PRINCIPLES OF CREATION AND RECONSTRUCTION
OF DOT-MATRIX HOLOGRAMS

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A method for calculation of dot-matrix holograms is presented, whose distinction from other methods is simplicity of the formulas for the parameters of each pixel element in a hologram. At the Innovation Spectroscopy Center of DU a device for recording dot-matrix holograms has been designed, with a program using the proposed method for their calculation. Holograms 50×50 mm in size well corresponding to the initial graphic design were obtained.

Key word: *dot-matrix holography.*

1. INTRODUCTION

Nowadays, dot-matrix holograms are widely used for identification of industrial products and documents and their protection against counterfeiting as well as for packing and decorating purposes. A dot-matrix hologram is composed of thousands of fine diffraction grating dots. The fringe space and orientation of each diffraction grating dot is computer-controlled against a pattern. These diffraction grating dots can be exposed onto a photoresist plate, forming a relief hologram that is used for mass production with embossing technique.

The dot-matrix holograms are obtained by decomposition of a hologram image into a two-dimensional array of elementary pixels. The pixel size is usually in the range of 10–200 μm and depends on the technology of optical recording. In each elementary pixel of a hologram there is a diffraction grating with definite parameters, such as the period and the orientation angle in the plane of hologram surface (Fig. 1). The size and shape of pixels may also change. The shape influences the factor of filling the whole hologram with pixels containing diffraction gratings. For example, the filling factor can reach 100% for rectangular pixels, whereas for spherical ones it is only 80%. Consequently, in the first case the relative diffraction efficiency of a hologram will be about 20% higher. By varying the size of pixels or the exposure time during optical recording it is possible to attain modulation of diffracted light intensity at the hologram reconstruction, i.e., to change the visual brightness of pixels. There exist technologies of digital hologram records: laser-beam or e-beam lithography, classical two-beam dot-matrix holography [1] and one-beam holography using SLM (spatial light modulator) micro-display [2]. However, in all these cases the main idea of the hologram synthesis remains the same. A hologram consists of a two-dimensional array of diffraction elements having a micron-level size, with parameters that need mathe-

matical calculation. Therefore the problem of search for an optimal calculation model for dot-matrix digital holograms remains topical.

Fig. 1. SEM image of a dot-matrix hologram at different magnifications:
a – dot-matrix structure of the hologram; b – image of one pixel; c – diffraction grating in the pixel.

2. THE METHOD OF DOT-MATRIX HOLOGRAM CALCULATION

The main idea underlying the process of dot-matrix hologram synthesis is creation of the algorithm that would correlate each dot of a graphic design having definite coordinates and color with the diffraction pixel of a hologram with calculated parameters. In this chapter, the physical background of dot-matrix holography is described and the formulas for the operating algorithm of such transformation are derived. The general task is to define conditions when a definite element of a hologram would direct a given spectral portion of diffracted light towards the observer, who would be able to see diffracted light of each element of the hologram only at a certain angle. The total perception of diffraction of all the elements makes a visual effect that corresponds to the initial graphic design. If it is necessary to make the incident light diffract in a given direction, one may start with recording the pixel diffraction grating with two beams. The former should be directed from the light source to the pixel, while the latter – along the direction of observation. In this case, during the reconstruction of a hologram the beam of light incident along one of these directions will diffract towards the second one. Figure 2 shows the situation when the observer sees the light diffracted from a hologram pixel with coordinates (x, y) . Here, it is important to understand that the parameters of a diffraction grating (such as period and orientation) in this place will correlate with those at recording a hologram in the X – Y plane using beams with wave vectors $\vec{k}_1$ and $-\vec{k}_2$ if the coherence and monochromaticity condition $|\vec{k}_2| = |\vec{k}_1|$ of convergent beams is met. Such an approach makes it possible, first, to calculate the parameters of each diffraction element of a hologram if we know positions of the observer and the light source, and then to specify the settings of optical record system in such a way that an element with calculated parameters could be obtained on the photoresist surface.

It is obvious that the main factors for reconstruction of the whole hologram are the orientation and period of diffraction grating in each pixel as well as the position of the observer and the light source. Under such conditions, the holographic image will correspond to the initial graphic design. If positions of the

observer and the light source are defined, period d and orientation angle ψ of the grating completely determine the conditions when the incident white light can diffract within a given spectral region towards the observer. In this case the color of a hologram element corresponds to that of a dot in the initial graphic image. The model underlying the algorithm of the program for calculating the diffraction pixel parameters is presented below in detail.

Fig. 2. Formation of the image by diffractive element.

We may separate a narrow section from the spectral structure of incident light with wave vector $\vec{k}_1$ having the following direction cosines: $(\cos \alpha_1, \cos \beta_1, \cos \gamma_1)$. Thus, in the chosen coordinate system

$$\vec{k}_1 = |\vec{k}_1| (\vec{e}_x \cos \alpha_1 + \vec{e}_y \cos \beta_1 + \vec{e}_z \cos \gamma_1). \quad (1)$$

The equiphasal wave surfaces are represented by a set of planes K_1 perpendicular to vector $\vec{k}_1$ and can be determined by the equation:

$$x \cos \alpha_1 + y \cos \beta_1 + z \cos \gamma_1 = \text{const}_j. \quad (2)$$

The distance between phase planes is equal to the wavelength λ .

Intersections of planes K_1 form a series of parallel straight lines with the X - Y plane where the hologram is situated. These lines can be determined using the equation resulting from (2) if $z = 0$:

$$y = -(\cos \alpha_1 / \cos \beta_1)x + \text{const}_j. \quad (3)$$

The angle of intersection of such a line with the X -axis will be

$$\varphi_1 = \arctg(\cos \alpha_1 / \cos \beta_1), \quad (4)$$

and the distance between neighboring lines in the set –

$$d_1 = \lambda / \sin \gamma_1. \quad (5)$$

Analogously, intersection lines of the X - Y plane and planes K_2 related to the diffraction directed towards vector $\vec{k}_2$, form with the X -axis the angle

$$\varphi_2 = \arctg(\cos \alpha_2 / \cos \beta_2). \quad (6)$$

The distance between neighboring straight lines is

$$d_2 = \lambda / \sin \gamma_2. \quad (7)$$

Fig. 3. Geometrical configuration illustrating derivation of formulas for the parameters of a diffraction element.

Two sets of parallel lines related to vectors $\vec{k}_1$ and $\vec{k}_2$ define the third set of parallel lines K_3 that form the lines of diffraction grating. This series is characterized by period d and orientation angle ψ (Fig. 3a). One can also consider the set K_3 as a result of interference of waves $\vec{k}_1$ and $-\vec{k}_2$ in the X - Y plane. In this case the position of intensity maximums is defined by K_3 . Figure 3b shows geometrical constructions that explain the derivation of formulas for the parameters of a diffractive element. For facets a , b , c , corresponding to the incident and diffraction waves and the diffraction grating, we obtain the following:

$$a = d_2 \sin \theta = \frac{\lambda}{\sin \gamma_2 \sin (\varphi_2 - \varphi_1)}; \quad (8)$$

$$b = d_1 \sin \theta = \frac{\lambda}{\sin \gamma_1 \sin (\varphi_2 - \varphi_1)}; \quad (9)$$

$$c^2 = a^2 + b^2 - 2ab \cos \theta. \quad (10)$$

Here $\theta = \pi + \varphi_1 - \varphi_2$ is the angle between the lines of K_1 and K_2 sets.

The final formula for calculating the orientation angle of a grating will be:

$$\psi = \varphi_1 + \arcsin(b \sin \theta / c). \quad (11)$$

After the substitution of (8)–(10) in (11) we will have:

$$\psi = \varphi_1 + \arcsin \left(\frac{\sin \gamma_2 \sin(\varphi_2 - \varphi_1)}{\sqrt{\sin^2 \gamma_1 + \sin^2 \gamma_2 + 2 \sin \gamma_1 \sin \gamma_2 \cos(\varphi_2 - \varphi_1)}} \right). \quad (12)$$

When analyzing formula (12) we can see that ψ is a function of direction angles (α, β, γ) of vectors $\vec{k}_1$ and $\vec{k}_2$ in the coordinate system. If positions of the light source and the observer remain unchanged, the orientation angle $\psi = \psi(x, y)$ will be only a coordinate function of a pixel in the hologram (Fig. 2).

For the period of diffraction grating in a pixel we obtain the following formula:

$$d = a \sin(\psi - \varphi_1) = \frac{\lambda \sin(\psi - \varphi_1)}{\sin \gamma_2 \sin(\varphi_2 - \varphi_1)}. \quad (13)$$

From (13) it follows that the period of grating is defined not only by the position of a pixel in the hologram but also by its visible color, thus being a function of three arguments: $d = d(x, y, \lambda)$.

For calculating the parameters of a diffraction element by Eqs. (12) and (13) some assumptions could be made for the sake of simplification. For this, we will take a coordinate system with the origin in the center of a hologram lying in the X - Y plane. The observer and the source of light are in the Y - Z plane, and the observation is directed along the Z -axis. Since the distance to the light source is usually much larger than that between the hologram and the observer (R), we can consider the source as infinitely remote and $\vec{k}_1 = \text{const}$ (Fig. 3). This means that $\alpha_1 = 90^\circ$ and $\varphi_1 = 0$. The size of a hologram does not usually exceed 2–4 cm and is much smaller than the distance between the hologram and the observer, $R = 30$ –50 cm, therefore $\gamma_2 = 0$ for the central element of a hologram and $\gamma_2 = -\sqrt{x^2 + y^2} / R$ for an element with coordinates (x, y) . According to the assumptions made we can rewrite formula (12) as

$$\psi \approx \frac{\gamma_2 \sin \varphi_2}{\sin \gamma_1}, \quad (14)$$

where for small angles $\sin \gamma_2 \approx \gamma_2$ and $\sin \gamma_2 \ll \sin \gamma_1$.

For a diffraction element with coordinates (x, y) :

$$\cos \alpha_2 = -x / R, \quad \cos \beta_2 = -y / R, \quad \gamma_2 = \sqrt{x^2 + y^2} / R. \quad (15)$$

By virtue of (6) and (14) we have:

$$\varphi_2 = -\arctg(x / y), \text{ and therefore } \sin \varphi_2 = x / \sqrt{x^2 + y^2}. \quad (16)$$

Substituting the values from (15) and (16) into formula (14) we will obtain:

$$\psi = x / (R \sin \gamma_1) = Kx. \quad (17)$$

Constant K in formula (17) depends on the positions of the light source and the observer. It has experimentally been defined that a good choice for calculations is $K \approx 1^\circ/\text{mm}$.

Substitution of (15)–(17) into formula (13) transforms it to the following:

$$d = \frac{\lambda R}{x} \sin\left(\frac{x}{R \sin \gamma_1}\right) \approx \frac{\lambda}{\sin \gamma_1}. \quad (18)$$

The obtained formulas (17) and (18) have a simple form and are basic for the computerized calculation of pixel parameters. We propose the following visual physical interpretation of these formulas. At a definite position of a hologram the observer will see only the pixels whose orientation is related to their positions through Eq. (17). The color of each pixel in this ensemble is defined only by the period of grating in accordance with formula (18). If the observer turns the hologram situated in the X – Y plane by angle $\Delta\psi$, another ensemble of pixels will become visible. For the orientation angle of these pixels, the following equity is used:

$$\psi = -\Delta\psi + Kx. \quad (19)$$

We have performed two holographic recordings under different conditions to prove that formula (19) works correctly. Figure 4 presents photographs of two 10×10 mm dot-matrix holograms with recovered image. In Fig. 4a the orientation of all pixels is the same, and the intensity of light diffracted in the direction towards the observer (photo camera in this case) decreases when pixels are moving away from the center of a hologram. In Fig. 4b the orientation of elements changes according to formula (19), and the intensity of diffraction of all elements in the observer direction is the same.

Fig. 4. Photographs of an image recovered from dot-matrix hologram:
a – orientation angle of diffraction grating of all pixels $\psi = 0$;
b – orientation angle of diffraction grating depends on the position of pixels $\psi = Kx$.

3. RESULTS AND DISCUSSION

The potentialities of the dot holographic recording that arise from Eqs. (17–18) are as follows. A programmed realization of kinetic visual effects is possible when by turning a hologram the visible images are changing. Another example is the possibility to imitate three-dimensional images when for each eye an individual image is formed corresponding to the changed viewing angle of an object (the perspective). It is also possible to obtain a holographic image in real colors. In this

case each dot of a graphic image corresponds to three diffraction elements, and an R–G–B model of basic color mixing is realized in the direction towards the observer. It should be mentioned that formula (18) contains a multiplier that depends on the position of the light source in relation to a hologram. Therefore the correlation of pixel colors with Eq. (18) can be seen only under the condition of a definite position of the light source. At the IS Center of DU, we have created a dot-matrix holographic recording device in which only the orientation of pixel diffraction grating can be changed by the program. The beam convergence angle in the optical scheme is constant, being chosen in such a way that the period of diffraction gratings corresponds to the middle of spectra. At $\lambda = 550$ nm and $\gamma_1 = 30^\circ$ the diffraction grating frequency, according to formula (18), will be $f = d^{-1} \approx 1000/\text{mm}$. The pixel parameters are calculated based on the graphic image stored as a graphics file in **.bmp** computer format. The color organization of the file is the following: a black-and-white image, with a shade of gray, of an 8-bit intensity. To each graphic dot with coordinates (w, h) , $w \in [0, \text{width}]$, $h \in [0, \text{height}]$, a diffraction pixel corresponds with the same coordinates in the hologram. The gray level value $g \in [0 \dots 255]$ of the image dots corresponds to the angle $\theta_g \in \left[\frac{g\theta_{\max}}{255} \right]$ of grating orientation in the X – Y plane. After the hologram is

turned by angle θ_g , the observer will be able to see only those elements of a hologram that correspond to the image dots of color g . Such a result can be obtained if the pixel grating orientation angle is defined by formula (19) or as a function of the computer image parameters:

$$\psi(w, h, g) = -\frac{g\theta_{\max}}{255} + \frac{Kw}{P_{\text{res}}}, \quad (20)$$

where P_{res} is the linear density of diffraction pixels forming a hologram.

The above formula describes the mechanism of graphic file image transformation into a holographic one.

4. CONCLUSIONS

The results of the work allow for the following conclusions.

A method for calculation of dot-matrix holograms has been developed. As compared with other ones (see, e.g. [3, 4]), in this method simpler formulas are used to calculate the parameters of each pixel element in a hologram. Based on the method, a transformation algorithm has been realized. It was proved that the computer-controlled dot-matrix device developed and constructed at the IS Center (DU) can successfully be used for holographic recording. In the framework of the developed model, a formula was derived for describing the mechanism of transformation of a graphic file image into a holographic one. The model gives good results for hologram dimensions not exceeding 50×50 mm, which is sufficient for the majority of scientific and applied tasks. For larger holograms, color deviations from the initial image become visible, which has been attained due to the simplification of known formulas. The obtained high-quality dot-matrix holograms can be used for production of holographic labels.

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PUNKTU MATRICAS HOLOGRAMMU VEIDOŠANAS UN REKONSTRUKCIJAS PRINCIPI

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K o p s a v i l k u m s

Rakstā apskatīta punktu matricas hologrammas aprēķina teorētiskā metode. Metodes īpatnība un atšķirība no citām ir galīgo formulu vienkāršība, kuras lieto pikseļu parametru aprēķināšanai. Daugavpils universitātes Fizikas katedras Inovātīvās spektroskopijas centrā izstrādāta un izveidota punktu matricas hologrammu ieraksta ierīce ar programmu, kurā hologrammas aprēķinam izmantots aprakstītais teorētiskais modelis. Iegūtas hologrammas ar izmēriem līdz 50×50 mm, kuras labi atbilst sākotnējam grafiskajam dizainam.

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