SCHEDULING PRODUCTION ORDERS,
TAKING INTO ACCOUNT DELAYS AND WASTE

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Abstract
The article addresses the problem of determining the sequence of entering orders for production in a flexible manufacturing system implementing technological operations of cutting sheet metal. Adopting a specific ranking of production orders gives rise to the vector of delays and waste in the form of incompletely used sheets. A new method was postulated for determining the optimal sequence of orders in terms of two criteria: the total cost of delays and the amount of production waste. The examples illustrate the advantages of the proposed method compared with the popular heuristic principles.

Keywords
flexible manufacturing systems, production control, scheduling algorithms.

Introduction

According to the current paradigm of agile manufacturing, the most important task for manufacturing organizations is to meet customer expectations by ensuring timely execution while maximizing profit [1–4]. This task can be accomplished by fully integrating management, planning and control subsystems. A practical solution to such requirements is a flexible manufacturing system, allowing for the manufacture of products in an automatic cycle while maintaining the possibility of rapid adjustment to changing customer needs [3, 4]. While analyzing the operation of flexible production systems, it should be noted that the effective work decisively depends on the proper operation of the control subsystem and more specifically on the control algorithms implemented in the subsystem. It is thanks to these algorithms, client orders are scheduled in the correct order, items from warehouses are taken, transport trucks are controlled, and control and diagnostics operations are performed [6, 7]. One of the most important tasks solved by the algorithms controlling the operation of flexible manufacturing systems is to determine the order of tasks provided for production (Scheduling problem) [8, 9]. An analysis of the operation of the post-production systems leads to the conclusion that the order of placing tasks affects the broadly defined efficiency of the whole production system. This efficiency is understood as ensuring both customer satisfaction by minimizing delay times of the orders executed, and taking into account economic and environmental aspects of the production process. One of the most important parameters affecting the efficiency and environmental aspects of the production process is the amount of waste [10].

The problems associated with the development of efficient algorithms to control the operation of flexible manufacturing systems are subject to research by many scientific papers. Two concepts are often mentioned: dynamic scheduling and reactive scheduling [11].

Dynamic scheduling is based on the successive selection of orders entered into production based on
the current information about ongoing production processes \cite{11,12}. This information may include, e.g. the time of completion of processing for each individual machine, the stock in station-side storages, tool changing times, equipment failures, delays in delivery of material and the results of interoperative control. Due to the very strict time limits, heuristic method and simple priority rules are most often used in this case \cite{11}.

The idea of reactive scheduling is to prepare the entire production schedule for stationary conditions and then modify it in the event of unforeseen events \cite{11,13}.

Particular interest is invested in developments taking into account the need to obtain acceptable solutions in an actual industrial environment. In \cite{3} a methodology for real-time design of production schedules with the use of priority rules and simulation studies has been presented. One may also notice a tendency to include more than one criterion, which allows obtaining solutions more suited to conditions prevailing in production systems \cite{7,14,15}.

In practice, production planning often involves priority rules due to their simple structure and low computational complexity \cite{4,12}. One can mention such rules as: a rule scheduling orders according to their date of entry in the system (FIFO), a rule scheduling orders by the date of their execution (EDD), a rule scheduling orders according to the shortest processing time (SPT), a rule scheduling orders according to the total time of operations left to perform (LWR), a rule scheduling orders by their assigned delay penalties (DDP).

In terms of the NP-hard nature of the problems associated with scheduling production orders often, the use of solutions applying tabu search approach, genetic, immune algorithms and neural networks is proposed \cite{5,16,17}. Due to the time limits, the methods applicable in this case do not include such exact methods as branch-and-bound (B&B) or integer linear programming (ILP). On the other hand, algorithms based on evolutionary methods do not guarantee reaching optimal solutions and therefore may prevent the full utilization of a system’s production capacity \cite{18}. Therefore, the application taboo search algorithms, which allow receiving the optimal solutions in acceptable time, may be particularly promising.

The article presents an algorithm which allows finding \(\varepsilon\)-optimal solutions for scheduling production orders in a flexible manufacturing system implementing technological operations of cutting sheet metal. An \(\varepsilon\)-optimal solution in this case stands for a solution that is not worse than the optimal one by more than a predetermined value \(\varepsilon\) (for \(\varepsilon = 0\), an \(\varepsilon\)-optimal solution is optimal). Adopting a specific ranking of production orders causes the appearance of the delay vector and waste in the form of incompletely used sheets. A new method to determine the optimal sequence of orders has been proposed in terms of two criteria: the total cost of delays and the amount of production waste. The examples illustrate the advantages of the proposed method compared with the simple heuristic rules.

**Description of the problem of scheduling production orders**

The technological process carried out in an automated, flexible manufacturing system, which implements cutting sheet metal components has been shown in Fig. 1. The most important element of the analyzed system is the M1 jet cutting machine and the automated M\textsubscript{in} racking sheet store, which stores various sheets provided as input material to the production process. Production orders are passed by the client to the parent planning subsystem via the Internet, and are then automatically scheduled and subsequently transferred for implementation. After processing, the executed orders are stored in the M\textsubscript{out} output store. In terms of technical limitations associated with the total automation of the process of generating the code for CNC jet cutting, it has been assumed that the input store will only hold full sheets of metal with standard dimensions 1200 \(\times\) 800 mm. Partly used sheets are treated as waste, and their possible further use takes place outside the analyzed process.

Based on preliminary simulation research, it can be concluded that the sequence in which orders are transmitted to the production system largely affects not only the completion date of the various production orders but also the volume of waste sheets used in the process.

The main task of the process of scheduling production orders is determining their sequence of execution in such a way as to prevent delays in their im-
Scheduling production orders, taking into account the cost of delays and total waste

This section proposes a method to determine the sequence of production orders, taking into account delay times of orders and total waste. It consists of four main stages. In a first stage, the base sequence is determined, which gives the smallest maximum delays per job. In the second stage, the sequence of orders giving the minimum total delay is determined. In the third stage, for a fixed \( \varepsilon \), the \( \varepsilon \)-optimal sequences are determined, i.e. those with the sum of delays no greater than the solution with the minimum sum of the delays plus \( \varepsilon \). In the fourth step, \( \varepsilon \)-optimal solutions are searched through for a schedule capable of producing as little waste as possible.

It is assumed that for a given list of orders \( \{z_1, z_2, \ldots, z_n\} \), the following information is available:
- \( t_0(i) \) – processing time of the order \( z_i \),
- \( t_d(i) \) – deadline for order \( z_i \),
- \( ab(i) \) – area required to cut the job.

Let \( S_o \) mean the total processing time of a given list of \( n \) orders:

\[
S_o = \sum_{i=1}^{n} t_0(i).
\] (1)

One of the simplest methods is the EDD method, where the orders are sorted by increasing deadline for completion \( t_d \) (from the one with the earliest deadline to the one with the latest deadline).

Maximum delays for a single order in the sequence obtained by this method are not greater than in all the other sequences. Unfortunately, this sequence does not necessarily give the smallest sum of delays (see example Table 2).

This section proposes a method that ensures appointing \( \varepsilon \)-optimal sequences of orders in terms of the minimum amount of delay, i.e. the optimal sequence giving the minimum amount of delays and sequences worse than optimal by no more than the set \( \varepsilon \) value. This method is a generalization of the \( MOpt \) method proposed by the authors in [19], i.e. for \( \varepsilon = 0 \) the presented method is used to determine optimal solutions, such as in \( MOpt \). Determination has been divided into three stages. In the first step, a sequence is obtained according to the one obtained using the EDD method obtained. In the second step, the optimal sequences are obtained in terms of the minimum total delay, and in the third - \( \varepsilon \)-optimal sequences. Let us denote the presented method an \( MOpt-\varepsilon \).
MOpt-$\varepsilon$ method:

Determination of the solutions is in the form of a tree.

Stage I:

At the beginning, in block 1 (the root of the tree), for each order a number is determined

$$p(i) = S_o - t_s(i),$$

which determines the delay in the case, when the order $z_i$ is done as a last. If $p(i) \leq 0$ for every $i = 1, \ldots, n$, this means that every one of all possible sequences gives total delay equal 0 (end). In the case where not every $p(i)$ is less than or equal 0, the order $z_j$ is first selected, where $p(j)$ is the smallest, and placed at the end of the queue. This order has a predetermined time required to process the remaining orders

$$S'_o := S_o - t_s(j)$$

and the delay that arises as a result of placing the order at the end

$$o_p = \max \{0, p(j)\}.$$  \hspace{1cm} (4)

Then, in block 2 (the successor of block 1 in the solution tree), the same is performed as in the root of the tree, although skipping the order that has already been put at the end of the queue. The currently selected order $z_k$ is put into the queue as the second last, updating the time $S'_o$ ($S'_o := S'_o - t_s(k)$) and the total delay $o_p$ ($o_p := o_p + \max \{0, p(k)\}$).

Then, the operations performed in block 2 are repeated until reaching back $n$ (leaf in the tree), from which the order is put at the beginning of the queue. This way the base branch in the tree is obtained (with preset sequence of all orders), for which the sequence of orders is in line with the one acquired using the EDD method. If the sum of delays for this branch $Sop = 0$, then the resulting sequence is optimal in terms of total delay.

Stage II:

If the base branch $Sop > 0$, then since block 1 it is verified whether selecting the next order (in terms of the minimum $p(i)$), would result in exceeding the sum of delays $Sop$ from the base solution. If so, another branch will not be formed. If not, the selected order is inserted into the appropriate place in the queue in the next block (the successor of block 1), etc.

If another leaf in the tree is reached (with a preset sequence of all orders), the sum of delays in this sequence is not greater than the one in the base solution. The $Sop$ is therefore updated and further branches are verified until no more can be developed ($Sop$ is exceeded). Finally, the optimal solutions are found in the leaves with the lowest $Sop$ possible; let us mark them as $Sop_m$.

Stage III:

If $\varepsilon > 0$, it is verified in blocks, which are not leaves, whether selecting the next order (in terms of the minimum $p(i)$ value) will not result in exceeding $Sop_m + \varepsilon$. If so, this branch is not developed any further. If not, the selected task is set in the next block in the appropriate place relative to the end of the queue, etc.

If we reach new leaves of the tree, the total delay corresponding to the sequences of these leaves are not greater than $Sop_m + \varepsilon$. Finally, $\varepsilon$-optimal solutions are found in the leaves, where $Sop \leq Sop_m + \varepsilon$.

Using the algorithm proposed below, one can determine the total waste and the number of used sheets for $\varepsilon$-optimal sequences determined using the MOpt-$\varepsilon$ method.

Stage IV:

The algorithm for determining total waste and the number of sheets used.

Designations:

- $ob$ – area used for the cut sheet,
- $reserve$ – free area in the cut sheet, $(ob + reserve = 100\%)$
- $waste$ – total waste from cut sheets,
- $sheets$ – number of sheets used,
- $n$ – number of orders,
- $obi = ob(U(i))$ – area required to cut the $i$-th order in the given sequence (in %), $i = 1, \ldots, n$.

Algorithm:

```plaintext
ob := 0; reserve := 100%; waste := 0; sheets := 1; for i:=1 to n do
    obi := ob(U(i));
    if obi = reserve then
        ob := ob + obi;
        reserve := reserve - obi;
    else
        waste := waste + reserve;
        sheets := sheets +1;
        ob := obi;
        reserve := 100% - obi;
    end
    waste := waste + reserve;
end
```

Example

Data for the example is presented in Table 1 This example assumes four orders, for which the processing time $t_s$ has been defined, as well as the scheduled deadline $t_1$, slack time of performance $z$ ($z = t_o - t_s$) and the area of cut sheet $o_b$ required to carry out the order relative to the overall surface of the sheet. It was assumed that individual orders consist of single
items, which means that each order needs to be cut from a single metal sheet.

<table>
<thead>
<tr>
<th>Orders</th>
<th>Processing time ( t_w ) [min]</th>
<th>Deadline ( t_t ) [min]</th>
<th>Slack time ( z ) [min]</th>
<th>Sheet area ( o_k ) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_1 )</td>
<td>10</td>
<td>150</td>
<td>140</td>
<td>50</td>
</tr>
<tr>
<td>( z_2 )</td>
<td>20</td>
<td>30</td>
<td>10</td>
<td>40</td>
</tr>
<tr>
<td>( z_3 )</td>
<td>100</td>
<td>110</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>( z_4 )</td>
<td>50</td>
<td>60</td>
<td>10</td>
<td>70</td>
</tr>
</tbody>
</table>

To determine \( \varepsilon \)-optimal sequences, the MOpt-\( \varepsilon \) method was used, as presented in p. 3. In the first stage, the base sequence \( U_1 (z_2, z_4, z_3, z_1) \) was established, where the total delay was set at \( Sop = 100 \) min. This is a sequence ordered by increasing deadlines. Then, in the second stage, the optimal sequence has been determined in terms of total delay \( U_2 (z_2, z_4, z_1, z_3) \) with the sum of delays \( Sop_m = 80 \) min. In the third stage a tolerance of \( \varepsilon = 10 \) min has been assumed, where \( \varepsilon \) means the limit value, by which the \( \varepsilon \)-optimal solution may differ from the optimal one in terms of the total cost of delays. In this step, two more sequences were determined: \( U_3 \) and \( U_4 \), where \( Sop \leq Sop_m + \varepsilon \) (see Table 2). Finally, out of the four sequences established during the initial three stages, sequences \( U_2 \), \( U_3 \) and \( U_4 \) are \( \varepsilon \)-optimal solutions for a given \( \varepsilon \).

These three schedules were classified into the fourth stage, where total waste and the number of sheets used (last two columns in Table 2) was determined using the presented algorithm (Stage IV). Table 2 also gives the amount of waste and the number of sheets used in sequence \( U_1 \) (it is not \( \varepsilon \)-optimal for \( \varepsilon = 10 \) min). As it turns out, sequences \( U_3 \) and \( U_4 \) result in waste of about 20% and 2 used sheets. To implement a sequence with as little delay as possible \( U_2 \), 3 sheets are required, and the waste will be greater by a whole sheet compared to sequences \( U_3 \) and \( U_4 \). In the optimal solution \( U_2 \), the cutting process begins with the order \( z_2 \). Since the next order in the sequence is \( z_4 \), which required 70% of the sheet’s area, it was necessary to enter a new sheet, which resulted in 60% waste from the first sheet. After cutting the order \( z_4 \), it was necessary to enter a new sheet, which resulted in 30% waste. The next two orders \( z_1 \) and \( z_3 \) could be cut from a single sheet with 30% waste. In total, sequence \( U_2 \) therefore requires using three sheets and generates total waste of 120% of the total sheet area.

The improper sequence of orders may give rise to a lot more total delays than the optimal solution – even as much as 150 min, e.g. for \( U_5 (z_2, z_3, z_4, z_1) \) as well as much more waste than the best of \( \varepsilon \)-optimal solutions in terms of this criterion (for \( U_5 \), total waste is 1.2 sheets, as in the case of \( U_1 \) and \( U_2 \)).

### Summary

Choosing the order of the tasks executed significantly affects the efficiency of a flexible production system. Simple methods for determining the order of tasks, dependent only on the processing time (e.g. SPT method) or the deadline of the order (e.g. method EDD) do not guarantee an optimal solution due to the sum of delays. These methods also do not allow for assessment as to how the solution reached is worse than the optimum. At the cost of slightly more difficult implementation, while taking into account the processing times and deadlines, the method proposed in the article does not have these major flaws.

First of all the method proposed in the article allows determining the optimal sequence of orders in terms of total delays. The resulting waste is also an important factor in determining the sequence. The method thus also allows determining sequences, which are not optimal and result in total delays greater than the optimal solution, but remaining under the fixed time limit at most \( (\varepsilon \text{-optimal solutions}) \).

As shown in the presented examples, by taking into account the \( \varepsilon \)-optimal solutions in terms of the sum of delays, one does not lose solutions that are...
much better in terms of the other criterion - the amount of waste generated.

Taking into account the ε-optimal solutions is also justified by the fact that the cost of delays for an ε-optimal solution is often equal or slightly greater than those for the optimal solution. However, the cost related to the amount of resulting waste is important. Therefore, the proposed method allows obtaining a compromise between the cost of delays and the cost of waste.

In a further study, the authors plan to develop a method that would take into account the costs of delays instead of the delay times.

References