DE GRUYTER OPEN

International Conference KNOWLEDGE-BASED ORGANIZATION
Vol. XXI No 3 2015

ANALYTICAL MODEL FOR APPROXIMATELY CALCULATING OF THE SECOND FREQUENCY OF CROSS VIBRATIONS FOR COMPLEX SHAPE BARREL OF THE SMALL ARM

Conyu Grigorov CONEV*, Krasimir Stoianov DAVIDOV**

***"Vasil Levski" National Military University, Shoomen, Bulgaria,**
****"Nikokola Vaptsarov" Naval Academy, Varna, Bulgaria,**
coni19@abv.bg, krasi_dav@abv.bg

Abstract*: The report scrutinizes an analytical model for approximately calculating of the second natural frequency of cross vibrations for complex shape barrel of the small arm. The model is based on known formulas, used for calculating on the second natural frequency of cross vibrations for cylindrical barrels. In suggested analytical model a coefficient, that gives an influence on form of the complex barrel is added. An experimental investigation is made in order to prove the workability of the analytical method.*

Keywords: small arms; vibrations

1. Introduction

In the last decade by using recent investigations it is concluded that cross vibrations bring influence to bear on grouping. It is discovered that the grouping of gunshot hits the single shooting influence of the second natural frequency of cross vibrations on the barrel of the small arm.

The analytical models used for calculating of the second natural frequency of cross vibrations on the barrel that are made so far, are very composite in cases when the barrel has complex shape (the barrel shape is not cylindrical or cone on all length).

The report scrutinizes an analytical model for approximately calculating of the second natural frequency of cross vibrations for a complex shape barrel (the barrel has definited number of cylindrical and cone sections) of the small arm. The main advantage of the suggested model is a large facilitation of calculating of second natural frequency of cross vibrations of barrel with a complex shape.

2. Analytical model

In order to achieve model workability the following restrictions are established:

- it is accepted that the barrel is outrigger with a complex shape;
- the influence of the vibrations of the barrel box is eliminated;
- the influence of the resistance is eliminated;
- it is accepted that the barrel composition is homogeneous.

The model is based on known formulas *1*, used for calculating the second natural frequency of cross vibrations for cylindrical barrels [1, 2, 3]. In the suggested analytical model is added a coefficient that gives an influence on the form of the complex barrel.

DOI: 10.1515/kbo-2015-0136

$$n = \frac{4{,}694^2}{2.\pi.l^2}.\sqrt{\frac{E.I}{S.\rho}} \text{ [Hz]} \qquad (1)$$

where: n – second natural frequency of cross vibrations of cylindrical barrel [Hz];
l – barrel length [m];
ρ– barrel material density [kg/m^3];
S – cross section area of the barrel [m^2];
I - area moment of inertia of the barrel cross section [m^4];
E – modulus of elasticity [Pa].

In order to calculate of the coefficient, that gives an influence of the form on the complex barrel, first it is necessary to define the relation between frequencies of two barrels with equal lengths, but different diameters. To do this it is possible to use the formula (2):

$$k_f = \frac{n_1}{n_2} \qquad (2)$$

where: κ_f – correction coefficient between the secondary frequencies of two cylindrical barrels, with equal lengths, but different diameters;
n_1 – second natural frequency of the first barrel [Hz];
n_2 - second natural frequency of the second barrel [Hz];
After replacing the frequencies of formula (1) in formula (2) the result is:

$$k_f = \frac{\frac{4{,}694^2}{2.\pi.l^2}.\sqrt{\frac{E.I_1}{S_1.\rho}}}{\frac{4{,}694^2}{2.\pi.l^2}.\sqrt{\frac{E.I_2}{S_2.\rho}}} = \frac{\frac{4{,}694^2}{2.\pi.l}.\sqrt{\frac{E.I_1}{S_1.\rho.l^2}}}{\frac{4{,}694^2}{2.\pi.l}.\sqrt{\frac{E.I_2}{S_2.\rho.l^2}}} \qquad (3)$$

where: I_1 - area moment of inertia of the first cylindrical barrel [m^4];
S_1 - cross section area of the first cylindrical barrel [m^2];
I_2 - area moment of inertia of the second cylindrical barrel [m^4];
S_2 - cross section area of the second cylindrical barrel [m^2];

But $S_1.\rho.l = m_1$ and $S_2.\rho.l = m_2$ (4)

where: m_1 – mass of the first cylindrical barrel [kg];
m_2 – mass of the second cylindrical barrel [kg].
And so the formulas (3) can be written:

$$k_f = \frac{\frac{4{,}694^2}{2.\pi.l}.\sqrt{\frac{E.I_1}{m_1.l}}}{\frac{4{,}694^2}{2.\pi.l}.\sqrt{\frac{E.I_2}{m_2.l}}} = \frac{\sqrt{\frac{I_1}{m_1.l}}}{\sqrt{\frac{I_2}{m_2.l}}} = \sqrt{\frac{I_1.m_2}{I_2.m_1}} \qquad (5)$$

Using formula (5) a conclusion can be made, that the values of the area moments of inertia and masses of two barrels influence upon the correction coefficient - κ_f.
The practical application of the correction coefficient between the second frequencies of two cylindrical barrels, with equal lengths, but different diameters can be used for barrel designing.
The next step after calculating the correction coefficient is to accept that the complex barrel is cylindrical with a diameter that has to be definited. The basic used frequency is a frequency that matches the cylindrical barrel with a diameter equal to the diameter of complex barrel on the fixing point.
To calculate the mass and moment of inertia of the barrel the following formulas can be used:

$$m = V.\rho = \pi.\frac{D^2}{4}.l.\rho \qquad (6)$$

$$I = \frac{\pi}{64}.\left(D^4 - d^4\right) \qquad (7)$$

where: m – mass of cylindrical barrel [kg];
D – external barrel diameter [m];
V – barrel capacity [m^3];
d – barrel opening diameter [m].

To calculate the barrel capacity the following formula can be used:

$$V = V_1 - V_2 = \pi.\frac{D^2}{4}.l - \pi.\frac{d^2}{4}.l = \pi.\frac{D^2 - d^2}{4}.l \qquad (8)$$

where: V_1 – barrel capacity without opening [m^3];
V_2 – capacity of the opening [m^3].
After replacing formulas (6,7 and 8) in formula (5) the result is:

$$k=\sqrt{\frac{I_r.m_{ts}}{I_{ts}.m_r}}=$$

$$=\sqrt{\frac{\left(\frac{\pi}{64}.\left(D_r^4-d^4\right)\right).\left(\pi.\frac{D_{ts}^2-d^2}{4}.l.\rho\right)}{\left(\frac{\pi}{64}.\left(D_{st}^4-d^4\right)\right).\left(\pi.\frac{D_r^2-d^2}{4}.l.\rho\right)}}=$$

$$=\sqrt{\frac{\left(D_r^4-d^4\right).\left(D_{ts}^2-d^2\right)}{\left(D_{ts}^4-d^4\right).\left(D_r^2-d^2\right)}}= \qquad (9)$$

$$=\sqrt{\frac{\left(D_r^2-d^2\right).\left(D_r^2+d^2\right).\left(D_{ts}^2-d^2\right)}{\left(D_{ts}^2-d^2\right).\left(D_{st}^2-d^2\right).\left(D_r^2-d^2\right)}}=$$

$$=\sqrt{\frac{D_r^2+d^2}{D_{ts}^2+d^2}}$$

where: k - correction coefficient, that gives an influence on the form of the complex barrel;
I_r – area moment of inertia of the cylindrical barrel, towards which the complex barrel is reduced [m^4];
I_{ts} - area moment of inertia of the cylindrical barrel, that has a diameter equal to the diameter of the complex barrel in the fixing point [m^4];
m_r – mass of cylindrical barrel, towards which complex barrel is reduced [kg];
m_{ts} – mass of cylindrical barrel, that has diameter equal to diameter of the complex barrel in the fixing point [kg];
D_r – external diameter of the cylindrical barrel that is equalized to the complex barrel [m];
D_{ts} – external diameter of the cylindrical barrel with a diameter equal to the diameter of complex barrel in the fixing point [m].

The calculation of the diameter D_r can be made by using the formula (6), because the mass of complex barrel and mass of equalized cylindrical barrel are the same. And so the diameter D_r can be calculated by using the formula:

$$D_r=\sqrt{\frac{4.m_r}{\rho.\pi.l}+d^2} \qquad (10)$$

After replacing formula (10) in formula (9) the result is:

$$k=\sqrt{\frac{D_r^2+d^2}{D_{ts}^2+d^2}}=$$

$$=\sqrt{\frac{\left(\frac{4.m_r}{\rho.\pi.l}+d^2\right)+d^2}{D_{ts}^2+d^2}}= \qquad (11)$$

$$=\sqrt{\frac{4.m_r+(2.d^2.\rho.\pi.l)}{\rho.\pi.l.\left(D_{ts}^2+d^2\right)}}$$

The mass of the complex barrel can be calculated by summing the masses of all barrel sectors. The formula (6) can be used to calculate the masses of all sectors.
The final formula for calculating the second natural frequency of cross vibration for complex small arm barrel:

$$n=\sqrt{\frac{4.m_r+(2.d^2.\rho.\pi.l)}{\rho.\pi.l.\left(D_{ts}^2+d^2\right)}}.\frac{4{,}694^2}{2.\pi.l^2}.\sqrt{\frac{E.I}{S.\rho}} \qquad (12)$$

Initial data for analytical calculation:
- modulus of elasticity – 2,16.10^5 [MPa];
- barrel material density - 7840 [kg/m^3].

Table 1 Ballistic barrel data

Barrel №	*Ballistic barrel description*	*Barrel mass [kg]*	*Barrel length [m]*	*External diameter of barrel in the fixing point [m]*	*Barrel opening diameter [m]*
1	The barrel consists of 3 cylindrical and 2 cone sectors.	0,746	0,499	0,027	0,00762
2	The barrel consists of 4 cylindrical and 2 cone sectors.	0,434	0,250	0,021	0,00762

Barrel №	Ballistic barrel description	Barrel mass [kg]	Barrel length [m]	External diameter of barrel in the fixing point [m]	Barrel opening diameter [m]
3	The barrel consists of 4 cylindrical and 1 cone sectors.	0,840	0,510	0,026	0,00762
4	The barrel consists of 3 cylindrical and 1 cone sectors.	0,684	0,482	0,026	0,00762
5	The barrel consists of 9 cylindrical and 1 cone sectors.	0,610	0,415	0,023	0,00762

An experimental investigation was made. It includes testing the frequencies of 5 different barrels with a complex shape (three-dimensional model is present on figure 1).

The devices used in the experiment include:
- DAQ - measurements plate NI USB-6211 – National Instruments;
- measurements amplifier Type 2635– Brüel & Kjær;
- censor for vibrations measurement– piezoelectrical accelerometer;
- computer (program "Lab View" 8.5);
- impulse dynamometer Type PE03MGD.

The analytical calculation of the second natural frequency of cross vibrations of the barrels is made whit program Mat Lab.

The results of the experimental investigations and analytical model are presented on table 2.

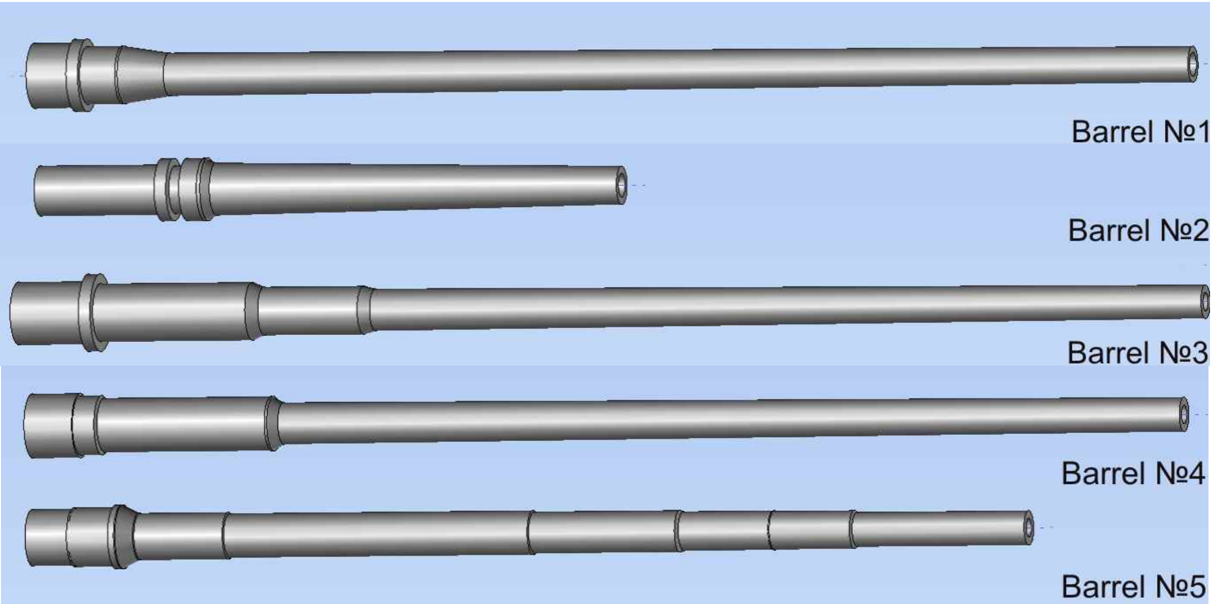


Figure 1: Ballistic barrels three-dimensional model.

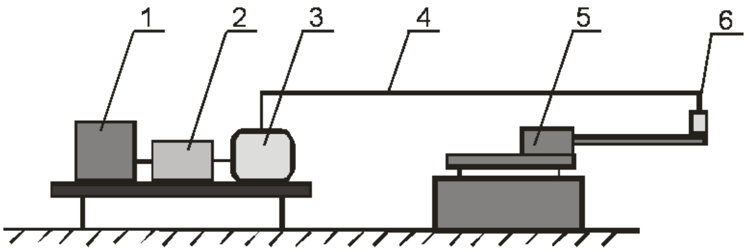
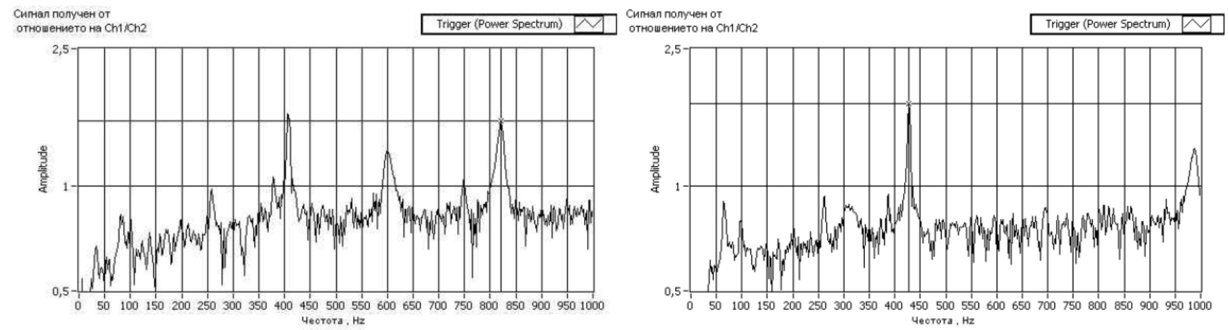


Figure 2: Scheme of experimental investigation devices disposal.
1- computer (program “Lab View 8.5”); 2- DAQ - measurements plate NI USB-6211 – National Instruments; 3- measurements amplifier Type 2635 – Brüel & Kjær; 4- cable; 5- base for ballistic barrel; 6- censor for vibrations measurement.

Table 2 Results of the experimental investigations and the analytical model

Results of the analytical model				*Results of the experimental investigation*		*Rrelative error [%]*
Barrel №	*Second natural frequency [Hz]*	*Correction coefficient - k*	*External diameter of the cylindrical barrel that is equalized to the complex barrel [m]*	*Barrel №*	*Second natural frequency [Hz]*	
1	350	0,67541	0,017349	1	392	10,7
2	1470	0,89325	0,018443	2	1567	6,2
3	347	0,72303	0,018047	3	407	14,7
4	369	0,68725	0,016990	4	428	13,8
5	503	0,77759	0,017231	5	580	13,3



A *B*

Figure 3: Barrel natural frequencies.

A- Barrel №3: B- Barrel №4.

The average error between the analytical model and the experimental data for the tested barrel is 11,7%.
The statistics hypothesis can be checked by comparing of the dispersions of the experimental and analytical results. The zero hypothesis is that the dispersion of data, received from the analytical model is commensurable with the dispersion of the experimental data.

Formula (13) can be used to check the statistics hypothesis [4]:

$$\chi_0^2 = \frac{SS}{\sigma_0^2} \tag{13}$$

where:

$SS = \sum_{i=1}^{n}\left(y_{Ei} - \bar{y}_E \right)^2$ - corrected sum of the squares of the experimental investigation data;

- $\sigma_0^2 = \frac{1}{n-1} . \sum_{i=1}^{n}\left(y_i - \bar{y} \right)^2$ - dispersion of the analytical model data;

- $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ - average analytical model data;

- $\bar{y}_E = \frac{1}{n}\sum_{i=1}^{n} y_{Ei}$ - average experimental investigation data;

n - data number.

The zero hypothesis is rejected in cases, when $\chi_0^2 > \chi^2_{\alpha/2;n-1}$ or $\chi_0^2 > \chi^2_{1-\alpha/2;n-1}$ [4].

Received results are present in table 3.

Table 3 Results of zero hypothesis checking

№	α	χ^2	$\chi^2_{(0,025;4)}$	$\chi^2_{(0,975;4)}$
1	0,05	13,70937	23,3366	4,40778

3. Conclusions

The results presented in table 3 show that the zero hypothesis can be accepted as true and the analytical model can be used for practical calculation of the second natural frequency of cross vibrations of a small arm barrel, that has a complex shape.

References

[1] Brosh J. T., *Mechanical Vibration and Shock Measurements*, Bruel & Kjar, 1980, 63.

[2] Кирилов В. М., *Основи за устройството и проектирането на стрелковото оръжие*, София, ДВИ, 1975, 303.

[3] Пановко Я.Г., *Основы прикладной теории колебаний и удара,* Ленинград, масиностроене, 1976, 120.

[4] Монтгомери Д.К., *Планирование експеримента и анализ данных*, Ленинград, Судостроене, 1980, 30.