FUSED DEPOSITION MODELLING AS RAPID PROTOTYPING FOR STRUCTURAL MATERIAL IMPROVEMENT: ANALYTICAL SOLUTION

I. Brensons, S. Polukoshko

Ventspils University College,
101a Inženieru Str., Ventspils, LV-3601, LATVIA
e-mail: ilmars.brensons@venta.lv ; pol.svet@inbox.lv.

Fused deposition modelling (FDM) is one of the most effective rapid prototyping (RP) techniques due to its low cost, available materials and versatility. In FDM, a part of material (usually plastic) is made by heating this material to the molten state, and from the melt it is extruded through a nozzle and deposited on a surface. In the article, an alternative RP method is considered for improvement of the mechanical properties of a rapid prototype. The authors propose an analytical solution which allows for achievement of this purpose via advanced technologies. The base materials applied in RP technology can be combined with liquid resin which solidifies after a definite time. This makes it possible to create a channel through the prototype and fill it with another material having better mechanical properties. The optimal channel sizes can be chosen in order to raise the strength of material parts.

Keywords: 3D printing technologies, fused deposition modelling, (FDM), Rapid Prototyping (RP), optimal channel proportion

1. INTRODUCTION

Fused deposition modelling (FDM) is one of the most popular and widely spread 3D printing technologies. It enables the fabrication of complex geometric details, which can be made directly from CAD models without special preparation or a complicated setup of machinery. The 3D printing uses inexpensive machinery and durable materials. During the process, a thermoplastic material is heated to a semi-molten state and then extruded through a nozzle in the form of a very thin filament and deposited on a surface to build a detail.

The extrusion nozzle moves in a tool-path defined by the cross-sectional boundary of the detail to be made, and the material is deposited on top of the existing layer. The heat is dissipated by conduction and forced convection, and the temperature fall in these processes causes the material to quickly solidify on the surrounding filaments. Bonding between the filaments is caused by local remelting of the previously solidified material and diffusion [1].

Unlike traditional manufacturing systems, which usually require constant human supervision and instructions during the fabrication process, a rapid prototyping (RP) system controls the fabrication operations using numerical
control (NC) tool-path files generated directly from the CAD model. Human presence is only needed when loading the materials, mounting the support structures, or removing the finished parts.

FDM systems can build complex geometric parts from various thermoplastic materials, such as acrylonitrile butadiene styrene (ABS) plastic, investment casting wax or elastomer.

Critical limitations for rapid prototyping are currently available materials and their compatibility with RP technologies [2]. One option could be to develop new materials that have better characteristics than conventional ones and are compatible with such a technology. Another option that many researchers choose is: to adjust the relevant parameters during the fabrication stage in order to improve them [3].

When building the FDM-based design of a part (a detail), the most important is to take into account the anisotropic nature of its properties, since different layering strategies will result in different material strengths. A regular additive manufacturing (AM) machine builds parts in layers. Each layer is made as a 2D object and is located on the x-y plane. Each additional layer is made in the same way, on top of a previous layer along the z-axis. A typical tool-path makes parts that have isotropic properties on the x-y plane, unless it is set to deposit preferentially along a particular direction: in this case such properties will also be anisotropic. In almost each case, a part’s strength in the z-direction is measurably less than its strength on the x-y plane. This means that, if a potential part to be produced by FDM method undergoes stress in a particular direction, the best way is to align the part so that the stress lines are mostly on the x-y plane rather than in the z-direction [4].

There are many advantages and opportunities offered by additive technologies. First of all, AM makes it possible to break the constraints caused by tooling. While conventional fabrication limits the designing freedom, AM allows complex geometries to be adopted at the design phase, since moulding limitations no longer exist. For instance, traditional high-pressure die-casting requires that there are considered the draft angles, parting line locations, wall thickness, aspect ratio, filleted corners, etc., whereas AM does not. In addition, AM components are free from flashing along the parting line, thus the aesthetic appearance is here better. Moreover, without tooling cost to be amortized into the parts produced, each component can be different, potentially allowing for true customization of every product. This is one of the most thoughtful implications of AM on the design process [5].

Nevertheless, there are some limitations which should be considered in regard to the additive processes [5]:

- Support design and removal are limited to some techniques such as selective laser sintering (SLS) of metals.
- Dimensional accuracy, tolerances, surface finish and minimum wall thickness are highly dependent on the specific technique as well as on the stair-stepping appearance.
- Narrow range of currently available materials and their properties.
• Mechanical properties of the additive part are dependent on the building direction (anisotropy).
• Component size is limited by the building volume of available AM machines.

2. FUNDAMENTAL EQUATIONS OF A CONTINUOUS MEDIUM

The classical equations of medium deformation (assuming that the medium is continuous, homogeneous and isotropic) are given in [6, 7]. The medium is assumed to be linearly deformable (Hooke's law is valid), with the displacement and deformation being small enough. The equations are written in Cartesian’s coordinates. To obtain them in the vicinity of some point of a body, the elementary parallelepiped with dimensions \(dx\), \(dy\), \(dz\) is selected (Fig. 1).

Equations of the first group express the equilibrium conditions of the selected element of medium (the equilibrium or static equations).

Equations of the second group link the deformation of a medium element to the functions expressing the movement of its points (the geometrical equations).

Finally, equations of the third group take into account the mechanical properties of material, and express the relationship between stress and strain (the physical equations).

**Equilibrium equations**

![Equilibrium state parameters](image)

The first group (equilibrium) equations are as follows.

\[
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + X = 0
\]

\[
\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + Y = 0
\]

\[
\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + Z = 0
\]
where $X,Y,Z$ are the equilibrium state body forces.

Conditions on the surface are:

$$\sigma_x l + \tau_{xy} m + \tau_{xz} n = p_x$$

$$\tau_{xy} l + \sigma_y m + \tau_{yz} n = p_y ,$$

$$\tau_{xz} l + \tau_{yz} m + \sigma_z n = p_z ,$$

where $l, m, n$ are the normal vector components,

and $p_x, p_y, p_z$ are the surface traction components.

**Geometrical equations**

Point displacement components parallel to the axes are:

$$u = u(x, y, z); \quad v = v(x, y, z); \quad w = w(x, y, z).$$

The relative deformation is expressed as

$$\varepsilon_x = \frac{\partial u}{\partial x}; \quad \varepsilon_y = \frac{\partial v}{\partial y}; \quad \varepsilon_z = \frac{\partial w}{\partial z};$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial y}{\partial x}; \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial z}{\partial y}; \quad \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial x}{\partial z}.$$

**Physical equations**

The generalized Hooke’s law equations for linear elastic isotropic bodies in a direct form are:

$$\varepsilon_x = \frac{1}{E} (\sigma_x - \mu \sigma_y - \mu \sigma_z);$$

$$\varepsilon_y = \frac{1}{E} (-\mu \sigma_x + \sigma_y - \mu \sigma_z);$$

$$\varepsilon_z = \frac{1}{E} (-\mu \sigma_x - \mu \sigma_y + \sigma_z);$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}; \quad \gamma_{yz} = \frac{\tau_{yz}}{G}; \quad \gamma_{zx} = \frac{\tau_{zx}}{G}; \quad G = \frac{E}{2(1+\mu)};$$

Hooke’s law in the inverse form is described by the equalities:

$$\sigma_x = 2G\varepsilon_x + \lambda \theta; \quad \sigma_y = 2G\varepsilon_y + \lambda \theta; \quad \sigma_z = 2G\varepsilon_z + \lambda \theta;$$

$$\tau_{xy} = G\gamma_{xy}; \quad \tau_{yz} = G\gamma_{yz}; \quad \tau_{zx} = G\gamma_{zx};$$

where $\theta = \varepsilon_x + \varepsilon_y + \varepsilon_z$. 

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and \( \lambda = \frac{\mu E}{(1 - 2\mu)(1 + \mu)} \) is the Lamé parameter.

The generalized Hooke’s law equations for a linear elastic orthotropic body are:

\[
\begin{align*}
\varepsilon_x &= \frac{\sigma_x}{E} - \mu_{xy} \frac{\sigma_y}{E_y} - \mu_{xz} \frac{\sigma_z}{E_z}; \\
\varepsilon_y &= -\mu_{yx} \frac{\sigma_x}{E} + \frac{\sigma_y}{E_y} - \mu_{yz} \frac{\sigma_z}{E_z}; \\
\varepsilon_z &= -\mu_{zx} \frac{\sigma_x}{E} - \mu_{zy} \frac{\sigma_y}{E_y} + \frac{\sigma_z}{E_z}; \\
\gamma_{xy} &= \frac{\tau_{xy}}{G_{xy}}; \\
\gamma_{yz} &= \frac{\tau_{yz}}{G_{yz}}; \\
\gamma_{zx} &= \frac{\tau_{zx}}{G_{zx}}; \\
\frac{\mu_{xy}}{E_x} &= \frac{\mu_{yx}}{E_y}; \\
\frac{\mu_{yz}}{E_y} &= \frac{\mu_{zy}}{E_z}; \\
\frac{\mu_{zx}}{E_z} &= \frac{\mu_{xz}}{E_x}; \\
\end{align*}
\]

while these equations for an anisotropic body are:

\[
\begin{align*}
\varepsilon_x &= a_{11} \sigma_x + a_{12} \sigma_y + a_{13} \sigma_z + a_{14} \tau_{yz} + a_{15} \tau_{zx} + a_{16} \tau_{xy}; \\
\varepsilon_y &= a_{21} \sigma_x + a_{22} \sigma_y + a_{23} \sigma_z + a_{24} \tau_{yz} + a_{25} \tau_{zx} + a_{26} \tau_{xy}; \\
\tau_{xy} &= a_{61} \sigma_x + a_{62} \sigma_y + a_{63} \sigma_z + a_{64} \tau_{yz} + a_{65} \tau_{zx} + a_{66} \tau_{xy}; \\
\end{align*}
\]

Here \( a_{ik} \) are the elastic constants depending on the material properties, their number being 36.

Due to their identity, this number reduces to 15.

In the general case of an anisotropic elastic property there is 21 independent constant.

It is possible to consider a composite material as a continuous medium only when the detail sizes are significantly larger than the filaments or cells.

The above equation of solid mechanics, together with the conditions on the surface, forms a complete statement of the problem in differential form.

The problem of defining the \( \tilde{\sigma}, \tilde{\varepsilon}, \tilde{u} \) functions may be reduced to defining the integral of a particular type of these functions named functional, and the desired functions are found from the extremum of this functional.

The functional expresses the total potential energy of a deformed body, and the load acting on it is of great importance:

\[
E = U + \Pi,
\]

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where $U, \Pi$ is the potential of internal and external forces, respectively.

In the case of a linear stress, the specific strain energy is:

$$U_0 = \frac{1}{2}(\sigma_x \epsilon_x + \sigma_y \epsilon_y + \sigma_z \epsilon_z + \tau_{xy} \gamma_{xy} + \tau_{yz} \gamma_{yz} + \tau_{xz} \gamma_{xz})$$

The volume deformation energy is:

$$U = \iiint U_0 dV$$

The potential of external forces is:

$$\Pi = -\iint (p_x u + p_y v + p_z w) dS + \iiint (Xu + Yv + Zw) dV$$

where $p_x, p_y, p_z$ are the projections of surface forces on coordinate axes.

$X, Y, Z$ are the projections of volume forces.

Thus $E = E(u,v,w)$, with $\delta E = 0$ being the necessary condition for a local extremum of the functional.

For the calculation of a loaded deformed body it is necessary to consider the most dangerous point identifying the main stresses on the surface of corresponding slope, and to determine the possibility of destruction in accordance with one of the strength theories.

3. CALCULATION OF A REINFORCED BODY.

When performing the reinforcement of material for a detail of another (more durable) material, it is necessary to consider both the medium geometry and the physical properties as well as the conditions of their combined effects.

**Consideration of the axially stretched detail**

In the proposed model of hardening (Fig. 2), the polymer matrix material is reinforced by the element of more durable polymer material, which occupies the area from 10 to 50% of the square matrix.

![Fig. 2. Body reinforcement model (l_c – the distance between cracks).](image)
As confirmed by experiments, the destruction of central tension bodies occurs through cracks formed in the matrix material; this matrix is excluded from the cracking operation, so the stress in reinforcement reaches the ultimate limit.

It is assumed that the stress is uniformly distributed over a cross-sectional area before cracking; then for the first group (Equilibrium equations) we have:

\[ \sigma_y = \frac{N}{A}, \quad \sigma_x = \sigma_z = 0, \quad \tau_{xy} = \tau_{xz} = \tau_{zy} = 0, \]

where \( A \) is the reduced cross-section area:

\[ A = \frac{E_{ar}}{E_m} A_{ar} + A_m, \quad A = E_m. \]

with \( A_{ar}, A_m \) being the area of reinforcement and matrix, respectively.

For the third group (Physical equations) we obtain:

\[ \varepsilon_x = \frac{1}{E_m}(-\mu_y \sigma_y) = -\frac{\mu_y \sigma_y}{E_m}; \]

\[ \varepsilon_y = \frac{\sigma_y}{E_m}; \quad \varepsilon_z = \frac{1}{E_m}(-\mu_z \sigma_y) = -\frac{\mu_z \sigma_y}{E_m}; \]

\[ \gamma_{xy} = \frac{\tau_{xy}}{G} = 0; \quad \gamma_{yz} = \frac{\tau_{yz}}{G} = 0; \quad \gamma_{zx} = \frac{\tau_{zx}}{G} = 0; \]

The tensile force after cracking will be:

\[ \sigma_y = \frac{N}{A_{ar}} \]

The distances between the cracks are determined based on the condition that the difference of the strain in tensile reinforcement in the sections with cracks and between cracks is balanced by the force of bond:

\[ \sigma_y A_{ar} \cdot \frac{l_{cr} \cdot p \cdot \tau}{}, \]

where \( \tau \) is the bond stress,

\( l_{cr} \) is the distance between cracks,

and \( p \) is the perimeter of reinforcement.

Break of a body occurs when the tensile stress in reinforcement reaches the failure level.

While the above equations give an insight into the nature of an enforced detail, another equation was used to calculate the theoretical ultimate tensile strength gain by enforcing 3D printed details: it is the Rule of Mixtures [8] adapted for calculating the ultimate tensile strength of composite materials with continuous and unidirectional fibres [9, 10];
\[
\left( \frac{f}{\sigma_{UTS,f}} + \frac{1-f}{\sigma_{UTS,m}} \right)^{-1} \leq \sigma_{UTS,e} \leq f \sigma_{UTS,f} + (1-f) \sigma_{UTS,m}
\]

where \( f \) is the volume fraction;

\( \sigma_{UTS,m} \) is the ultimate tensile strength for the relevant matrices, fibres, and composites.

4. RESULTS AND DISCUSSION

The additive manufacturing offers many advantages and opportunities, which share a common line as creating 3D objects of unlimited geometry, building the detail rapidly and allowing it to be modified between the building stages. When it comes to practical applications, a detail’s physical properties is what matters most. This means that, if a part to be produced by FDM method undergoes stress in a particular direction the best way is to align it accordingly to anisotropic properties.

5. CONCLUSIONS

The method described in this article is new and easily applicable; it does not require any modification for the existing 3D printing machines and gives significant gain in regard to the ultimate tensile strength of a detail to be produced. To better understand the behaviour of enforced 3D printed details, deeper experimental research is needed. Overall, the method shows a great potential for improving the strength of 3D printed parts beyond the currently offered.

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ĀTRĀS PROTOTIPĒŠANAS AR KAUSĒŠANAS METODI STRUKTURĀLĀ UZLABOJUMA ANALĪTISKS RISINĀJUMS.

I. Brensons, S. Polukoshko

Kopsavilkums

Darbā tiek apskatīts ātrās prototipēšanas veids, kura pamatā ir detalās veidošana, izmantojot kausētu materiālu parasti plastmasu. Šī detalās veidošanas metode ir kļuvusi par vienu no visizplatītākajām tās zemo izmaksu, pieejamo materiālu un daudzpusības dēļ.

Šī raksta mērķis ir izpētīt alternatīvu veidu, kā uzlabot prototipu mehāniskās īpašības, tādējādi palielinot printētu detalju izmantošanu kā galu produktu. Raksts piedāvā analītisko risinājumu, kā uzlabot ātro prototipu mehāniskās īpašības, uzlabojot tehnoloģiskos procesus, kas iesaistīti detalju izgatavošanā.

Darba pamatā tiek izmantota 3D printēšanas tehnoloģijas iespēja veidot iekšējus kanālus bez ģeometriskiem ierobežojumiem, kā rezultātā ir iespējams izveidot iekšēju kanālu shēmu, ko pēc tam piepildā ar citu materiālu, kam ir labākas mehāniskās īpašības kā pamata materiālam. Pildīšanai izmantotais materiāls ir epoksīda sveķi, kas pieļauj vieglu iepildīšanu šķidrā fāzē, un sniedz labas mehāniskās īpašības pēc sacietēšanas.

Analītiskais risinājums šajā rakstā piedāvā viegli realizējamu variantu kā būtiski uzlabot prototipu mehāniskās īpašības.

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