Jerzy Boryczka, Maria Stopa-Boryczka

A MATHEMATICAL MODEL OF POLAND'S CLIMATE

The purpose of this paper is to describe spatial-time changes in Poland's climate with mathematical formulae.

So far Poland's climate has been presented with isorithms of long-term values of individual meteorological element (Gumiński 1952, Romer 1949, Okołowicz 1968, Climatic Atlas of Poland 1973, Wiszniewski, Chelchowski 1975). And in works made by the Climatological Division of the Physico-Geographical Sciences at Warsaw University Poland's climate was described with empirical formulae, expressing dependence of meteorological elements on longitude, latitude and on height above sea-level (Stopa-Boryczka 1973, 1974, 1976, 1980, 1986, Boryczka 1977, 1984).

In the model $y = f(\varphi, \lambda, H)$ only most important meteorological elements are taken into consideration, namely air temperature and precipitation with utilization of data from the observation network of the Institute of Meteorology and Water Management.

1. APPROXIMATION OF FIELDS OF AIR TEMPERATURE AND PRECIPITATION. CONSTRUCTION OF OBJECTIVE MAPS OF ISOLINES

Fields of meteorological variables are quite well approximated with equations of hyperplanes of regression. The spatial distribution $y = f(\varphi, \lambda, H)$ of the most important meteorological elements in Poland is characterized by the following equations:

Air temperature

$$\begin{aligned} 1881-1960 \quad y &= -0.5126 \varphi - 0.1144 \lambda - 0.5901 H + 37.42, \quad n = 57 \\ 1931-1960 \quad y &= -0.4768 \varphi - 0.1169 \lambda - 0.5781 H + 35.67, \quad n = 57 \\ 1951-1960 \quad y &= -0.4767 \varphi - 0.0091 \lambda - 0.5625 H + 35.05, \quad n = 57 \\ 1951-1960 \quad y &= -0.3297 \varphi - 0.0810 \lambda - 0.4903 H + 27.07, \quad n = 522 \end{aligned}$$

Precipitation

$$\begin{aligned} 1951-1970 \quad y &= -4.230 \varphi - 7.639 \lambda + 74.29 H + 863.7, \quad n = 757 \\ 1951-1960 \quad y &= -4.434 \varphi - 5.327 \lambda + 60.79 H + 374.6, \quad n = 60 \end{aligned}$$

where: φ — latitude, λ — longitude, H — height above the sea level — hm., n — number of meteorological stations.

Better than equations of hyperplanes of regression are hypersurfaces of regression of the second step — the second term of expansion of field into Taylor series (Boryczka, Stopa-Boryczka, Wawer 1984). The spatial distribution of mean yearly air temperature and total of precipitation are presented by hypersurfaces of regression:

Air temperature

1951—1960, $n = 322$

$$y = -5.082 \varphi + 0.147 \lambda - 4.228 H + 0.04427 \varphi^2 + \\ + 0.0008279 \varphi \lambda + 0.06507 \varphi H - 0.0082 \lambda^2 + 0.01925 \lambda H + \\ + 0.007299 H^2 + 152.3$$

Precipitation

1951—1970, $n = 757$

$$y = -2245 \varphi - 16.17 \lambda + 892.3 H + 21.75 \varphi^2 + \\ - 0.4503 \varphi \lambda - 13.91 \varphi H + 1.111 \lambda^2 - 6.704 \lambda H + \\ - 0.9838 H^2 + 58760$$

An optimum function approximating mean fields of air temperature in Poland (data from 322 stations) is a polynomial of the fourth step in relation to φ , λ , H — four terms of the Taylor series.

$$y = -0.000001 \varphi - 0.001649 \lambda + 0.000053 H - 0.000085 \varphi^2 \\ - 0.000201 \varphi \lambda - 0.000129 \varphi H - 0.000437 \lambda^2 - 0.000494 \lambda H \\ + 0.000317 H^2 - 0.001293 \varphi^3 + 0.002561 \varphi^2 \lambda + 0.002381 \varphi^2 H \\ - 0.001321 \varphi \lambda^2 - 0.010130 \varphi \lambda H - 0.007532 \varphi H^2 - 0.001912 \lambda^3 \\ - 0.007912 \lambda^2 H - 0.003395 \lambda H^2 - 0.0013170 H^3 + 0.000042 \varphi^4 \\ - 0.000299 \varphi^3 \lambda + 0.000053 \varphi^3 H + 0.001091 \varphi^2 \lambda^2 - 0.000572 \varphi^2 \lambda H \\ + 0.000064 \varphi^2 H^2 - 0.001949 \varphi \lambda^3 + 0.002043 \varphi \lambda^2 H + 0.0003919 \varphi \lambda H^2 \\ + 0.00080 \varphi H^3 + 0.001368 \lambda^4 - 0.001593 \lambda^3 H - 0.000001 \lambda^2 H^2 \\ - 0.000308 \lambda H^3 - 0.000033 H^4 + 39.77$$

And the optimum function approximating fields of total of precipitation in Poland (data from 757 stations) is a polynomial of the third step in relation to φ , λ , H — three terms of the Taylor series.

$$y = -0.8886 \varphi - 55.23 \lambda + 59.83 H - 20.32 \varphi^2 +$$

$$\begin{aligned}
 & - 24.95 \varphi \lambda + 64.34 \varphi H + 45.67 \lambda^2 - 174.8 \lambda H + \\
 & + 226.4 H^2 + 0.2587 \varphi^3 + 0.3437 \varphi^2 \lambda - 1.482 \varphi^2 H + \\
 & - 0.5215 \varphi \lambda^2 + 4.790 \lambda H - 4.167 \varphi H^2 - 0.2590 \lambda^3 + \\
 & - 2.002 \lambda^2 H - 1.001 \lambda H^2 - 0.06838 H^3 + 22.200
 \end{aligned}$$

The measure of accuracy of approximation of temperature field are: mean square difference ε between values measured at n stations and calculated from formulae, determination coefficient and standard error δ :

Number of terms of Taylor series

	ε	R^2	δ
$\xi = 1$ hypersurface of regression	0.370°C	91.70%	0.373°C
$\xi = 2$ hypersurface of second step	0.328	93.5	0.333
$\xi = 3$ hypersurface of third step	0.284	95.1	0.294
$\xi = 4$ hypersurface of fourth step	0.273	95.5	0.289
$\xi = 5$ hypersurface of fifth step	0.276	95.4	0.304

Thus, the mean square difference between results of measurements of temperature and the values calculated from the equation of hypersurface of regression of the fourth step has the lowest value $\varepsilon = 0.273^\circ\text{C}$. This equation also has the lowest standard error $\delta = 0.289^\circ\text{C}$ and the highest determination coefficient $R^2 = 95.5\%$.

Accuracy of approximation of fields of precipitation totals is the highest in the case of a polynomial of the third step.

Number of terms of Taylor series

	ε	R^2	δ
$\xi = 1$	88.07	77.60%	88.30 mm
$\xi = 2$	74.81	83.9	75.31
$\xi = 3$	72.44	84.8	73.42

Since the distribution of differences $\varepsilon_i = y_i - f(\varphi_i, \lambda_i, H_i)$ between measured and theoretical values is similar to Gaussian distribution:

	Air temperature $\varepsilon = 0.273^\circ\text{C}$	Precipitation $\varepsilon = 72 \text{ mm}$
$\varepsilon_i/\varepsilon$		
< -3	1	7

—3 — —2	10	13
—2 — —1	26	60
—1 — 0	118	295
0 — 1	128	307
1 — 2	26	47
2 — 3	11	19
> 3	2	9

then it is known with probability of 97.7% that the estimated values of temperature and precipitation totals in any place are in the confidence interval $f(\varphi, \lambda, H) \pm 3\delta$.

The formulae can be used to determine mean annual values of temperature and annual amount of precipitation in any place in Poland — also in those places where measurements have never been taken.

For instance, differences are given between measured and calculated values of temperature from the equation of hypersurface of regression of the fourth step in some selected places:

	φ	λ	H	y	$f(\varphi, \lambda, H)$	$y-f(\varphi, \lambda, H)$
Kołobrzeg	54.2	15.6	0.03	7.5	7.77	—0.27°C
Warszawa	52.2	21.0	1.08	7.8	7.68	0.12°C
Kasprowy Wierch	49.2	20.0	19.91	—0.8	—0.87	0.07°C

Also examples are given of the annual amount of precipitation estimated from the equation of hypersurface of the third step and measured in the same places

	y	$f(\varphi, \lambda, H)$	$y-f(\varphi, \lambda, H)$
Kołobrzeg	640	678	— 38 mm
Warszawa	497	525	— 28 mm
Kasprowy Wierch	1721	1757	— 36 mm

Such a recording of the field of air temperature of precipitation in Poland makes it possible to construct objective maps of isorithms. For this purpose a geographical network of meridians and parallels was made, dividing Poland's territory into squares with sides $\Delta\varphi = 0.2^\circ$ and $\Delta\lambda = 0.2^\circ$, that is rectangles measuring 22.2 km and 22.2 cos φ km. In 1260 nodes of the network from hypsometric maps of Poland heights above sea level were read (1 : 300 000 — for mountains, 1 : 500 000 — for lowlands, highlands) and then temperature values were calculated from

the formula. Thus an optimum map of isotherms for a year on the scale 1:1000 000 was prepared. A four-time increase of the number of points in relation to the number of stations, where temperature values were calculated, makes it possible to take into account thermal differentiation not only on regional scale, but also on local scale.

The advantage of this method is the fact that the account is taken of real — vertical and horizontal changes of temperature when drawing isorithms, which can be determined very easily once the approximating function is known. There is also a possibility to draw isorithms with a smaller scale, e.g. 1:500 000 through suitable condensation of the number of points (nods of the net). The proposed method can also be used in the case of smaller areas, e.g. north-eastern part of Poland, the mountains, highlands, lowlands and even regions and mesoregions.

2. GEOGRAPHICAL GRADIENTS OF TEMPERATURE AND PRECIPITATION — MAPS OF ISOGRADIENTS

Isogradients defined by Romer (1949) are lines connecting squares (meshed of the net) with the same number of crossings of isotherms, isohyets etc. Thus they are a relative measure of spatial differentiation of meteorological variables. They depend on the distance between isorithms and size of net „mesh”. Moreover, the field of such a defined gradient is scalar.

And fields of gradients $\frac{\partial f}{\partial \varphi}, \frac{\partial f}{\partial \lambda}, \frac{\partial f}{\partial H}$, defined through approximating functions $f(\varphi, \lambda, H)$ are vectorial. Components of the gradient — horizontal and hypsometric — indicate how large changes are of a given element along with growth of latitude by 1° and longitude by 1° as well as altitude by 100 m. Constant gradients: $\frac{\partial f}{\partial \varphi} = \text{const}, \frac{\partial f}{\partial \lambda} = \text{const}, \frac{\partial f}{\partial H} = \text{const}$, determined by the equation of hypersurface of regression, are at the same time a measure of influence of basic geographical factors.

Thus, air temperature falls towards the north of Poland by $0.33\text{---}0.51^\circ\text{C}/1^\circ\varphi$, drops towards the east by $0.08\text{---}0.11^\circ\text{C}/1^\circ\lambda$, and falls down along with growth of altitude by $0.49\text{---}0.59^\circ\text{C}/100\text{ m}$.

Horizontal gradients of precipitation in Poland are on the average: $-4.5\text{---}4.4\text{ mm}/1^\circ\varphi$, $-7.7\text{---}5.3\text{ mm}/1^\circ\lambda$, and the hypsometric gradient equals $60.8\text{---}74.2\text{ mm}/100\text{ m}$.

Horizontal and vertical gradients show dependence on the season of

the year. The range of their changes within a year is as follows: air temperature from -0.14 to $-0.67^{\circ}\text{C}/1^{\circ}\varphi$, -0.29 — $0.16^{\circ}\text{C}/1^{\circ}\lambda$, -0.38 — $-0.69^{\circ}\text{C}/100$ m, precipitation — -4.39 — 3.06 mm/ $1^{\circ}\lambda$, -1.50 — 0.50 mm/ $1^{\circ}\lambda$, 3.83 — 11.45 mm/100 m.

A peculiar feature of Poland's climate is change of the sign of the gradient of air temperature with respect to geographical longitude — minus in the cool half-year and plus in the warm half-year. This is a proof of transitional character of Poland's climate, that is with more thermal features of a sea climate in the west and continental climate in the east.

Moreover, the influence of latitude on temperature (drop towards the north) is more visible in summer months than in winter. Thus, the temperature gradient with regard to latitude is not constant but ranges from -0.14 in winter months to $-0.67/1^{\circ}\varphi$ in summer.

The results obtained concerning vertical gradients of air temperature (from -0.38 to -0.69°C per 100 m) show that it is impossible to accept with monthly reductions the value equal to $0.5^{\circ}\text{C}/100$ m temperature to the sea level of constant drop $0.5^{\circ}\text{C}/100$ m. The hypsometric gradient used so far is overvalued in the case of winter months and undervalued in summer months.

Spatial distribution of precipitation in Poland depends primarily on the height above sea-level, the hypsometric gradient of annual totals reaches the value of 74 mm/100 m. Hypsometric gradients show sinusoidal annual changes with a maximum of about 10 mm/100 m in summer and a minimum — over 3 mm/100 m in winter.

Horizontal gradients with regard to geographical latitude change within a year — they are above zero in the cool half-year and below zero — in the warm half-year. Amount of precipitation drops in summer (after elimination of the height above the sea level) towards the north. For instance, in July the precipitation gradient is over 3 mm/ $1^{\circ}\varphi$.

And gradients determined by polynomials of the fourth step (temperature) and the third one (precipitation) in each place in Poland do not only give information about influence of geographical latitude (the angle of sunrays and length of the day), longitude (distance from the Atlantic Ocean and the centre of Asia) and the height above sea level. They additionally inform about the influence of local factors, e.g. the relief and cover of terrain of smaller water reservoirs, larger towns etc.

Here is an example of calculations from the field of air temperature:

	φ	λ	H	$\frac{\partial f}{\partial \varphi}$	$\frac{\partial f}{\partial \lambda}$	$\frac{\partial f}{\partial H}$
Kołobrzeg	54.2	15.6	0.03	-0.22	-0.15	-0.36
Warszawa	52.2	21.0	1.08	-0.50	-0.15	-0.74
Kasprowy Wierch	49.2	20.0	19.91	4.58	1.81	-0.80

For instance, near Warsaw the mean annual temperature of air drops towards the north of the country — by $0.54^{\circ}\text{C}/1^{\circ}\varphi$, towards the east — by $0.16^{\circ}\text{C}/1^{\circ}\lambda$ and falls down along with growth of altitude by $0.74^{\circ}\text{C}/100$ m. This gradient in Kołobrzeg $\frac{\partial f}{\partial \varphi}$ may result from the warming impact of the Baltic in the autumn-winter season.

The field of air temperature in Poland (Fig. 1) is characterized by a minus horizontal component $-0.61 \leq \frac{\partial f}{\partial \varphi} \leq -0.01$ and hypsometric $-0.93 \leq \frac{\partial f}{\partial H} \leq -0.08$. And the other horizontal component $-0.93 \leq \frac{\partial f}{\partial \lambda} \leq -0.08$ is close to zero and changes within a year. Stations in high mountains are an exception: Kasprowy Wierch, Zakopane, Śnieżka — where horizontal gradients are above zero. This is obvious since they are the main minima of the field of air temperature in Poland ($dy > 0$).

It should be added that horizontal components on the surface of the Earth, expressed per 1 km are as follows: $\frac{180}{\pi R} \frac{\partial f}{\partial \varphi}$, $\frac{180}{\pi R \cos \varphi} \frac{\partial f}{\partial \lambda}$ where R is the mean radius of the Earth. Thus in Warsaw horizontal components after conversion amount to: $-0.45^{\circ}\text{C}/100$ km $-0.23^{\circ}\text{C}/100$ km

As an example also a fragment of calculation from the field of precipitation totals is given:

	$\frac{\partial f}{\partial \varphi}$	$\frac{\partial f}{\partial \lambda}$	$\frac{\partial f}{\partial H}$
Kołobrzeg	140.0	-43.75	28.71
Warszawa	-2.68	0.014	51.32
Kasprowy Wierch	-1520	-741.0	26.54

The field of precipitation in Poland is characterized by usually minus horizontal components: $-1520 \leq \frac{\partial f}{\partial \lambda} \leq 164$, $-741 \leq \frac{\partial f}{\partial \lambda} \leq 4$ and a plus hypsometric component $-40 \leq \frac{\partial f}{\partial H} \leq 93$.

It should be stressed that for small areas of the Earth, latitude and

longitude have primarily a geometrical sense and it is better to express them in km. They are simply Cartesian coordinates.

3. EQUATIONS OF SPATIAL — TIME CHANGES OF TEMPERATURE AND PRECIPITATIONS

Since gradients show periodical changes, the field of meteorological elements can be modelled with application of sinusoidal functions. For instance annual changes of air temperature and precipitation in Poland are quite well described by the following equations:

Air temperature 1881—1960

$$y = 37.390 + 9.1575 \sin (\omega t - 1.3935) \\ + [-0.5120 - 0.09492 \sin (\omega t - 0.6384)] \varphi \\ + [-0.1110 - 0.2186 \sin (\omega t + 1.4282)] \lambda \\ + [-0.5889 - 0.1320 \sin (\omega t - 1.3417)] H$$

Precipitation 1951—1960

$$y = 16.763 + 208.475 \sin (\omega t - 1.07934) \\ + [0.6653 - 3.6842 \sin (\omega t - 1.00109)] \varphi \\ + [-0.4953 - 0.2161 \sin (\omega t - 0.4318)] \lambda \\ + [4.989 + 1.0605 \sin (\omega t - 1.1241)] H$$

$$\text{where } \omega = \frac{2\pi}{365.24}$$

Thus they express the most important features of Poland's climate — annual oscillations of the field of temperature and the field of precipitation totals. According to these formulae, annual amplitudes of horizontal gradients $\frac{\partial f}{\partial \varphi}$, $\frac{\partial f}{\partial \lambda}$ and the hypsometric component $\frac{\partial f}{\partial H}$ are as follows:

field of temperature — $0.18^\circ\text{C}/1^\circ \varphi$, $0.44^\circ\text{C}/1^\circ \lambda$, $0.26^\circ\text{C}/100 \text{ m}$

field of precipitation — $7.3 \text{ mm}/1^\circ \varphi$, $0.42 \text{ mm}/1^\circ \lambda$, $2.1 \text{ mm}/100 \text{ m}$

The formulae can be used to estimate mean monthly values of temperature or monthly precipitation totals in any place in Poland — also in places where no measurements have been taken. For instance, the mean air temperature in July in Warsaw calculated from the formula ($\varphi = 52.2$, $\lambda = 21.0$, $H = 1.08 \text{ km}$, $t = 197.8$) is 18.59°C and the measured — 18.5°C .

Temperature in Poland: horizontal in relation
 to longitude ———— $^{\circ}\text{C}/1^{\circ}\text{L}$
 ———— $^{\circ}\text{C}/100\text{ m}$

4. TIME TREND OF AIR TEMPERATURE. PROGNOSIS OF CHANGE IN POLAND'S CLIMATE

So far the time trend $f(t)$ of meteorological variables has been expressed by the Fourier series in which fictional periods are assumed (harmonic and spectral analysis), functionally dependent on the length of the measuring series (Landsberg 1979, 1980; Kaczorowska 1962; Stachy 1970).

Thus it is important to introduce in the model — in the function of the time trendy $y = f(t)$ real periods, optimum in the sense of smallest squares. They can be discovered treating the period of a sinusoid $y = a + b \sin\left(\frac{2\pi}{T}t + C\right)$ as a changing parameter, e.g. $T = 1, 2, \dots$ or $T = 0.1, 0.2, \dots$

The permanent tendency of these variables $\frac{\partial y}{\partial t} = A = \text{const}$ was calculated assuming a linear dependence $y = A_0 + At$.

Time anomalies of Poland's climate and their reasons were determined on the basis of measurements of air temperature in Warsaw (1779—1979), precipitation in Wrocław (1859—1979), as well as activity of the sun (1700—1978) and meridian circulation from the period 1881—1967 (Boryczka, Stopa-Boryczka 1984).

Regression lines of mean annual air temperature and Wolf numbers in relation to time t have the following equations:

$$y = 6.877 + 0.005811 t$$

$$y = 36.10 + 0.08026 t$$

Positive, though high value of the regression coefficient $A = 0.6^\circ\text{C}/100$ years suggests a dominant influence of carbon dioxide CO_2 (warming effect of the atmosphere) and expansion of towns on the temperature. That is why „catastrophic” prognoses have appeared recently in literature — about a constant growth of temperature on the Earth.

It should be mentioned, however, that the activity of the Sun is also characterized by a positive trend of changes of 8.03 per 100 years and may have caused growth of temperature within the past two centuries — through higher frequency of meridian circulation of the air.

There is an analogy between periods of mean annual temperature and activity of the Sun:

Air temperature

$$y = 7.466 + 0.233 \sin\left(\frac{2\pi}{89}t + 2.336\right)$$

$$y = 7.467 + 0.568 \sin \left(\frac{2\pi}{217} t - 2.969 \right)$$

Wolf numbers

$$y = 47.22 + 18.15 \sin \left(\frac{2\pi}{95} t - 2.69 \right)$$

$$y = 46.51 + 9.856 \sin \left(\frac{2\pi}{180} t - 1.32 \right)$$

Fig. 2 Apparent (regression line 1) and real (regression line 2) anthropogenic change of air temperature in Warsaw (1779–1979)

Temperature time minima in the cycle of 89 years occur in: 1813, 1902, 1991 and activity of the Sun within a 95-year cycle in the years: 1812, 1907, 2002. Thus the cycles are synchronic. Those too high „catastrophic” coefficients of regression A: winter — $A = 0.6\text{--}1.5^\circ\text{C}/100$ years, year — $0.6^\circ\text{C}/100$ years are simply a result of approximation with a straight line of arches of long-term sinusoids with quite large amplitudes. This is proven by diverging tendencies $\frac{\partial f}{\partial t} = a + \dots$ defined by the linear element $a_0 + at$ of the time trend $f(t)$ of temperature — after taking into account sinusoids with real periods of 89 and 217 years

(Fig. 2.) By eliminating periodical changes it is possible to distinguish anthropogenic changes of air temperature resulting e.g. from concentration of dust ($a < 0$) and carbon dioxide CO_2 ($a > 0$). Then the result is: winter $a = -0.1$ and $-0.6^\circ\text{C}/100$ years. But in the warm half-year the temperature tendency is close to zero ($a \approx 0$). Probably, the change of the sign in the temperature tendency $\frac{\partial f}{\partial t} = a + \dots$ is conditioned by adverse influence of CO_2 and dust (radiation absorption) on the climate.

Time trends $y = f(t)$ of air temperature, activity of the Sun and frequency of meridian circulation were obtained by assembling sinusoids with real periods — verified by the Fisher-Snedecor test at the significance level 0.05. In the reconstruction-prognosis model the linear element was eliminated as insignificant. Thus, the prognoses disregard the anthropogenic component climate changes.

Here are time trends of air temperature (January, July, year), Wolf numbers and frequency of meridian circulation: January ($F_0 = 2.667$, $\delta = 3.258$)

$$\begin{aligned} y = & 0.677 \sin \left(\frac{2\pi}{7} t + 2.865 \right) + 0.793 \sin \left(\frac{2\pi}{10} t + 2.767 \right) + \\ & + 0.714 \sin \left(\frac{2\pi}{13} t + 0.756 \right) + 0.567 \sin \left(\frac{2\pi}{15} t + 0.658 \right) + \\ & + 0.658 \sin \left(\frac{2\pi}{22} t - 2.000 \right) + 0.404 \sin \left(\frac{2\pi}{27} t - 1.513 \right) + \\ & + 0.538 \sin \left(\frac{2\pi}{44} t + 0.887 \right) + 0.563 \sin \left(\frac{2\pi}{64} t + 0.299 \right) + \\ & + 1.402 \sin \left(\frac{2\pi}{205} t - 2.782 \right) - 4.084 \end{aligned}$$

July ($F_0 = 1.621$, $\delta = 1.471$)

$$\begin{aligned} y = & 0.206 \sin \left(\frac{2\pi}{3} t - 2.725 \right) + 219 \sin \left(\frac{2\pi}{7} t - 3.040 \right) + \\ & + 0.268 \sin \left(\frac{2\pi}{15} t + 1.265 \right) + 0.206 \sin \left(\frac{2\pi}{23} t + 0.015 \right) + \\ & + 0.057 \sin \left(\frac{2\pi}{52} t + 0.432 \right) + 0.398 \sin \left(\frac{2\pi}{75} t + 0.350 \right) + \\ & + 0.276 \sin \left(\frac{2\pi}{156} t + 2.290 \right) + 18.56 \end{aligned}$$

Year ($F_0 = 3.657$, $\delta = 0.856$)

$$y = 0.157 \sin \left(\frac{2\pi}{4} t + 1.140 \right) + 0.102 \sin \left(\frac{2\pi}{9} t - 0.355 \right) +$$

$$\begin{aligned}
& + 0.195 \sin \left(\frac{2\pi}{13} t + 0.280 \right) + 0.148 \sin \left(\frac{2\pi}{17} t + 1.542 \right) + \\
& + 0.114 \sin \left(\frac{2\pi}{19} t + 0.302 \right) + 0.128 \sin \left(\frac{2\pi}{22} t + 1.445 \right) + \\
& + 0.104 \sin \left(\frac{2\pi}{41} t + 0.769 \right) + 0.143 \sin \left(\frac{2\pi}{60} t + 0.961 \right) + \\
& + 0.143 \sin \left(\frac{2\pi}{89} t + 2.463 \right) + 0.544 \sin \left(\frac{2\pi}{217} t - 2.907 \right) + 7.450
\end{aligned}$$

Wolf numbers ($F_o = 24.31$, $\delta = 23.38$)

$$\begin{aligned}
y = & 28.87 \sin \left(\frac{2\pi}{11.1} t - 1.525 \right) + 4.863 \sin \left(\frac{2\pi}{21.4} t + 0.952 \right) + \\
& + 15.78 \sin \left(\frac{2\pi}{94} t - 2.806 \right) + 5.322 \sin \left(\frac{2\pi}{179} t - 1.06 \right) + \\
& + 19.79 \sin (0.633 t + 1.098) + 0.909 \sin (0.499 t - 1.189) + \\
& + 10.00 \sin (0.601 t - 2.724) + 12.58 \sin (0.531 t - 2.504) + \\
& + 1.312 \sin (0.860 t + 1.861) + 0.864 \sin (0.273 t + 1.173) + \\
& + 41.03
\end{aligned}$$

Meridian circulation ($F_o = 7.301$, $\delta = 18.09$)

$$\begin{aligned}
y = & 9.606 \sin \left(\frac{2\pi}{3} t + 1.666 \right) + 9.101 \sin \left(\frac{2\pi}{8} t - 0.518 \right) + \\
& + 5.128 \sin \left(\frac{2\pi}{11} t + 1.898 \right) + 12.76 \sin \left(\frac{2\pi}{16} t - 1.093 \right) + \\
& + 6.786 \sin \left(\frac{2\pi}{100} t - 0.826 \right) + 89.92
\end{aligned}$$

The model of constant amplitudes and phases: $b = \text{const}$, $c = \text{const}$ (used so far) proved imprecise in case of Wolf numbers. That is why to the function of trend $f(t)$ components have been introduced which account for modulation of the sinusoid with a period of 11.1 years in the other cycles.

Curves in Figure 3 are the graphs of approximating functions $y = f(t)$.

Time trends of air temperature are characterized by high accuracy — they are significant at the level of 0.01 since the calculated value F_o is higher than F_{cr} — critical, read from the Fischer-Snedecor distribution tables. The formula for a year is an exception which is significant at the level of 0.10. The mean square difference between the measured and calculated temperatures are: January — 3.1°C, July — 1.4°C, year — 0.8°C.

The curve of time anomalies of the mean annual air temperature in Warsaw in the period of 1,000 years between 1500 and 2500 has deep „minima” in the years: date: 1632, 1812, 2056, 2244, 2452; $y_{\min}$: 6.2, 6.1, 6.3, 6.3, 6.5

Fig. 3 Temperature trend in Warsaw (January, July, year)

— according to instrumental records

— according to mathematical models $y = f(t)$

Hypothetical trend is in confidence interval $f(t) \pm \Delta$ with probability of 90%.

The first two dates conform with the time of occurrence of the known „manunder minimum” of the activity of the Sun — absolute disappearance of spots on the Sun (Lendsberg 1980). The year 1811 is the bottom limit of the weakest cycle of 11 years of the Sun activity. About that time the coolest summers were recorded in Warsaw: 1805 — 5.3°C, 1812 — 5.6°C, 1829 — 4.8°C. And the absolute maximum of 8.8°C of time trend of the mean annual temperature falls in the year 1950 approximated to the absolute maximum of Sun spots — 1958. Thus there is quite good conformability between extremes of the mean air temperature and extremes of the activity of the Sun.

At present we are passing from a mesoepoch of the warm climate

1890—1980 to a mesoepoch of cool climate 1980—2100 with a forecast main minimum 6.1°C in the year 2056.

It is also interesting to compare the dates of the coolest Januaries (y_o) which occurred in the period 1779—1979 in Warsaw with the dates of century minima ($y_{\min}$) of the trend $f(t)$:

	y_o		$y_{\min}$
1803	—17.8	1801	—8.6°C
1823	—14.1	1822	—7.0
1860	—11.6	1860	—7.6
1893	—13.1	1892	—7.1
1940	—12.1	1841	—7.0

The absolute minimum of the trend: 1801, -8.6°C is only two years apart from the date of the coolest January recorded in Warsaw: 1803, -17.8°C .

A fragment of extrapolated values of time trends $f(t)$ in the time interval 1986—2005 is presented below:

	January	July	Year		January	July	Year
1985	—4.4	18.0	7.1	1996	—4.8	18.1	7.0
86	—3.4	18.3	7.5	97	—4.9	17.7	7.2
87	—3.2	18.7	7.5	98	—4.7	17.9	7.4
88	—3.3	18.3	7.3	99	—4.3	18.5	7.3
89	—4.8	18.2	7.3	2000	—3.9	18.4	7.1
1990	—5.6	18.5	7.4	01	—3.9	18.4	7.1
91	—5.8	18.3	7.2	02	—4.2	18.6	7.3
92	—5.4	18.4	6.9	03	—4.6	18.3	7.2
93	—4.9	18.7	6.9	04	—4.8	18.3	6.9
94	—4.5	18.3	7.1	2005	—4.6	18.8	6.9
1995	—4.6	18.0	7.1				

Differences between the measured values of temperature and the ones calculated from the formulae have distributions similar to the Gaussian distribution with parameters $0, \delta$. That is why hypothetical time trends are contained within confidence intervals $f(t) \pm 3\delta$ with probability of 99.7%.

REFERENCES

- Boryczka J., 1977. *Empiryczne równania klimatu Polski* [Empirical equations of Poland's climate], Warszawa

- Boryczka J., 1984. *Model deterministyczno-stochastyczny wielookresowych zmian klimatu* [A deterministic-stochastic model of long-term climate changes], Warszawa
- Boryczka J., Stopa-Boryczka M., Wawer J., 1984. *Aproksymacja pola temperatury powietrza w Polsce* [Approximation of field of air temperature in Poland]. Contribution of Polish science to the World Climatological Programme, Skierniewice
- Boryczka J., 1984. *Trend temperatury powietrza w Warszawie (1500—2500)* (Time trend of air temperature in Warsaw), Materiały I Międzynarodowej Konferencji „Klimat i bioklimat miast”, Łódź
- Gumiński R., 1952. „Materiały do poznania genezy i struktury klimatu Polski” [Materials to recognize genesis and structure of climate of Poland], *Przegląd geograficzny* XXIV
- Kaczorowska Z., 1962. „Opady w Polsce w przekroju wieloletnim” [Precipitation in Poland in the long-term section], *Prace Geograficzne PAN* No 33
- Landsberg H. E., 1980. „Variable Solar Emission, the „Maunder Minimum” and Climatic Temperature Fluctuation”, *Archiv für Meteorologie. Geophysic und Bioclimatologie*, Ser. B, 28
- Okołowicz W., 1968. „Regiony klimatyczne, Polska” [Climatic regions, Poland], *Atlas Geograficzny*, Warszawa
- Romer E., 1949. „Regiony klimatyczne Polski” [Poland's climatic regions], *Prace Wrocławskiego Towarzystwa Naukowego*, No. 16
- Stachy J., 1970. *Wieloletnia zmienność odpływów rzek polskich* [Long-term variability of outflow of Polish rivers], PIHM, Warszawa
- Stopa-Boryczka, M., 1973, *Cechy termiczne klimatu Polski* [Thermic features of Poland's climate], Warszawa
- Stopa-Boryczka M., Boryczka J., 1973—part I, 1976—part II, 1980—part III, *Atlas współzależności parametrów meteorologicznych i geograficznych w Polsce* [Atlas of Interdependences of meteorological and geographical parameters in Poland], Warszawa
- Stopa-Boryczka M., et al. 1986. *Atlas klimatu Północno-wschodniej Polski* [Atlas of climate in north-eastern Poland], Warszawa
- Wiszniewski W. (ed.), 1972. *Atlas Klimatyczny Polski* (The Climatic Atlas of Poland's climate), Warszawa
- Wiszniewski W., Chełchowski W., 1975. *Charakterystyka klimatu i regionalizacja klimatyczna Polski* [Characteristics of climate and climatic regionalisation of Poland], Warszawa