the algorithm, the *MRO* model 01, was written to find the *n-th* best solutions inside of the full factorial design. The algorithm is processed according to this logic: first it discretizes the input values (factors) in the *j-th* possible solution and then it uses the data matrix and the sub-code developed with the *MLR* model to calculate the deviations of a *j-th* prediction. Then, by means of (7) the values of the general objective functions are calculated and, finally, compared, ranked and written in descending order are the *n-th* possible solutions with the respective input values. Thus, it searches for the minimum value of the general solution vector (minimizes O_j). This procedure was developed and implemented using MATLAB® software.

Table 5. presents the 05 best results. From the table it is evident that only one (01) set of factors/parameters is presented as the best solution to minimize deviations, having a value of O_j equal to 0.18, and factors A = 90 s, B = 100 %, C = 100 bar, D = 7.2 s and E = 15 mbar.

A second attempt was made to find optimal solutions. For this the solution space of the input variables was expanded to values beyond those used in the main experimental procedure, and also, smaller limits were defined for the discretization. For this new search the *MRO* model 02 Algorithm was programmed. Table 6. presents the 05 best results.

As shown in Table 6., the *MRO* model 02 algorithm can predict other *n-th* configurations of input variables, which minimize the O_j estimator. We see that several configurations have the same value of *Oj* and very close values, which were already predicted when dealing with a problem with multiple solution spaces. However, analyzing Fig.4., we see that in general, for the set of deviations, factor “A” has better results in levels ≥ 85, factor “B” in levels ≥ 95, since factor “C” improves next at levels ≤ 92.5, factor “D” at mean levels ≥ 8.1, and factor “E” close to level ≥ 12.5. From this follows that the first solution from Table 6. is the most appropriate solution to the problem.

Table 5. Summary of the 05 best minimum of O_j value for the 1st variation of the optimization algorithm.

O_j value	**Factor**				
	A (s)	***B (%)***	***C (bar)***	***D (s)***	***E (mbar)***
0.18	90	100	100	7. 2	15.0
0.25	90	100	100	7. 2	12. 5
0.27	90	100	93	7. 2	15.0
0.27	81	100	85	9.0	10.0
0.28	90	100	100	8. 1	15.0

Table 6. Summary of the 05 best minimum of O_j value for the 2nd variation of the optimization algorithm.

O_j value	**Factor**				
	A (s)	***B (%)***	***C (bar)***	***D (s)***	***E (mbar)***
0.05	94. 5	97. 5	92. 5	6. 3	15
0.06	94. 5	95	96. 25	6. 3	15
0.06	94. 5	95	100	7. 2	15
0.08	81	105	92. 5	7. 2	7. 5
0.08	78. 75	105	92. 5	7. 2	7. 5

3.1. Validation tests

New experimental tests were performed to validate the MRO algorithms and to test their efficiencies in predicting the multiple errors in two different search conditions developed. For the development of these validation tests, two test sequences were performed, using five samples of each type, according to the selection of factors (parameters) developed, respectively. Also, the same experimental conditions, infrastructure and material were preserved. Afterwards, the samples were inspected, adopting the same procedures already described and the errors previously calculated. Table 7. and Table 8. show the errors measured in the pieces for each test model.

Table 7. Deviations: samples of 1st *MRO* model.

Sample	**Results**			
	DEV 1 (mm)	DEV 2 (mm)	DEV 3 (º)	DEV 4 (mm)
1st	-0.269	0.240	0.201	-0.521
2nd	-0.367	0.215	0.308	-0.536
3nd	-0.113	0.108	0.259	-0.423
4nd	-0.084	0.246	0.114	-0.460
5nd	-0.272	0.142	0.169	-0.520

Table 8. Deviations: samples of 2nd *MRO* model.

Sample	**Results**			
	DEV 1 (mm)	DEV 2 (mm)	DEV 3 (º)	DEV 4 (mm)
1st	-0.138	0.260	0.154	-0.308
2nd	-0.084	0.217	0.221	-0.287
3nd	-0.092	0.194	0.115	-0.406
4nd	-0.122	0.164	0.129	-0.342
5nd	-0.302	0.107	0.106	-0.339

As evidenced by the values of the tables, the tests produced parts within the tolerance limits defined in this study; even the lower and upper limits of the deviations were at acceptable levels. The values of the errors are shown in Fig.5. For comparative purposes, the data of the best performing test pair (01 and 11) are shown with A = 90 s, B = 100 %, C = 90 Bar, D = 7,2 s and D = 10 mBar.

As shown in Fig.5., even at different levels, all the deviations follow the same trend, independently of the optimal configurations of the models. Also, we see that, on average, the results of the *MRO* model 02 samples present errors in smaller values when compared with the parts produced with the parameters of the *MRO* model 01, and in general, a significant improvement when compared with the best samples of the experimental test.

To compare the efficiency of the predictions, Table 9. and Table 10. present the results of the expected value (or mean) of the deviations for samples in the validation tests, at the 95 % confidence interval (IC) on the mean (α = 0.05). The predictions of the models and also the results of the best samples in the main experimental tests, samples 01 and 11, are shown (Table 2.).

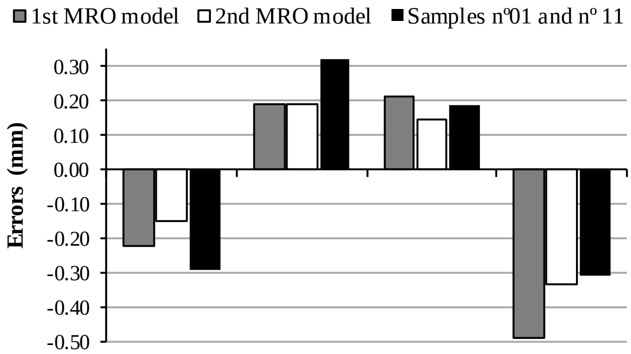

Fig.5. Comparison of mean value of the errors in the samples.

Table 9. General comparative of results for the 1st *MRO* model.

Error type	**Samples of validation**			***MRO* model 01**	**Samples nº01 and nº 11**
	Mean	***95% CI***		***predicted***	***Mean***
DEV 01	-0.221	-0.117	-0.325	-0.075	-0.293
DEV 02	0.190	0.136	0.244	0.273	0.323
DEV 03	0.210	0.144	0.276	0.084	0.188
DEV 04	-0.492	-0.449	-0.534	-0.297	-0.310
O_j	0.26	0.18	0.34	0.18	0.31

Table 10. General comparative of results for the 2nd *MRO* model.

Error type	**Samples of validation**			***MRO* model 01**	**Samples nº01 and nº 11**
	Mean	***95% CI***		***predicted***	***Mean***
DEV 01	-0.148	-0.070	-0.226	-0.006	-0.293
DEV 02	0.188	0.138	0.238	0.029	0.323
DEV 03	0.145	0.104	0.185	0.055	0.188
DEV 04	-0.336	-0.297	-0.376	-0.230	-0.310
O_j	0.20	0.13	0.26	0.05	0.31

Finally, based on these results, we conclude that the validation tests produced samples within the tolerance limits defined in this study with a significant improvement in product quality.

4. DISCUSSION AND CONCLUSIONS

The proposed procedure can be considered a new method to mutually model the manufacturing parameters and to predict and minimize dimensional and geometric errors in products during the vacuum thermoforming process, using a small number of tests in a laboratory.

As already presented by other authors [7], [8], [11], [30], [26], the simultaneous analysis of parameters and errors of products does not allow us to select a single set of optimal values. This is because different levels of one factor could be optimal levels for different response variables (e.g., factor E). Consequently, it is necessary to use a multi-objective optimization technique to find the set of optimal levels for the problem.

The analysis of the interaction between product errors and parameters of manufacture presented us new information, such as:

- The parameters of heating influence all types of geometrical and dimensional errors in a more significant way, considering all their different levels.
- In addition to mold characteristics [15], [36] the parameters of mold can influence geometric errors;
- The interaction of two factors can influence the deviations of the product by reducing the value of dimensional errors, but a significant increase of geometric errors can occur;
- The manufacturing parameters interact simultaneously with the errors in a non-linear and non-proportional model;

  In the analysis of the deviation value as a function of the variation of factor levels, we can conclude that:
- The factors “Heating Time” and “Heating Power” at high levels result in smaller dimensional deviations of height and lateral angles;
- In relation to the reduction of deviations the parameters “Vacuum Time” and “Vacuum Pressure” for some deviations have direct correlation and to others, inversely, “Vacuum Time” and “Mold Pressure” have direct correlation in all deviations;
- The parameter “heating time” in value equal to or greater than 90 seconds, produces smaller deviation values.
- In general, among the analyzed factors, the “Mold Pressure” has the lowest ratio rate in its levels and, for the “*DEV 02*”, the factors have an inverse correlation behavior in relation to the other deviations.

As for the *MLR* models, it can be concluded from the R^2 values that they are valid to represent the sample data and that this technique is valid to model errors in this process.

From Table 9. and Table 10., we conclude that within the confidence interval, the implemented models have indeed been able to find new improved solutions and have a significant gain in the overall reduction of errors of validation test and of value of objective function.

The *MRO* models have been able to find a set of *n-th* possible solutions that altogether minimize errors and these solutions are in values outside the limits of test of the main experiment.

In addition, by the analysis of Fig.5. and Table 9. and Table 10., we conclude that the minimization solution found by the optimization of model 02 is the best configuration of factor levels for the problem.

However, as shown in Table 10., there were failures in the predicted values in the new range of values [7], and from this we conclude that the solutions of optimization models for the errors are neither linear nor curved in the surface model. Thus, when using a linear model (*MLR*) with center point to model an (01) error, we obtain a representative model in relation to slope and direction [27], but this model is not able to represent all the local minima and the global minimum of a solution surface in this problem.

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