Application of artificial neural networks (ANN) in Lake Drwęckie water level modelling

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Abstract: This paper presents an attempt to model water-level fluctuations in a lake based on artificial neural networks. The subject of research was the water level in Lake Drwęckie over the period 1980-2012. For modelling purposes, meteorological data from the weather station in Olsztyn were used. As a result of the research conducted, the model M_Meteo_Lag_3 was identified as the most accurate. This artificial neural network model has seven input neurons, four neurons in the hidden layer and one neuron in the output layer. As explanatory variables meteorological parameters (minimal, maximal and mean temperature, and humidity) and values of dependent variables from three earlier months were implemented. The paper claims that artificial neural networks performed well in terms of modelling the analysed phenomenon. In most cases (55%) the modelled value differed from the real value by an average of 7.25 cm. Only in two cases did a meaningful error occur, of 33 and 38 cm.

Key words: neural networks, postglacial lake, East Central Europe, Lake Drwęckie

Introduction

As an element of the environment, lakes undergo qualitative changes (e.g. in chemical composition) and quantitative changes (e.g. fluctuations in water level and resources, recasting of lake basin geometry). Individual transformations are interconnected or interdependent. The ability to determine the influence of specific environmental factors on the changes which occur in lakes is a significant and current issue within the field of limnology.

Among the above-mentioned quantitative changes, water level fluctuation is the most dynamic in nature. Moreover, it determines the shape and regime of lakes. In numerous papers authors point out the existence of miscellaneous elements which have an impact on water supply and outflow, governing the scale of water level changes. Generally, one may distinguish natural and anthropogenic factors. At the same time, it is important to note that these factors often come into play concurrently, and it is therefore not an easy task to distinguish and assign a hierarchy to their impacts

(Marszelewski et al. 2011). From a global perspective, currently observed climate fluctuations are considered by many scientists to be the main cause underlying changes in lake water levels (Kadioğlu et al. 1997; Sen et al. 1999).

Many authors have investigated issues relating to water-level fluctuations in Polish lakes. Their research focused mainly on amplitude (Paślowski 1975, Borowiak 2000), cyclicity (Skibniewski 1953; Paślowski 1972; Dąbrowski and Węglarczyk 2005) and tendency (Bajkiewicz-Grabowska 2001; Dąbrowski 2004) of water level changes. The most extensive study on water-level fluctuations in Polish lakes is the work of Borowiak (2000). The author has considered 48 lakes, the vast majority of whose measured time series exceed 30 years of observations.

The aim of this work is to present the possibility of modelling water-level changes using artificial neural networks. The ANN method has previously been applied in research on broadly-defined water resources (Jain et al. 2001; Cancelliere et al. 2002; Muleta and Nicklow 2005; Sivapragasam and Muttill 2005;

Chelmecki et al. 2003; Pociask-Karteczka 1998; Skalo-
va et al. 2011). So far only a sparse number of publica-
tions in this research area have been related to lakes
(Ondimu and Murase 2006; Çimen and Kisi 2009;
Coulibaly 2010). What is more, authors have focused
on large lakes (with areas of several thousand square
kilometres), or those located in warm climate areas. So
far, post-glacial lakes of relatively small surface areas
and lying in a temperate climate have not been the
subject of such studies.

Object of study

ANN has been used to model the water level
changes of Lake Drwęckie in northern Poland (Fig.
1). Lake Drwęckie, similarly to the majority of Polish
lakes, was formed by the erosive and accumulative ac-
tivities of the Baltic continental ice sheet. The River
Drwęca is its main tributary, with an average flow to
Lake Drwęckie amounting to $2 \text{ m}^3 \text{ s}^{-1}$ (at the Idzbark
station) and an outflow of over $7 \text{ m}^3 \text{ s}^{-1}$ (at the Sam-
borowo station). The highest water levels at both sta-
tions are observed in spring (March-April) and the
lowest during late autumn and winter (November-
December).

Lake Drwęckie itself has an area of around 8.8 km^2 , and the average depth is 5.7 metres. The lake
shape is very characteristic and consists of two elon-
gated gullies which coincide at a sharp angle, broken
by a necking, over which a viaduct has been built. The
north-eastern gully is about 5 kilometres long and has
a relatively large, high and forested island. In the east-

ern part of the gully is the town of Ostróda with 34,000
inhabitants. The second gully has a parallel course and
a length of about 12 kilometres. The western part has a
variegated lakeside, which is generally flat and marshy,
whereas in the southern lakeshore it is high and steep
(WIOS Olsztyn 2006).

The total catchment area is rather vast, at about
 1015.2 km^2 . The direct catchment area is 8.5 km^2 and is
covered with forest (67.7%). The remaining part con-
sists of arable land, idle land and buildings. The lake is
connected, by means of watercourses and canals, with
several smaller lakes (with areas less than 1 km^2), in-
ter alia: Ilińsk, Sajmino, Pauzeńskie, Ilgi, Smordy and
Morliny.

Materials and methods

In this research, monthly mean values of the
water level in Lake Drwęckie for the period 1980-2012
were used (Fig. 2). These values were determined by
daily observations conducted as a part of the monitor-
ing services of the Institute of Meteorology and Water
Management, National Research Institute (IMGW-
PIB). Selected meteorological parameters recorded
by the Olsztyn meteorological station for the years
1980-2012 were taken as independent variables. Mean
monthly values for the following meteorological pa-
rameters were used: (i) average air temperature (T),
(ii) maximal temperature (T_Max), (iii) minimal tem-
perature (T_Min), (iv) relative humidity (R_H), (v)
sum of rainfall (Rain), (vi) wind speed (W_S).

Fig. 2. Lake Drwęckie water level fluctuations in hydrological years 1981-2012

Fig. 1. Subject of study location: 1 – Idzbark, 2 – Samborowo

Modelling of any kind of environmental phenomenon requires the acceptance of a number of simplifications and assumptions (Soczyńska 1997; Ozga-Zielińska and Brzeziński 1997). In the case under analysis, it has been assumed that the following elements remain stable: size and catchment development, shape of lake basin and afforestation rate. The aforementioned meteorological parameters and lagged values of water level (Lag 1-4) have been used as input variables determining the water level of the investigated lake. Table 1 presents statistical parameters of endogenous (water level (W_L)) and exogenous variables.

Selection of explanatory variables

There is no universal method of input selection for ANN models (Babel et al. 2011). The literature shows that, so far, authors have used methods of linear cross-correlation, sensitivity analysis and heuristic ap-

proaches (Bowden et al. 2005). In this study, the input selection has been made on the basis of expert knowledge and presumption based on the correlation coefficient between dependent and independent variables (Table 2). On the initial assumption that all variables have a significant impact on water level, the whole model may be expressed by the following function (1):

$$W_L = (T, W_S, T_{Max}, T_{Min}, RH, Rain, W_{L(Lag)}) \quad (1)$$

The magnitude of correlation coefficients indicates that meteorological parameters have low interdependence with water level. However, there is a strong correlation between water level and its values from previous periods. In particular for Lag_1 and Lag_2, whereas for Lag_3 the value is not very beneficial but still remains statistically significant. Against this background, it has been assumed that initial ANN

Table 1. Statistical parameters of dependent and independent variables (nomenclature explained in the text above)

Variable	Xmax	Xmin	Xavg	STDx	Var.(unit ²)	Skew	Kurtosis
T [°C]	21.2	13.8	7.5	7.6	57.3	0.2	1.0
W _S [m s ⁻¹]	4.8	1.4	2.8	0.6	0.4	0.4	0.0
T _{Max} [°C]	27.7	10.4	11.4	8.7	75.4	0.1	1.2
T _{Min} [°C]	15.5	17.9	3.5	6.6	43.6	0.3	0.7
R _H [%]	98.6	52.2	79.3	9.0	80.6	0.4	0.6
Rain [mm]	245.6	0.0	43.4	33.3	1107.3	1.4	3.9
W _L [cm]	518.0	422.0	456.7	14.7	215.0	0.7	1.2

Table 2. Correlation matrix for dependent and independent variables

	W _L	W _L Lag 1	W _L Lag 2	W _L Lag 3	W _L Lag 4	T	T _{Max}	T _{Min}	W _S	R _H	Rain
	[cm]	[cm]	[cm]	[cm]	[cm]	[°C]	[°C]	[°C]	[m s ⁻¹]	[%]	[mm]
W _L	1.000	0.642	0.360	0.194	0.090	0.157	0.162	0.145	0.001	0.230	0.049
W _L Lag 1	0.642	1.000	0.642	0.358	0.192	0.296	0.309	0.266	0.154	0.360	0.015
W _L Lag 2	0.360	0.642	1.000	0.641	0.356	0.352	0.357	0.331	0.175	0.314	0.056
W _L Lag 3	0.194	0.358	0.641	1.000	0.640	0.326	0.315	0.329	0.132	0.162	0.119
W _L Lag 4	0.090	0.192	0.356	0.640	1.000	0.205	0.185	0.227	0.109	0.016	0.103
T	0.157	0.296	0.352	0.326	0.205	1.000	0.994	0.985	0.442	0.634	0.339
W _S	0.001	0.154	0.175	0.132	0.109	0.442	0.455	0.404	1.000	0.254	0.068
T _{Max}	0.162	0.309	0.357	0.315	0.185	0.994	1.000	0.963	0.455	0.695	0.326
T _{Min}	0.145	0.266	0.331	0.329	0.227	0.985	0.963	1.000	0.404	0.524	0.353
R _H	0.230	0.360	0.314	0.162	0.016	0.634	0.695	0.524	0.254	1.000	0.023
Rain	0.049	0.015	0.056	0.119	0.103	0.339	0.326	0.353	0.068	0.023	1.000

Note: Boldface black type indicates a correlation coefficient of P value less than 0.05.

models will be built on the basis of statistically significant meteorological parameters (T, T_Max, T_Min, R_H), and for subsequent models, lagged water level values will be gradually added.

Artificial neural networks

Not all dependencies existing in nature can be described by linear models. What is more, the application of linear regression models is limited by their weak potential for generalization and extrapolation. This means that results obtained are often satisfactory only within the range of parameters used for their creation. Correlation coefficient values from Table 2 indicate that, for the analysed phenomenon, there is no possibility to identify one variable which will enable us to build a satisfactory linear model.

The artificial neural network is a method which can be used to model such a complex phenomenon. The ANN is a mathematical reflection of how neurons work in the human brain. By exploiting networks of

artificial neurons organized in layers (Fig. 3), an input signal is passed down, and then, based on transformation functions, is converted into an output signal. The characteristic feature of ANNs is their ability to learn based on a teaching set, whereas ANNs' ability to generalize is guaranteed by a testing set. Its aim is to govern the learning process in a way which excludes learning by heart.

To build ANN models Statistica 10 software was used (Table 3). Its features allowed the creation of multilayer perceptron (MLP) networks with the aid of an automatic network creator, which looks through a user-defined number of networks and then selects those which have the best (smallest) error values for teaching and validating sets.

Model performance criteria

The created artificial neural network models have been assessed based on the following: the correlation coefficient between observed and modelled

Fig. 3. Three-layer perceptron with connections between neurons, for three input and one output variables, based on Tadeusiewicz (1993)

Table 3. Artificial neural network model characteristics

MODEL	Architecture	Error (set)			Teaching algorithm	Activation function	
		Training	Testing	Validation		Hidden layer	Output layer
M_Meteo	MLP 4-7-1	89.27	73.38	136.68	BFGS 29	Exponential	Logistic
M_Meteo_Lag_1	MLP 5-3-1	55.13	51.16	53.80	BFGS 57	Tanh	Linear
M_Meteo_Lag_2	MLP 6-5-1	54.14	52.60	52.58	BFGS 55	Tanh	Logistic
M_Meteo_Lag_3	MLP 7-4-1	50.70	49.62	47.47	BFGS 66	Tanh	Logistic

values of water level (CC); the criterion of model efficiency (E, formula 2; Nash et al. 1970); mean absolute percentage error (MAPE, formula 3); coefficient of determination (R^2 , formula 4); and root mean squared error (RMSE, formula 5):

$$E = 1 - \frac{\sum_{i=1}^n (W_L \cdot obs_i - W_L \cdot mod_i)^2}{\sum_{i=1}^n (W_L \cdot obs_i - W_L \cdot avg)^2}, \quad (2)$$

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left| \frac{W_L \cdot obs_i - W_L \cdot mod_i}{W_L \cdot obs_i} \right|, \quad (3)$$

$$R^2 = \frac{\sum_{i=1}^n (W_L \cdot mod_i - W_L \cdot avg)^2}{\sum_{i=1}^n (W_L \cdot obs_i - W_L \cdot avg)^2}, \quad (4)$$

$$RMSE = \sqrt{\left(\frac{1}{n} \sum_{i=1}^n (W_L \cdot mod_i - W_L \cdot obs_i)^2 \right)}. \quad (5)$$

The criterion of model efficiency E takes values in the range from minus infinity to plus one, inclusive. A value of one corresponds to a perfect match of modelled data to observed data, whereas 0 indicates that the model is as accurate as the mean of the observed data. When $E < 0$, the observed mean is a better way to model data than the analysed model.

Mean absolute percentage error takes values in the range from zero to plus infinity. Models with lower MAPE values more precisely map the course of investigated phenomena variability.

The coefficient of determination R^2 informs us to what extent the variability of an input variable has been explained by the created model. This criterion takes values in the range of 0-1, where values less than 0.5 suggest that, from a modelling perspective, this model is unsatisfactory.

The RMSE is a balanced evaluation of errors, where, for a perfect model, the expected value of the RMSE would be 0. The final selection of the model has been done for a testing set, based on the criteria described above.

Results

Four different artificial neural network models were built. In each model all statistically significant ($p < 0.05$) meteorological parameters were used, whereas in models denoted as (Lag_1 etc.) an additional variable was introduced describing the water level in former periods. However, Lag_1 means that only information about the water level from one month earlier has been used, but Lag_2 denotes that the water level in period T has been modelled using values from periods Lag_1 and Lag_2, and so on. An architecture of created ANN models is presented in Table 1. For example, in the case of the model named M_Meteo_Lag_2, an MLP-type of network has been used which has six input neurons – five neurons in a hidden layer and one neuron in the output layer. This network was taught by means of the BFGS 55 teaching algorithm, while hyperbolic tangent and logistic function are used as activation functions.

In order to select a model which is capable of most precisely reflecting the analysed phenomenon, all ANNs were assessed based on the following criteria (CC, E, MAPE, R^2 and RMSE) for a testing set (Table 4).

It has been found that model M_Meteo_Lag_3 outperforms the other models in terms of applied quality measures. It has the lowest value of MAPE and RMSE errors and the highest value of R^2 and E criteria. The model based exclusively on meteorological data performed only slightly worse in MAPE error. However, from CC, R^2 and E it can be seen that a significant part of phenomenon variability has still not been explained. In Figure 4, real values and those generated by model M_Meteo_Lag_3 are presented. The distribution of absolute error has been visualized by means of a histogram (Fig. 5).

Table 4. Values of quality measures for model fit

MODEL	Performance criterion				
	MAPE	CC	R2	E	RMSE [cm]
M_Meteo	2.22%	0.3725	0.1092	0.1270	13.6805
M_Meteo_Lag_1	1.67%	0.7054	0.4336	0.4949	10.4390
M_Meteo_Lag_2	1.70%	0.7082	0.4774	0.5011	10.3746
M_Meteo_Lag_3	1.63%	0.7317	0.5118	0.5347	10.0190

Fig. 4. Observed and modelled lake water levels obtained from model M_Meteo_Lag_3

Fig. 5. Histogram of absolute errors

Khatibi et al. (2014) tested model performance by means of the coefficient of correlation values, assuming that: $CC > 0.95$ = good performance; $0.95 < CC < 0.9$ = satisfactory performance; $0.8 < CC < 0.9$ = poor performance; $CC < 0.8$ = very poor performance. Assuming that CC is a bounding criterion, the performance of all the models detailed in this paper is less than satisfactory. However, this is not the only way to assess models. In the case of RMSE, we obtained values only 4 cm less beneficial than the best ANN models created in the above-mentioned paper. What is more, MAPE value is very promising because the difference between our models and those in the refer-

enced paper is not greater than 2 percentage points. It is worth mentioning that the MAPE criterion is very sensitive to the values of the time series. Therefore, in the paper mentioned, in the case of Salt Lake (USA) and Lake Van (Turkey), where average measured water level values are about 1280 metres and 1648 metres, the value of the MAPE criterion is close to 0. To sum up, results obtained do not diverge in terms of quality from those generated by R. Khatibi et al., particularly when comparing model performance for Lake Drwęckie with models created for lakes with a similar or smaller catchment area.

Conclusions

Based on the study conducted it can be stated that modelling water level changes by means of artificial neural networks gives satisfactory results.

Weak dependence between meteorological parameters and water level forced us to include the water level from previous months in modelling values. As a result, we have included the long-term effect of changing meteorological parameters on water level.

The best model for Lake Drwęckie, *M_Meteo_Lag_3*, is a multilayer perceptron ANN with seven neurons in the input layer, four neurons in the hidden layer and one neuron in the output layer. The following meteorological parameters were used as independent variables: temperature (T), minimal and maximal temperature (T_Min, T_Max) and relative humidity (T_H.). But this model has performed very well, mainly because of values of dependent variables from former periods (Lag_1, Lag_2 and Lag_3).

In most (55%) cases of the testing set, the modelled value was smaller or greater than the observed value by less than 7.25 cm (average error). Only in two cases were greater errors observed, namely 33 and 38 cm.

Regarding results obtained by other authors in similar studies, ANN models for Lake Drwęckie performed on a very similar level. Special attention has been paid to differences when it comes to evaluation measures, and to the coefficient of correlation in particular. It has been observed that some differences may have resulted from the fact that some lakes may be prone to greater water level fluctuations as a result of external factors. What is more, Lake Drwęckie is much smaller than other lakes investigated so far. Additionally, in winter periods, it is usually covered by an ice sheet, which distorts the water level, especially during spring melt.

The value of the R^2 criterion indicates that 45% of time series variability in the case of the best model *M_Meteo_Lag_3* has not been explained by exogenous variables. Therefore, further analysis of water level changes should include meteorological parameters from more than one meteorological station, and the water level of main tributaries.

In future research it is planned to pay more attention to the role of catchment area in water level modelling. Comparison of our results with those obtained by other authors revealed some interesting dependencies which require further detailed analysis.

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