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RESEARCH INTO THE 3D ROUGHNESS OF A ROUGH SURFACE

A. Avisane, J. Rudzitis, G. Springis

Riga Technical University,
Institute of Mechanical Engineering,
6 Ezermalas Str., Riga, LATVIA
e-mail: anita.avisane@rtu.lv

One of the most important parameters in determination of the deformation associated with roughness is its height on the surface. The authors study the density of probability distribution as related to the surface peak height (SPH) and estimate the mathematical expectation (ME) of SPH for the roughness values above a determined deformation level. In the contact theory, the surface is modelled as a normal random field described by the Nayak SPH formula. Since this formula is practically inapplicable in the engineering tasks, the authors propose to replace it by a simpler distribution law. For this purpose the former is compared with two other formulas obeying the most known probability distribution laws: of normal distribution (Gauss') law and Rayleigh's law. Comparison of these three formulas made it possible to derive a simpler yet sufficiently precise one. In the work, the numerical values of the density of SPH probability distribution and the relevant ME values at different deformation levels for all three formulas.

Keywords: *rough surface, surface peak height, SPH distribution laws*

1. INTRODUCTION

The roughness deformation arising in the contact of rough surfaces is the subject of many investigations. In particular, it affects the accuracy of measurements if contacting components are made of elastic materials. One of the most important parameters characterizing deformation of the type in contact problems is the height of surface roughness. This parameter is measured from the midline (in the case of profile) or from the mid-plane (in the case of a 3D surface), which means that the maximum values of random process or random function are sought-for. In this article we will consider the mathematical expectation (ME) of the surface peak height (SPH) for all roughness peaks (including those being above the determined deformation level) with the density of probability distribution (DPD) for the SPHs established preliminarily.

2. SURFACE CONTACT DIAGRAM

In previous works [1, 2] the roughness deformation is measured from the top, while in the relevant statistics all roughness parameters are measured from the mid-plane. The roughness deformation diagram is presented in Fig. 1, which shows the relation between deformation a_i and deformed peak height $h_p(u)$ measured from level u.

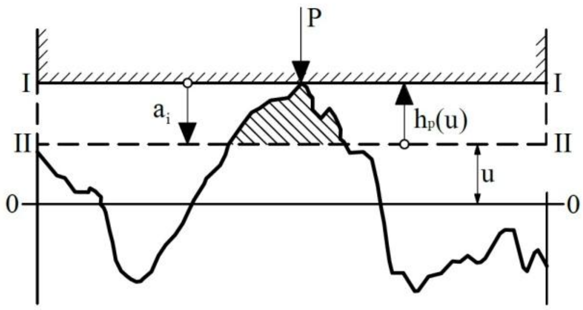

Fig. 1. Diagram of the contact between perfect plane and rough surfaces.

The deformation diagram shown in the figure relates to such a contact between the ideal plane and a rough surface where under applied force P the ideal plane shifts from position I – I to position II – II, reaching a balance between the external force and the micro-roughness deformation resistance. In this latter position the distance between the ideal plane and the mid-plane of rough surface 0 – 0 is equal to u.

3. DISTRIBUTION DENSITY OF SURFACE ROUGHNESS HEIGHT

In the contact theory (see, e.g. [3-5]) it is often assumed that the level of surface roughness deformation does not exceed some standard level – the mid-plane 0 – 0 (Fig. 1) $\gamma=u/\sigma$, where σ is the random field standard deviation.

In this work, the emphasis is given to the character of surface roughness as related to the levels above γ. The SPH distribution law for an irregular 3D surface (mathematically – for a normal random field) was studied by P.R. Nayak [6], where according to the density of probability distribution the function $f_1(\xi_p)$ of SPHs could be found dividing the number of peaks on level $[\gamma, \gamma+d\gamma]$ by that of all surface peaks above level γ :

$$f_{1\gamma}(\xi_p) = \frac{E\{n_p(\gamma)\}}{E\{M_p(\gamma)\}}\ , \tag{1}$$

where

ξ_p	–	$\xi_p = \dfrac{h_p}{\sigma}$ – standardized value of peak height;
$E\{n_p(\gamma)\}$	–	ME of the number of rough peaks on level $[\gamma, \gamma+d\gamma]$;
$E\{M_p(\gamma)\}$	–	ME of the number of all surface peaks above level γ .

The first of the ME values can be determined by the expression [6]:

$$E\left\{n_p(\gamma)\right\} = \frac{E^2\{m_1\}}{3} c \left\{ \begin{array}{l} \frac{\lambda}{4}\sqrt{3\left(8-3\lambda^2\right)} \cdot \gamma e^{-\frac{4}{8-3\lambda^2}\gamma^2} \\ +\frac{3\sqrt{2\pi}}{4}\lambda^2\left(\gamma^2-1\right)e^{-\frac{\gamma^2}{2}} \cdot \phi\left(\sqrt{\frac{3\lambda^2}{8-3\lambda^2}}\gamma\right) \\ +4\sqrt{\frac{2\pi}{3\left(4-\lambda^2\right)}} e^{-\frac{2}{4-\lambda^2}\gamma^2} \cdot \phi\left(\sqrt{\frac{4\lambda^2}{\left(4-\lambda^2\right)\left(8-3\lambda^2\right)}}\gamma\right) \end{array} \right\} \quad (2)$$

where indices 1 and 2 refer to the direction of parameter setting, respectively: 1 – perpendicular to the treatment direction, 2 – perpendicular to direction 1.

$E\{m_1\}$	–	ME of surface's maxima per unit of length;
λ	–	dimensionless parameter $\lambda = \frac{E\{n_1(0)\}}{E\{m_1\}}$; $E\{n(0)\}$ is the ME of number of zeroes per unit of length;
c	–	coefficient of anisotropy $c = \frac{E\{n_1(0)\}}{E\{n_2(0)\}}$;
$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt$,	–	Laplacian function (the numerical values of which can be found in [7]).

The second of mentioned above ME values is [6]:

$E\left\{M_p\right\} = \int_{-\infty}^{\infty} E\left\{n_p\right\} \cdot d\gamma$. In turn, to determine the probability distribution density function $f_{1\gamma}\left(\xi_p\right)$ for SPHs we have to determine the ME for the number of $E\left\{M_p(\gamma)\right\}$ surface peaks above level γ:

$$E\left\{M_p(\gamma)\right\} = \int_{\gamma}^{\infty} E\left\{n_p(\gamma)\right\} \cdot d\gamma \quad (3)$$

Inserting into formula (3) coherence (2) and carrying out the integration we obtain:

$$E\left\{M_p(\gamma)\right\} = c \cdot \frac{2\pi}{3\sqrt{3}} E^2\{m_1\} \cdot \varkappa_1, \quad (4)$$

where $\varkappa_1$ is the function of roughness parameter λ and level γ:

$$\varkappa_1 = \frac{3\lambda}{8\pi}\sqrt{8-3\lambda^2}\cdot e^{-\frac{4\gamma^2}{8-3\lambda^2}} + \frac{3\sqrt{3}\lambda^2}{4\sqrt{2\pi}}\cdot\gamma\cdot e^{-\frac{\gamma^2}{2}}\phi\left(\sqrt{\frac{3\lambda^2}{8-3\lambda^2}}\gamma\right)+$$
$$+\left[1-\phi\left(\sqrt{\frac{4}{4-\lambda^2}}\gamma\right)\right]+2\sqrt{\frac{2}{\pi\left(4-\lambda^2\right)}}\cdot \tag{5}$$
$$\cdot\int_{\gamma}^{\infty}\phi\left(\sqrt{\frac{4\lambda^2}{\left(4-\lambda^2\right)\left(8-3\lambda^2\right)}}\gamma\right)\cdot e^{\frac{2\gamma^2}{4-\lambda^2}}\cdot d\gamma$$

The integral in (5) cannot be expressed in elementary functions; therefore, $\varkappa_1$ values shown in Table 1 are obtained by numerical integration.

Table 1

Numerical values of coefficient $\varkappa_1$

γ	λ				
	1.63	1.40	1.00	0.60	0
0	1.0000	0.9871	0.8974	0.7673	0.5000
0.5	0.9963	0.9234	0.7554	0.5813	0.3085
1.0	0.9215	0.7719	0.5479	0.3749	0.1587
1.5	0.6824	0.5209	0.3276	0.2000	0.0668
2.0	0.3746	0.2707	0.1565	0.0862	0.0028
2.5	0.1518	0.1064	0.0586	0.0296	0.0062
3.0	0.0461	0.0375	0.0170	0.0013	0.0013

By inserting expressions (2) and (4) into coherence (1) we obtain the probability distribution density function $f_{1\gamma}\left(\xi_p\right)$ for the peaks above level γ:

$$f_{1\gamma}\left(\xi_p\right) = \varkappa_1\cdot\frac{\sqrt{3}}{2\pi}\left\{\begin{array}{l} \frac{\lambda}{4}\sqrt{3\left(8-3\lambda^2\right)}\cdot\gamma\cdot e^{-\frac{4}{8-3\lambda^2}\gamma^2} + \\ +\frac{3\sqrt{2\pi}}{4}\lambda^2\left(\gamma^2-1\right)e^{-\frac{\gamma^2}{2}}\phi\left(\sqrt{\frac{3\lambda^2}{8-3\lambda^2}}\gamma\right)+ \\ +4\sqrt{\frac{2\pi}{3\left(4-\lambda^2\right)}}e^{-\frac{2}{4-\lambda^2}\gamma^2}\ . \\ \cdot\phi\left(\sqrt{\frac{4\lambda^2}{\left(4-\lambda^2\right)\left(8-3\lambda^2\right)}}\gamma\right) \end{array}\right\} \tag{6}$$

Since the formula proposed by P.R. Nayak is complicated for engineering calculations, it is necessary to find another distribution law and derive a simpler formula as a substitute for the former.

We will consider two best-known probability distribution laws: normal (Gaussian) distribution law and that of Rayleigh. According to these laws the density of probability distribution of surface peak height $f\left(\xi_p\right)$ can be determined as follows.

The truncated normal distribution law gives:

$$f_2\left(\xi_p\right)=\frac{2}{\sqrt{2\pi}}\cdot e^{-\frac{1}{2}\xi_p{}^2}, \qquad \xi_p \geq 0 \tag{7a}$$

and Rayleigh's law:

$$f_3\left(\xi_p\right)=\xi_p\cdot e^{-\frac{1}{2}\xi_p^2} \qquad \xi_p \geq 0 \tag{7b}$$

Since in our case the probability distribution density function $f\left(\xi_p\right)$ relates to the SPHs above level γ, this should be standardized so that it fulfils the condition:

$$\int_{\gamma}^{\infty} f\left(\xi_p\right)=1. \tag{8}$$

4. TRUNCATED NORMAL DISTRIBUTION LAW

As mentioned above, we need to obtain the function $f_{2\gamma}\left(\xi_p\right)$ for SPHs above level *γ*. The truncated normal distribution law is valid in the range $0-\infty$, and, therefore, should be standardized within the range $\gamma-\infty$. For this purpose we assume that

$$f_{2\gamma}\left(\xi_p\right)=C_2\cdot f_2\left(\xi_p\right), \tag{9}$$

where C_2 is the standardized multiplier:

$$C_2\cdot\int_{\gamma}^{\infty} f_2\left(\xi_p\right)d\xi_p=1,$$

thus $$C_2=\frac{1}{\int_{\gamma}^{\infty} f_2\left(\xi_p\right)d\xi_p}. \tag{10}$$

We will introduce another standardized multiplier: $A_2=\int_{\gamma}^{\infty} f_2\left(\xi_p\right)d\xi_p$.

To determine the standardized function $f_{2\gamma}\left(\xi_p\right)$ we should first determine the value of standardized multiplier:

$$A_2=\int_{\gamma}^{\infty}\frac{2}{\sqrt{2\pi}}e^{-\frac{1}{2}\xi_p{}^2}d\xi_p=\sqrt{\frac{\pi}{2}}\int_{\gamma}^{\infty}e^{-\frac{1}{2}\xi_p{}^2}d\xi_p.$$

Using integral tables [8] we will carry out the integration:

$$\int_{\gamma}^{\infty} e^{-\frac{1}{2}\xi_p{}^2} d\xi_p = \frac{\sqrt{\pi}\cdot erfc\left(\frac{\gamma}{\sqrt{2}}\right)}{2\cdot\sqrt{\frac{1}{2}}} = \frac{\sqrt{2\pi}}{2}\cdot erfc\left(\frac{\gamma}{\sqrt{2}}\right),$$

where

$erfc\left(\frac{\gamma}{\sqrt{2}}\right)$ is the error integral: $erfc(x) = \frac{2}{\sqrt{\pi}}\int_{x}^{\infty} e^{-t^2} dt$ [7].

We obtain:

$$A_2 = \frac{2}{\sqrt{2\pi}}\cdot\frac{\sqrt{2\pi}}{2}\cdot erfc\left(\frac{\gamma}{\sqrt{2}}\right) = erfc\left(\frac{\gamma}{\sqrt{2}}\right).$$

Thus the standardized multiplier is:

$$C_2 = \frac{1}{erfc\left(\frac{\gamma}{\sqrt{2}}\right)}. \tag{11}$$

Putting it into coherence (9) we will obtain:

$$f_{2\gamma} = \frac{1}{erfc\left(\frac{\gamma}{\sqrt{2}}\right)}\cdot\frac{2}{\sqrt{2\pi}}\cdot e^{-\frac{1}{2}\cdot\xi_p{}^2}. \tag{12}$$

If we verify expression (12) by inserting there $\gamma = 0$, we will find out that it corresponds to the previously mentioned truncated normal distribution law within the range $0-\infty$. This means that within the range $\gamma-\infty$ the probability distribution density function $f_{2\gamma}\left(\xi_p\right)$ for SPHs can be determined by expression (12).

5. RAYLEIGH'S DISTRIBUTION LAW

Similarly to the case of truncated normal distribution, we will proceed with Rayleigh's law in order to obtain the probability distribution density function $f_{3\gamma}\left(\xi_p\right)$ for SPHs above level γ.

Standardization within the range $\gamma-\infty$ gives:

$$f_{3\gamma}\left(\xi_p\right) = C_3 f_3\left(\xi_p\right),$$

where

$$C_3 = \frac{1}{\int_{\gamma}^{\infty} f_3\left(\xi_p\right) d\xi_p}. \tag{13}$$

We will mark $A_3 = \int_{\gamma}^{\infty} f_3\left(\xi_p\right) d\xi_p$ and write:

$$A_3 = \int_{\gamma}^{\infty} \xi_p \cdot e^{-\frac{1}{2}\xi_p{}^2} \cdot d\xi_p = -\int_{\gamma}^{\infty} \left(e^{-\frac{1}{2}\xi_p{}^2} \right)' d\xi_p = -e^{-\frac{1}{2}\xi_p{}^2} \Big|_{\gamma}^{\infty} = -\left[e^{-\infty} - e^{-\frac{1}{2}\gamma^2} \right] = e^{-\frac{1}{2}\gamma^2}$$

Thus: $$C_3 = \frac{1}{e^{-\frac{1}{2}\gamma^2}} = e^{\frac{1}{2}\gamma^2}, \qquad (14)$$

and the density of probability distribution according to Rayleigh' law, where $\gamma = 0 - \infty$, will be:

$$f_{3\gamma}\left(\xi_p\right) = e^{\frac{1}{2}\gamma^2} \cdot \xi_p \cdot e^{-\frac{1}{2}\xi_p{}^2} = \xi_p \cdot e^{\frac{1}{2}\left(\gamma^2 - \xi_p{}^2\right)}. \qquad (15)$$

Having carried out the control insertion in expression (12) $\gamma = 0$, we find that it corresponds to the above Rayleigh distribution law within the range $0 - \infty$ (and thus within the range $\gamma - \infty$), the density of probability distribution of surface roughness height $f_{3\gamma}\left(\xi_p\right)$ is to be determined according to expression (15).

Diagrams of the distribution density are given in Fig. 2. They show that, starting from the value $\xi > 1$, the relevant expressions are approaching each other and become closer to the precise expression in Rayleigh's distribution; therefore, the corresponding distribution density for the values $\xi > 1$ can be used for solving engineering tasks.

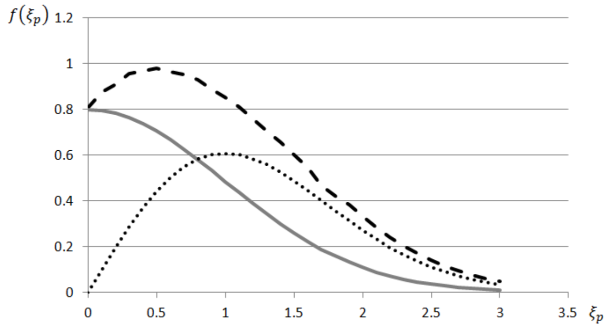

Fig. 2. Density of probability distribution of surface peak heights.

Formula by P.R. Nayak
Truncated normal distribution law
Rayleigh's distribution law

6. MATHEMATICAL EXPECTATION OF THE SURFACE PEAK HEIGHT

To determine roughness deformations and thus also the measurement error it is important to find the ME of surface peak height (SPH). Likewise as in the previous chapters where the probability distribution density of SPH was determined by three formulas, its ME should be found in the framework of three distribution laws (Nayak's, the truncated distribution law, and Rayleigh's).

6.1. Mathematical expectation of surface peak height by Nayak

In his research on the rough surface random processes P.R. Nayak [6] determines the ME of SPH by the formula:

$$E_1\left\{\xi_p\right\} = \int\limits_{-\infty}^{\infty} f_1\left(\xi_p\right)\cdot \xi_p \cdot d\xi_p \quad , \tag{16}$$

where

$f_1\left(\xi_p\right)$	–	density of probability distribution of rough surface's peak height
$\xi_p = \dfrac{h_p}{\sigma}$	–	standardized value of peak height;
σ	–	standard deviation of random field.

From Chapter 3 it follows that according to Nayak (see (1)) the density of probability distribution of SPH can be found dividing the number of peaks situated above level γ by the total number of peaks. As mentioned before, in the contact problems we should determine the ME for the height of the surface peaks that are above the set deformation level u or standardized deformation level γ. Thus, by transforming formula (16) for the ME of surface peak height above level γ we obtain:

$$E_{1\gamma}\left\{\xi_p\right\} = \int\limits_{\gamma}^{\infty} f_{1\gamma}\left(\xi_p\right)\xi_p d\xi_p \ , \tag{17}$$

Inserting into equation (17) expression for $f_{1\gamma}\left(\xi_p\right)$ gained previously, after integration [8] we will have:

$$E_{1_\gamma} = C_1 \cdot \frac{\sqrt{3}}{2\pi}\left\{\begin{aligned}&\frac{3\lambda^2\sqrt{2\pi}}{4}\cdot\left(\gamma^2+1\right)\cdot e^{-\frac{\gamma^2}{2}}\cdot\phi\left(\beta_6\gamma\right)+\frac{4\sqrt{\pi}\lambda}{\sqrt{3}}\cdot\left[1-\phi\left(\beta_7\gamma\right)\right]+\\&+\frac{\sqrt{6}\lambda}{\beta_7}\cdot\gamma e^{-\frac{\beta_7{}^2\gamma^2}{2}}+2\sqrt{\frac{\pi}{6}\left(4-\lambda^2\right)}\cdot e^{-\frac{2\gamma^2}{4-\lambda^2}}\cdot\phi\left(\beta_{8\gamma}\right)\end{aligned}\right\} \tag{18}$$

where C_1 – standardized multiplier;

$$\beta_6 = \sqrt{\frac{3\cdot\lambda^2}{8-3\lambda^2}}\ ; \qquad \beta_7 = \sqrt{\frac{8}{8-3\lambda^2}}\ ; \qquad \beta_8 = \sqrt{\frac{4\lambda^2}{\left(4-\lambda^2\right)\left(8-3\lambda^2\right)}}\ .$$

Coefficient C_1 is determined from the relation:

$$C_1\int\limits_{\gamma}^{\infty} f_1\left(\xi_p\right)d\xi_p = 1 \tag{19}$$

The value of coefficient C_1 changes depending on deformation level γ and roughness parameter λ (Table 2).

Table 2

Values of coefficient C_1

γ	λ				
	0	0.63	1.00	1.41	1.63
0	2.000	1.304	1.115	1.013	1.00
0.5	3.145	1.721	1.325	1.065	1.004
1.0	6.329	2.667	1.825	1.295	1.085
1.5	14.921	5.000	3.049	1.919	1.466
2.0	43.472	41.631	6.410	3.690	2.667
2.5	161.322	34.483	11.952	8.434	6.579
3.0	769.233	125.022	58.821	26.322	21.741

Similar to the determination of the probability distribution density of rough surface's peak height by Nayak's formula, that of ME for SPH is too complicated for engineering calculations; therefore, it would be better to use simpler distribution laws to replace the precise formula. For the ME determination we will apply the Rayleigh law and the normal distribution law.

6.2. Mathematical expectation of surface peak height according to the truncated normal distribution law

The ME of SPH using the truncated normal distribution law is determined as

$$E_2\left\{\xi_{p\gamma}\right\}=\int_{\gamma}^{\infty}\xi_p\cdot f_{2\gamma}\left(\xi_p\right)\cdot d\xi_p=\frac{2}{\sqrt{2\pi}\cdot erfc\left(\frac{\gamma}{\sqrt{2}}\right)}\cdot\int_{\gamma}^{\infty}\xi_p\cdot e^{-\frac{1}{2}\xi_p{}^2}d\xi_p\,. \qquad (20)$$

In this case, we will mark: $B_2=\int_{\gamma}^{\infty}\xi_p\cdot e^{-\frac{1}{2}\xi_p{}^2}d\xi_p\,.$

According to [8] the integration looks as

$$B_2=\int_{\gamma}^{\infty}\xi_p\cdot e^{-\frac{1}{2}\xi_p{}^2}\cdot d\xi_p=\frac{e^{-\frac{1}{2}\gamma^2}}{2\cdot\frac{1}{2}}=e^{-\frac{1}{2}\gamma^2}\,. \qquad (21)$$

Inserting the obtained result into (20) we derive the following ME of surface peak height for the truncated normal distribution law:

$$E_2\left\{\xi_{p\gamma}\right\}=\frac{2\cdot e^{-\frac{1}{2}\gamma^2}}{\sqrt{2\pi}\cdot erfc\left(\frac{\gamma}{2}\right)}\,. \qquad (22)$$

6.3. Mathematical expectation of surface peak height according to Rayleigh's law

Similarly, in the case of truncated distribution law the ME of SPH for Rayleigh's law is:

$$E_3\left\{\xi_{p\gamma}\right\} = \int_{\gamma}^{\infty} \xi_p \cdot f_3\left(\xi_p\right) d\xi_p = e^{\frac{1}{2}\gamma^2} \cdot \int_{\gamma}^{\infty} \xi_p{}^2 \cdot e^{-\frac{1}{2}\xi_p{}^2} \cdot d\xi_p. \tag{23}$$

Making integration in parts according to [8] we obtain:

$$E_3\left\{\xi_{p\gamma}\right\} = e^{\frac{1}{2}\gamma^2} \cdot \left[\sqrt{\frac{\pi}{2}} \cdot erfc\left(\frac{\gamma}{\sqrt{2}}\right) + \gamma \cdot e^{-\frac{1}{2}\gamma^2}\right] = e^{\frac{1}{2}\gamma^2} \cdot \sqrt{\frac{\pi}{2}} \cdot erfc\left(\frac{\gamma}{\sqrt{2}}\right) + \gamma \ . \tag{24}$$

The relevant ME diagrams are shown in Fig. 3. It could be seen that starting from $\gamma > 1$ the ME values of SPH are becoming closer to each other, with the value closer to the precise one being that corresponding to Rayleigh's law, which thus can be used in solving the engineering tasks for levels $\gamma > 1$.

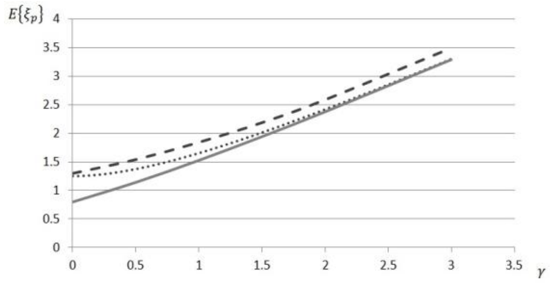

Fig. 3. Mathematical expectation of surface peak height

Formula by P.R. Nayak

Truncated normal distribution law

Rayleigh's distribution law

6.4. Asymptotics of mathematical expectation of surface peak height according to Rayleigh's law

As was previously found out, the closest to the precise (though complicated) Nayak's formula for determination of the ME for peak height is Rayleigh's distribution law. The ME diagrams presented in Fig. 3 show that Rayleigh's law is approximating the precise law at high levels ($\gamma > 1.5$) when according to (7) the following coherence is realized:

$$\sqrt{\frac{\pi}{2}} erfc\left(\frac{\gamma}{\sqrt{2}}\right) \approx \frac{e^{-\frac{\gamma^2}{2}}}{\gamma} \ .$$

Then from expression (24) we derive the following asymptotic formula for the ME:

$$E_4\left\{\xi_{p\gamma}\right\} \sim \gamma + \frac{1}{\gamma} \ . \tag{25}$$

The numerical values of all the distribution laws for determination of the peak height ME along with the precise formula and Rayleigh's law asymptotic values at different deformation levels γ are tabulated below.

Table 3

Comparison of mathematical expectation for different distribution laws

γ	Precise formula	Truncated distribution law		Rayleigh's law		Asymptotic law	
	$E_{1\gamma}\{\xi_p\}$	$E_{2\gamma}\{\xi_p\}$	Deviation %	$E_{3\gamma}\{\xi_p\}$	Deviation %	$E_{4\gamma}\{\xi_p\}$	Deviation %
0	1.3032	0.7979	39%	1.2533	4%	-	-
0. 5	1.5445	1.1411	26%	1.3764	11%	-	-
1.0	1.8254	1.5251	16%	1.6557	9%	-	-
1.5	2.1893	1.9387	11%	2.0158	8%	2.1667	1%
2.0	2.5897	2.3732	8%	2.4214	6%	2.5000	3%
2.5	2.6424	2.8227	7%	2.8543	8%	2.9000	10%
3.0	3.4922	3.2831	6%	3.3046	5%	3.3333	5%

7. CONCLUSIONS

Conclusions that could be drawn from the results of this work are as follows.

The precise Nayak's formula for determination of the SPH probability distribution density and of the peak height mathematical expectation (being too complicated for solving engineering tasks) can be replaced by a simpler one. The Rayleigh law has been found the most suitable for such replacement, while at high levels ($\gamma > 1.5$) the Rayleigh law asymptotics can be used.

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RAUPJAS VIRSMAS TRĪSDIMENSIJU NEGLUDUMU AUGSTUMA PĒTĪJUMI

A. Avišāne, J. Rudzītis, G. Spriņģis

K o p s a v i l k u m s

Lai noteiktu negludumu deformāciju, viens no būtiskākajiem parametriem ir virsmas negludumu augstums. Šajā rakstā apskatītas un salīdzinātas trīs dažādas formulas virsmas izciļņu augstuma varbūtību sadalījuma blīvuma aprēķināšanai un virsmas izciļņu augstuma matemātiskās sagaidāmās vērtības noteikšanai tiem nelīdzenumiem, kas atrodas virs nosacīta deformācijas līmeņa γ. Kontaktteorijā virsma tiek modelēta kā normāls gadījuma lauks. Šādam normālajam gadījuma laukam izciļņu augstuma varbūtību sadalījuma blīvuma likumu ir ieguvis P.R. Naijaks, taču šī izteiksme ir praktiski nepiemērojama inženieruzdevumu risināšanai, tāpēc šajā darbā ir noskaidrots, ka esošo formulu ir iespējams aizstāt ar vienkāršāku sadalījuma likumu. Ir apskatīta P.R. Naijaka formula un divi pazīstamākie varbūtību sadalījuma likumi: normālais sadalījuma (Gausa) likums un Releja likums. Salīdzinot šīs trīs formulas, ir atrasts vienkāršāks, bet pietiekami precīzs risinājums, ar ko aizstāt sarežģīto formulu. Darbā ir iespējams uzskatāmi redzēt, grafiski attēlotās, iegūtās virsmas izciļņu augstuma varbūtību sadalījuma blīvuma skaitliskās vērtības un virsmas izciļņu augstuma matemātiskās sagaidāmās vērtības pie dažādām γ vērtībām, visām trijām formulām, kā arī tabulā ir apkopotas iepriekšminēto varbūtību sadalījuma likumu maksimālās novirzes no precīzās formulas.

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