DETERMINING A PARTICULAR SOLUTION FOR N -TH ORDER LINEAR DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS THAT HAVE THE FREE TERM IN THE FOLLOWING FORM

$$e^{a \cdot x} \cdot P(x) \cdot \cos^n x \text{ or } e^{b \cdot x} \cdot Q(x) \cdot \sin^n x$$

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Abstract: A particular solution for the n -th order linear differential equations with constant coefficients that are free of term such as $P(x) \cdot e^{a \cdot x} \cdot \cos^n x$ or/and $Q(x) \cdot e^{b \cdot x} \cdot \sin^n x$, $n \in N$, can be determined based on two elements: the way in which $\cos^n x$ and $\sin^n x$ can develop, and, on the other hand on the way a particular solution for the free terms $P(x) \cdot e^{a \cdot x} \cdot \cos^n x$ or/and $Q(x) \cdot e^{b \cdot x} \cdot \sin^n x$, $n \in N$ is sought. We can, of course, write the way a particular solution looks also in the case we have a combination of the two terms or more terms of this kind.

Keywords: n -th order linear differential equations with constant coefficients, particular solution, general solution, free term of the following form $P(x) \cdot e^{a \cdot x} \cdot \cos^n x$ or/and $Q(x) \cdot e^{b \cdot x} \cdot \sin^n x$

Introduction

Various processes and phenomena from different technical domains can be modeled using the differential equations. Thus, through the differential equations, different physical or other types of laws are described, later on being determined, as the solutions of these equations, different links among the laws' parameters which described those processes and phenomena.

Of course, it is not possible to find a solution for any differential equation, and it is even more difficult to find an explicit solution, but, when we can do this, it is useful to identify the classes of these kinds of

differential equations and the way in which they can be solved.

It is the case of the present study in which, based on an observation, for a n -th order linear differential equation with constant coefficients in which the free term is $P(x) \cdot e^{a \cdot x} \cdot \cos^n x$ or/and $Q(x) \cdot e^{b \cdot x} \cdot \sin^n x$, $n \in N$, the way in which a particular solution can be obtained is presented.

We will start by solving the first problem, namely how $\cos^n x$ or $\sin^n x$, $n \in N$ develops. We have the following relationships:

$$\cos^n x = \frac{1}{2^{n-1}} \cdot \left[\sum_{\substack{r=0 \\ 2 \cdot r < n}} C_n^r \cdot \cos(n-2 \cdot r) \cdot x \right] + \frac{1}{2^n} \cdot C_n^{n/2}, n \text{ even}; \quad (1)$$

$$\cos^n x = \frac{1}{2^{n-1}} \cdot \left[\sum_{\substack{r=0 \\ 2 \cdot r < n}} C_n^r \cdot \cos(n-2 \cdot r) \cdot x \right], n \text{ odd}; \quad (2)$$

$$\sin^n x = \frac{(-1)^{n/2}}{2^{n-1}} \cdot \left[\sum_{\substack{r=0 \\ 2 \cdot r < n}} (-1)^r \cdot C_n^r \cdot \cos(n-2 \cdot r) \cdot x \right] + \frac{1}{2^n} \cdot C_n^{n/2}, n \text{ odd}; \quad (3)$$

$$\sin^n x = \frac{(-1)^{(n-1)/2}}{2^{n-1}} \cdot \left[\sum_{\substack{r=0 \\ 2 \cdot r < n}} (-1)^r \cdot C_n^r \cdot \sin(n-2 \cdot r) \cdot x \right], n \text{ odd}. \quad (4)$$

All of the above relationships are easily demonstrated using mathematical induction.

Problem description

Let us consider a n -th order linear differential equation with constant coefficients with the following form

$$a_0 \cdot y^{(m)}(x) + a_1 \cdot y^{(m-1)}(x) + \dots + a_n \cdot y(x) = f(x), \\ x \in I \subset \mathbf{R} \text{ an interval of } \mathbf{R}. \quad (5)$$

The homogenous equation attached to the equation (5) is

$$a_0 \cdot y^{(m)}(x) + a_1 \cdot y^{(m-1)}(x) + \dots + a_n \cdot y(x) = 0, \\ x \in I \subset \mathbf{R} \text{ an interval of } \mathbf{R}. \quad (6)$$

We will take in turn the 4 cases of free terms, cases generated by the development of terms $\cos^n x$ and $\sin^n x$.

The first situation corresponds to a free term $f(x)$ of the form

$$f(x) = P(x) \cdot e^{a \cdot x} \cdot \cos^n x, \quad (7)$$

where $a \in \mathbf{R}$, P is a polynomial with degree $P = s$ and n is even.

The second situation corresponds to a free term $f(x)$ of the form

$$f(x) = P(x) \cdot e^{a \cdot x} \cdot \cos^n x, \quad (8)$$

where $a \in \mathbf{R}$, P is a polynomial with degree $P = s$ and n is odd.

The third case corresponds to a free term $f(x)$ of the form

$$f(x) = Q(x) \cdot e^{b \cdot x} \cdot \sin^n x, \quad (9)$$

where $b \in \mathbf{R}$, Q is a polynomial with degree $Q = r$ and n is odd.

The last case corresponds to a free term $f(x)$ of the form

$$f(x) = Q(x) \cdot e^{b \cdot x} \cdot \sin^n x, \quad (10)$$

where $b \in \mathbf{R}$, Q is a polynomial with degree $Q = r$ and n is odd.

The particular solution of the equation (5), where the free term $f(x)$ is of the form (7), (8), (9) or (10) is constructed taking into account the developments of the terms $\cos^n x$ or $\sin^n x$, $m, n \in \mathbf{N}$ given in the previous chapter.

The form of the particular solution and the exemplification of its determination

We will build the particular solution for each of the 4 situations of free terms presented in the previous chapter.

We will consider

$$a_0 \cdot \lambda^m + a_1 \cdot \lambda^{m-1} + \dots + a_{m-1} \cdot \lambda + a_m = 0 \quad (11)$$

the characteristic equation attached to the equation (6).

We consider that the equation (11) has the following expansion:

$$\begin{aligned} a_0 \cdot \lambda^m + a_1 \cdot \lambda^{m-1} + \dots + a_{m-1} \cdot \lambda + a_m = a_0 \cdot (\lambda - a_1)^{n_1} \cdot (\lambda - a_2)^{n_2} \cdot \dots \cdot (\lambda - a_q)^{n_q} \cdot \\ \cdot [(\lambda - (b_1 + c_1 \cdot i)) \cdot (\lambda - (b_1 - c_1 \cdot i))]^{m_1} \cdot [(\lambda - (b_2 + c_2 \cdot i)) \cdot (\lambda - (b_2 - c_2 \cdot i))]^{m_2} \cdot \dots \cdot \\ \cdot [(\lambda - (b_p + c_p \cdot i)) \cdot (\lambda - (b_p - c_p \cdot i))]^{m_p} \end{aligned} \quad (12)$$

where $n_1 + n_2 + \dots + n_q + 2 \cdot m_1 + 2 \cdot m_2 + \dots + 2 \cdot m_p = m$.

For all 4 cases we also assume that $a + (n - 2 \cdot r_1) \cdot i$, $a + (n - 2 \cdot r_2) \cdot i$, ..., $a + (n - 2 \cdot r_p) \cdot i$, with $0 \leq 2 \cdot r_1 < 2 \cdot r_2 < \dots < 2 \cdot r_p < n$ and $r_1, r_2, \dots, r_p \in N$ are among the solutions of the equation (11), and have the multiplicity orders $k_1, k_2, \dots, k_p \in N$.

In addition, for cases 1 and 3 we have to see if a is the solution of the equation (11). In these situations, we have 2 sub-cases: the sub-case in which a is not a solution of the equation (11) and the sub-case in which a is a solution of the equation (11), and in this sub-

case we will assume that the multiplicity ordinal of the radical a is k .

Under these conditions, as in [1], the particular solution of the equation (5) is sought according to the rules established in that study, depending on the form of $f(x)$ and the solutions of the equation (11).

If $f(x)$ is of the form (7) and a is not a solution of the equation (11), the particular solution of the equation (5) must be constructed as follows:

$$\begin{aligned} y_p(x) = T(x) \cdot e^{a \cdot x} + \sum_{i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}} P_i(x) \cdot e^{a \cdot x} \cdot \cos(n - 2 \cdot r_i) + \sum_{i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}} Q_i(x) \cdot e^{a \cdot x} \cdot \sin(n - 2 \cdot r_i) + \\ + \sum_{j \in \{k_1, k_2, \dots, k_p\}} R_j(x) \cdot x^j \cdot e^{a \cdot x} \cdot \cos(n - 2 \cdot r_i) + \sum_{j \in \{k_1, k_2, \dots, k_p\}} S_j(x) \cdot x^j \cdot e^{a \cdot x} \cdot \sin(n - 2 \cdot r_i) \end{aligned} \quad (13)$$

where $P_i, Q_i, i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}$ and $R_j, S_j, j \in \{k_1, k_2, \dots, k_p\}$ are polynomials with degree $T = \text{degree } P_i = \text{degree } Q_i = \text{degree } R_j = \text{degree } S_j = \text{degree } P = S$, $i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}$ and $j \in \{k_1, k_2, \dots, k_p\}$.

The form of the solution (13) takes into account the expansion of the term $\cos^n x$, when n is even and a is not a solution of the equation (11).

If a is a solution of the equation (11), the particular solution of the equation (5), in which $f(x)$ is of the form (7), must be constructed as follows

$$\begin{aligned} y_p(x) = T(x) \cdot x^k \cdot e^{a \cdot x} + \sum_{i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}} P_i(x) \cdot e^{a \cdot x} \cdot \cos(n - 2 \cdot r_i) + \sum_{i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}} Q_i(x) \cdot e^{a \cdot x} \cdot \sin(n - 2 \cdot r_i) + \\ + \sum_{j \in \{k_1, k_2, \dots, k_p\}} R_j(x) \cdot x^j \cdot e^{a \cdot x} \cdot \cos(n - 2 \cdot r_i) + \sum_{j \in \{k_1, k_2, \dots, k_p\}} S_j(x) \cdot x^j \cdot e^{a \cdot x} \cdot \sin(n - 2 \cdot r_i) \end{aligned} \quad (14)$$

where $P_i, Q_i, i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}$ and $R_j, S_j, j \in \{k_1, k_2, \dots, k_p\}$ are polynomials with degree $T = \text{degree } P_i = \text{degree } Q_i = \text{degree } R_j = \text{degree } S_j = \text{degree } P = S$, $i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}$ and $j \in \{k_1, k_2, \dots, k_p\}$.

The form of the solution (14) takes into account the expansion of the $\cos^n x$, when n is even and a is the solution of the equation (11) with the order of multiplicity k .

In the second case, the particular solution of the equation (5), in which $f(x)$ is of the form (8), must be constructed as follows

$$y_p(x) = \sum_{i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}} P_i(x) \cdot e^{a \cdot x} \cdot \cos(n - 2 \cdot r_i) + \sum_{i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}} Q_i(x) \cdot e^{a \cdot x} \cdot \sin(n - 2 \cdot r_i) + \sum_{j \in \{k_1, k_2, \dots, k_p\}} R_j(x) \cdot x^j \cdot e^{a \cdot x} \cdot \cos(n - 2 \cdot r_i) + \sum_{j \in \{k_1, k_2, \dots, k_p\}} S_j(x) \cdot x^j \cdot e^{a \cdot x} \cdot \sin(n - 2 \cdot r_i) \quad (15)$$

where $P_i, Q_i, i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}$ and $R_j, S_j, j \in \{k_1, k_2, \dots, k_p\}$ are polynomials with degree $P_i = \text{degree } Q_i = \text{degree } R_j = \text{degree } S_j = \text{degree } P = s, i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}$ and $j \in \{k_1, k_2, \dots, k_p\}$.

The form of solution (15) takes into account the expansion of the term $\cos^n x$, when n is odd.

The third case, as specified above, has two sub-cases. If a is not a solution of the equation (11), the particular solution of the equation (5), in which $f(x)$ is of the form (9), must be constructed as follows

$$y_p(x) = T(x) \cdot e^{a \cdot x} + \sum_{i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}} P_i(x) \cdot e^{a \cdot x} \cdot \cos(n - 2 \cdot r_i) + \sum_{i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}} Q_i(x) \cdot e^{a \cdot x} \cdot \sin(n - 2 \cdot r_i) + \sum_{j \in \{k_1, k_2, \dots, k_p\}} R_j(x) \cdot x^j \cdot e^{a \cdot x} \cdot \cos(n - 2 \cdot r_i) + \sum_{j \in \{k_1, k_2, \dots, k_p\}} S_j(x) \cdot x^j \cdot e^{a \cdot x} \cdot \sin(n - 2 \cdot r_i) \quad (16)$$

where $P_i, Q_i, i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}$ and $R_j, S_j, j \in \{k_1, k_2, \dots, k_p\}$ are polynomials with degree $T = \text{degree } P_i = \text{degree } Q_i = \text{degree } R_j = \text{degree } S_j = \text{degree } Q = r, i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}$ and $j \in \{k_1, k_2, \dots, k_p\}$.

The form of the solution (16) takes into account the expression of the term $\sin^n x$, when n is even and the fact that a is not a solution of the equation (11).

If a is a solution of the equation (11), the particular solution of the equation (5), in which $f(x)$ is of the form (9), must be constructed in the following form

$$y_p(x) = T(x) \cdot x^k \cdot e^{a \cdot x} + \sum_{i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}} P_i(x) \cdot e^{a \cdot x} \cdot \cos(n - 2 \cdot r_i) + \sum_{i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}} Q_i(x) \cdot e^{a \cdot x} \cdot \sin(n - 2 \cdot r_i) + \sum_{j \in \{k_1, k_2, \dots, k_p\}} R_j(x) \cdot x^j \cdot e^{a \cdot x} \cdot \cos(n - 2 \cdot r_i) + \sum_{j \in \{k_1, k_2, \dots, k_p\}} S_j(x) \cdot x^j \cdot e^{a \cdot x} \cdot \sin(n - 2 \cdot r_i) \quad (17)$$

where $P_i, Q_i, i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}$ and $R_j, S_j, j \in \{k_1, k_2, \dots, k_p\}$ are polynomials with degree $T = \text{degree } P_i = \text{degree } Q_i = \text{degree } R_j = \text{degree } S_j = \text{degree } Q = r, i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}$ and $j \in \{k_1, k_2, \dots, k_p\}$.

The form of the solution (17) takes into account the expansion of the term $\sin^n x$, when n is even and the fact that a is the solution of the equation (11) with the order of multiplicity k .

In the latter case, the particular solution of the equation (5), in which $f(x)$ is of the form (10), must be constructed as follows

$$y_p(x) = \sum_{i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}} P_i(x) \cdot e^{a \cdot x} \cdot \cos(n - 2 \cdot r_i) + \sum_{i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}} Q_i(x) \cdot e^{a \cdot x} \cdot \sin(n - 2 \cdot r_i) + \sum_{j \in \{k_1, k_2, \dots, k_p\}} R_j(x) \cdot x^j \cdot e^{a \cdot x} \cdot \cos(n - 2 \cdot r_j) + \sum_{j \in \{k_1, k_2, \dots, k_p\}} S_j(x) \cdot x^j \cdot e^{a \cdot x} \cdot \sin(n - 2 \cdot r_j) \quad (18)$$

where $P_i, Q_i, i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}$ and $R_j, S_j, j \in \{k_1, k_2, \dots, k_p\}$ are polynomials with degree $P_i = \text{degree } Q_i = \text{degree } R_j = \text{degree } S_j = \text{degree } Q = r, i \in \overline{1, n} \setminus \{k_1, k_2, \dots, k_p\}$ and $j \in \{k_1, k_2, \dots, k_p\}$.

The form of the solution (18) takes into account the expansion of the term $\sin^n x$, when n is odd.

We will further exemplify the way in which the method described above must be applied.

Let us consider the 2nd order linear differential equation with constant coefficients

$$y'' - 2 \cdot y' + 2 = (4 \cdot x + 8) \cdot e^x \cdot \sin^3 x.$$

The attached homogenous equation is

$$y'' - 2 \cdot y' + 2 = 0.$$

We have:

$$y_p'(x) = [(A - 3 \cdot C)x + A + B - 3 \cdot D] \cdot e^x \cdot \sin 3x + [(3 \cdot A + C)x + 3 \cdot B + C + D] \cdot e^x \cdot \cos 3x + [(E - G)x^2 + (2 \cdot E + F - H)x + F] \cdot e^x \cdot \sin x + [(E + G)x^2 + (F + 2 \cdot G + H)x + H] \cdot e^x \cdot \cos x,$$

$$y_p''(x) = [(-8 \cdot A - 6 \cdot C)x + 2 \cdot A - 8 \cdot B - 6 \cdot C - 6 \cdot D] \cdot e^x \cdot \sin 3x + [(6 \cdot A - 8 \cdot C)x + 6 \cdot A + 6 \cdot B + 2 \cdot C - 8 \cdot D] \cdot e^x \cdot \cos 3x + [-2 \cdot Gx^2 + (4 \cdot E - 4 \cdot G - 2 \cdot H)x + 2 \cdot E + 2 \cdot F - 2 \cdot H] \cdot e^x \cdot \sin x + [2 \cdot Ex^2 + (4 \cdot E + 2 \cdot F + 4 \cdot G)x + 2 \cdot F + 2 \cdot G + 2 \cdot H] \cdot e^x \cdot \cos x.$$

By introducing these derivatives in the given equation we get:

$$y'' - 2 \cdot y' + 2 = (-8 \cdot Ax - 8 \cdot B - 6 \cdot C) \cdot e^x \cdot \sin 3x + (-8 \cdot Cx + 6 \cdot A - 8 \cdot D) \cdot e^x \cdot \cos 3x + (-4 \cdot Gx + 2 \cdot E - 2 \cdot H) \cdot e^x \cdot \sin x + (4 \cdot Ex + 2 \cdot F + 2 \cdot G) \cdot e^x \cdot \cos x = (3 \cdot x + 6) \cdot e^x \cdot \sin x - (x + 2) \cdot e^x \cdot \sin 3x.$$

By identifying the coefficients, we obtain the following system of linear equations:

$$\begin{cases} -8 \cdot A = -1 \\ -8 \cdot B - 6 \cdot C = -2 \\ -8 \cdot C = 0 \\ 6 \cdot A - 8 \cdot D = 0 \end{cases} \quad \begin{cases} -4 \cdot G = 3 \\ 2 \cdot E - 2 \cdot H = 6 \\ 4 \cdot E = 0 \\ 2 \cdot F + 2 \cdot G = 0 \end{cases}.$$

The solution of the above system is:

We will first determine the solutions of the characteristic equation:

$$\lambda^2 - 2 \cdot \lambda + 2 = 0.$$

The solutions to this equation are

$$\lambda_1, \lambda_2 = 1 \pm i.$$

Because $f(x)$ is written as

$$f(x) = (4 \cdot x + 8) \cdot e^x \cdot \sin^3 x = (3 \cdot x + 6) \cdot e^x \cdot \sin x - (x + 2) \cdot e^x \cdot \sin 3x,$$

and $1+i$ is the radical of the characteristic equation and given that $P(x) = 4 \cdot x + 8$ is of the first order, the particular solution must be as follows

$$y_p(x) = (Ax + B) \cdot e^x \cdot \sin 3x + (Cx + D) \cdot e^x \cdot \cos 3x + (Ex + F) \cdot x \cdot e^x \cdot \sin x + (Gx + H) \cdot x \cdot e^x \cdot \cos x,$$

where A, B, C, D, E, F, G and H are real constants to be determined by identification after the introduction of y_p derivatives up to the 2nd order in the given equation.

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In[1]:= m = {{-8, 0, 0, 0, 0, 0, 0, 0}, {0, -8, -6, 0, 0, 0, 0, 0}, {0, 0, -8, 0, 0, 0, 0, 0}, {6, 0, 0, -8, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
-4, 0}, {0, 0, 0, 0, 2, 0, 0, -2}, {0, 0, 0, 0, 4, 0, 0, 0}, {0, 0, 0, 0, 2, 2, 0, 0}}
v = LinearSolve[m, {-1, -2, 0, 0, 3, 6, 0, 0}]
Out[1]:= {{-8, 0, 0, 0, 0, 0, 0, 0}, {0, -8, -6, 0, 0, 0, 0, 0}, {0, 0, -8, 0, 0, 0, 0, 0}, {6, 0, 0, -8, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -4, 0}, {0, 0, 0, 0, 2,
0, 0, -2}, {0, 0, 0, 0, 4, 0, 0, 0}, {0, 0, 0, 0, 2, 2, 0, 0}}

Out[2]:= {1/8, 1/4, 0, 3/32, 0, 3/4, -3/4, -3}

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namely, $A = 1/8$; $B = 1/4$; $C = 0$; $D = 3/32$; $E = 0$; $F = 3/4$; $G = -3/4$; $H = -3$.

In conclusion, a particular solution of the given equation is

$$y_p(x) = (1/8 \cdot x + 1/4) \cdot e^x \cdot \sin 3x + 3/32 \cdot e^x \cdot \cos 3x + 3/4 \cdot x \cdot e^x \cdot \sin x - (3/4 \cdot x^2 + 3 \cdot x) \cdot e^x \cdot \cos x.$$

After determining the particular solution, we can write the general solution of the given equation.

Conclusions and discussions

This study comes to complete the range of free terms for which we can determine a particular solution for a n -th order linear differential equation with constant coefficients. As we mentioned in the abstract, it is a combination of two simple elements, but which helps us to extend the form that the free term can have, this term being able to also contain terms like $P(x) \cdot e^{a \cdot x} \cdot \cos^n x$ or $Q(x) \cdot e^{b \cdot x} \cdot \sin^m x$.

Another aspect we want to mention is related to determining the solutions of the

characteristic equation. This problem is not simple, but on the other hand, for the construction of the particular solution, we must know which of the values generated by the terms that appear in the expression of the function $f(x)$ are among the solutions of the characteristic equation and what orders of multiplicity they have. This aspect has a simple solution by using the Horner scheme or the derivatives of the characteristic equation for all the values generated by the terms that appear in the expression of the function $f(x)$.

Another problem that is not always easy to solve is that of determining the solution of the system of linear equations that results from identifying the coefficients and without which we cannot write the particular solution. As it can be noticed, to determine the solution of the system, especially when we have many unknowns, it is advisable to use a computer application, and in our case the application used to determine the solution was *Wolfram*.

References

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