MARKOV MODEL OF THE TRANSPORT DEVICES EXPLOITATION PROCESS

MARKOWSKI MODEL PROCESU EKSPLOATACJI ŚRODKÓW TRANSPORTU

Maciej Woropay, Klaudiusz Migawa, Mirosław Szubartowski

University of Technology and Life Sciences, Department of Machine Maintenance
ul. Kaliskiego 7, 85-799 Bydgoszcz, Poland
E-mails: kem@utp.edu.pl

Abstract: The article presents the model of transport means operation and maintenance process, realized within a system of urban bus transport. On the basis of identification of the investigated system and the realized in it process, a mathematical model of the operation and ensuring efficiency processes was built, assuming that models of these processes are homogenous Markov processes. On this bases an analysis of the considered controlled process had been made. All the considerations are presented on the example of a chosen real system of operation and maintenance of means of transport.

Keywords: transport system, exploitation process

Streszczenie. W artykule przedstawiono model procesu eksploatacji środków transportu realizowanego w systemie autobusowego transportu miejskiego. Na podstawie identyfikacji badanego systemu i realizowanego w nim procesu zbudowano matematyczny model procesu użytkowania oraz procesu zapewniania zdolności środków transportu, zakładając, że modelami tych procesów są jednorodne procesy Markowa. Na tej podstawie dokonano analizy rozpatrywanego procesu sterowanego. Całość rozważań przedstawiono na przykładzie wybranego rzeczywistego systemu eksploatacji środków transportu.

Słowa kluczowe: system transportowy, proces eksploatacji

1. Introduction

Transport systems aim at meeting the transport demands by realization of transports on given routes. The process of operation and maintenance of means of transport is a controlled process and can be roughly divided into the process of operation and ensuring efficiency. The aim of the operation process is to realize its basic function and thereby generate profits. The process of ensuring efficiency is to maintain operational efficiency of the used technical objects.

In order to provide high level of operational efficiency of the system, an appropriate identification of the process realized in a given system is indispensable, as well as its analysis and assessment. An analysis and assessment of controlled systems can be carried out on the basis of examining their models. The most frequently used projection of the operation and maintenance process model are graphs including directed graphs whose peaks are states of operation and maintenance, and whose arcs are possible transitions between the states. Due to a random character of factors determining the course and efficiency of the operation and maintenance process realized within a complex system of operation and maintenance, stochastic processes are used for mathematical modeling of the operation and maintenance process. Among random processes, Markov and semi-Markov processes have found wide application for modeling the operation and maintenance process [5, 9, 10]. Carrying out of modeling tests with the use of described models of the operation and maintenance process enables both an analysis of specific problems connected with operation and maintenance of technical objects and an analysis of relations occurring between the determined number of the model parameters. Markov and semi-Markov processes have been explicitly discussed in literature by many authors [eg. 2, 3, 4, 7]. However, there are not enough practical examples of applications of these models for the description of operation and maintenance processes realized in real transport systems, especially urban transport systems.

In the work an attempt of mathematical description and analysis had been made of both, the process of transport means operation and the process of ensuring efficiency, realized within a selected transport system, assuming that the model of these processes are homogenous Markov $X(t)$ processes. A system of operation and maintenance of an urban bus transport, being one of subsystems of the urban transport system, was chosen as the object of investigations.

2. Event Model of Transport Means Operation and Maintenance

The model of operation and maintenance was built on the basis of an analysis of space of the states and operational events concerning technical objects (urban buses) employed in the analyzed real transport system. In result of identification of a given transport system and, realized within it, multi-state process of technical objects operation and maintenance there were determined important states of this

process and possible transitions between the distinguished states. On this basis a graph of state changes of the operation and maintenance process was built which is demonstrated in Figure 1.

Fig. 1 Directed graph representing means of transport operation and maintenance process

S_1 – realization of the transport process, S_2 – damage to the technical object during realization of the transport task, S_3 – diagnosing the technical object by service workers, S_4 – repair of the technical object by a service unit without losing a course, S_5 – repair of the technical object by a service unit with losing a course, S_6 – stay of the technical object on a halting place of a bus depot, S_7 – refueling the tank, S_8 – servicing during the day of operation, S_9 – realization of periodic servicing, S_{10} – diagnosing before the repair within the system of ensuring efficiency, S_{11} – diagnosing after the repair within a subsystem of providing efficiency, S_{12} – repair of the technical object within a subsystem of ensuring efficiency

Directed graph of the transport means operation and maintenance process projection, demonstrated in Figure 1, is built from a big number of states and transitions between them. As a result, corresponding to it, matrixes of probabilities and matrixes of the process states change intensity are significantly extended, and analytical determinations of boundary probabilities p_i^* are impossible.

For the purpose of analytical determination of boundary probabilities p_i^* a directed graph presented in Figure 1, had been divided into two sub-graphs in which state S_6 is the shared state (when the technical object stays in the halting place in a bus

depot). The first sub-graph (Figure 2) corresponds to the operation process states, whereas the second sub-graph (Figure 3) corresponds to states of the process of ensuring transport means efficiency.

Fig. 2 Directed graph of the process of operation and maintenance of means of transport

$S_1 \div S_6$ – respectively as in Figure 1, S_A – the technical object being in the subsystem of ensuring efficiency (state fully aggregated from states $S_7 \div S_{12}$ – Figure 1)

Fig. 3 Directed graph of the ensuring means of transport efficiency process projection

S_B – the technical object being in the operation subsystem (state fully aggregated from states $S_1 \div S_5$ – Figure 1), $S_6 \div S_{12}$ – respectively as in Figure 1

3. Mathematical Model of Means of Transport Maintenance

In result of the carried out analysis of assumptions and limitations it was assumed that the model of operation and maintenance process (respectively operation and ensuring efficiency processes) of means of transport is Markov's process $X(t)$. For the purpose of determination of probabilities p_i^* and $p_i(t)$ it was assumed that the initial state is S_6 – when the technical object stays on the halting place in a bus depot.

3.1. Mathematical Model of Means of Transport Operation

On the basis of a directed graph, demonstrated in Figure 2, matrixes P_1 of probabilities of state changes and Λ_1 of intensity of $X(t)$ process transitions, were built:

$$P_1 = \begin{bmatrix} 0 & p_{12} & 0 & 0 & 0 & p_{16} & 0 \\ 0 & 0 & p_{23} & 0 & 0 & p_{26} & 0 \\ 0 & 0 & 0 & p_{34} & p_{35} & p_{36} & 0 \\ p_{41} & 0 & 0 & 0 & 0 & p_{46} & 0 \\ p_{51} & 0 & 0 & 0 & 0 & p_{56} & 0 \\ p_{61} & 0 & 0 & 0 & 0 & 0 & p_{6A} \\ 0 & 0 & 0 & 0 & 0 & p_{A6} & 0 \end{bmatrix} \quad (1)$$

$$\Lambda_1 = \begin{bmatrix} -\lambda_{11} & \lambda_{12} & 0 & 0 & 0 & \lambda_{16} & 0 \\ 0 & -\lambda_{22} & \lambda_{23} & 0 & 0 & \lambda_{26} & 0 \\ 0 & 0 & -\lambda_{33} & \lambda_{34} & \lambda_{35} & \lambda_{36} & 0 \\ \lambda_{41} & 0 & 0 & -\lambda_{44} & 0 & \lambda_{46} & 0 \\ \lambda_{51} & 0 & 0 & 0 & -\lambda_{55} & \lambda_{56} & 0 \\ \lambda_{61} & 0 & 0 & 0 & 0 & -\lambda_{66} & \lambda_{6A} \\ 0 & 0 & 0 & 0 & 0 & \lambda_{A6} & -\lambda_{AA} \end{bmatrix} \quad (2)$$

where: p_{ij} – probability of transition from the state S_i to the state S_j ,
 λ_{ij} – intensity of transition from the state S_i to the state S_j of the process $X(t)$.

On the basis of the matrixes (1) and (2) a system of linear equations was built

$$\sum_i \lambda_{ij} \cdot p_i^* = 0, \quad j = 1, 2, \dots, 6, A \quad (3)$$

In result of the solution of system of equations (3), boundary probabilities p_i^* of Markov's process (model of operation process) was obtained:

$$p_1^* = \frac{1}{1 + \frac{\lambda_{12}}{\lambda_{22}} \cdot \left[1 + \frac{\lambda_{23}}{\lambda_{33}} \cdot \left(1 + \frac{\lambda_{34}}{\lambda_{44}} + \frac{\lambda_{35}}{\lambda_{55}} \right) \right] + a \cdot \left(1 + \frac{\lambda_{6A}}{\lambda_{AA}} \right)} \quad (4)$$

$$p_2^* = \frac{\lambda_{12}}{\lambda_{22}} \cdot p_1^* \quad (5)$$

$$p_3^* = \frac{\lambda_{12} \cdot \lambda_{23}}{\lambda_{22} \cdot \lambda_{33}} \cdot p_1^* \quad (6)$$

$$p_4^* = \frac{\lambda_{12} \cdot \lambda_{23} \cdot \lambda_{34}}{\lambda_{22} \cdot \lambda_{33} \cdot \lambda_{44}} \cdot p_1^* \quad (7)$$

$$p_5^* = \frac{\lambda_{12} \cdot \lambda_{23} \cdot \lambda_{35}}{\lambda_{22} \cdot \lambda_{33} \cdot \lambda_{55}} \cdot p_1^* \quad (8)$$

$$p_6^* = a \cdot p_1^* \quad (9)$$

$$p_A^* = a \cdot \frac{\lambda_{6A}}{\lambda_{AA}} \cdot p_1^* \quad (10)$$

where:

$$a = \frac{\lambda_{11} - \frac{\lambda_{12} \cdot \lambda_{23}}{\lambda_{22} \cdot \lambda_{33}} \cdot \left(\frac{\lambda_{34} \cdot \lambda_{41}}{\lambda_{44}} + \frac{\lambda_{35} \cdot \lambda_{51}}{\lambda_{55}} \right)}{\lambda_{61}} \quad (11)$$

3.2. Mathematical Model of the Process of Ensuring Means of Transport Efficiency

On the basis of a directed graph, demonstrated in Figure 3, matrixes P_2 of probabilities of state changes and Λ_2 of intensity of $X(t)$ process transitions, were built:

$$P_2 = \begin{bmatrix} 0 & p_{B6} & 0 & 0 & 0 & 0 & 0 & 0 \\ p_{6B} & 0 & p_{67} & 0 & 0 & 0 & 0 & p_{612} \\ 0 & 0 & 0 & p_{78} & 0 & p_{710} & 0 & p_{712} \\ 0 & p_{86} & 0 & 0 & p_{89} & p_{810} & 0 & p_{812} \\ 0 & p_{96} & 0 & 0 & 0 & p_{910} & 0 & p_{912} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{1012} \\ 0 & 0 & p_{117} & 0 & 0 & 0 & 0 & p_{1112} \\ 0 & 0 & p_{127} & 0 & 0 & 0 & p_{1211} & 0 \end{bmatrix} \quad (12)$$

$$\Lambda_2 = \begin{bmatrix} -\lambda_{BB} & \lambda_{B6} & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_{6B} & -\lambda_{66} & \lambda_{67} & 0 & 0 & 0 & 0 & \lambda_{612} \\ 0 & 0 & -\lambda_{77} & \lambda_{78} & 0 & \lambda_{710} & 0 & \lambda_{712} \\ 0 & \lambda_{86} & 0 & -\lambda_{88} & \lambda_{89} & \lambda_{810} & 0 & \lambda_{812} \\ 0 & \lambda_{96} & 0 & 0 & -\lambda_{99} & \lambda_{910} & 0 & \lambda_{912} \\ 0 & 0 & 0 & 0 & 0 & -\lambda_{1010} & 0 & \lambda_{1012} \\ 0 & 0 & \lambda_{117} & 0 & 0 & 0 & -\lambda_{1111} & \lambda_{1112} \\ 0 & 0 & \lambda_{127} & 0 & 0 & 0 & \lambda_{1211} & -\lambda_{1212} \end{bmatrix} \quad (13)$$

On the basis of the matrixes (12) and (13) a system of linear equations was built

$$\sum_i \lambda_{ij} \cdot p_i^* = 0, \quad j = B, 6, 7, \dots, 12 \quad (14)$$

In result of the solution of system of equations (14), boundary probabilities p_i^* of Markov's process (model of ensuring efficiency process) was obtained:

$$p_B^* = \frac{\lambda_{6B}}{\lambda_{BB}} \cdot p_6^* \quad (15)$$

$$p_6^* = \frac{1}{1 + \frac{\lambda_{6B}}{\lambda_{BB}} + a \cdot \left(1 + \frac{\lambda_{88}}{\lambda_{78}} + \frac{\lambda_{89}}{\lambda_{99}}\right) + b \cdot \left(1 + \frac{\lambda_{1211}}{\lambda_{1111}}\right) + c} \quad (16)$$

$$p_7^* = a \cdot \frac{\lambda_{88}}{\lambda_{78}} \cdot p_6^* \quad (17)$$

$$p_8^* = a \cdot p_6^* \quad (18)$$

$$p_9^* = a \cdot \frac{\lambda_{89}}{\lambda_{99}} \cdot p_6^* \quad (19)$$

$$p_{10}^* = c \cdot p_6^* \quad (20)$$

$$p_{11}^* = b \cdot \frac{\lambda_{1211}}{\lambda_{1111}} \cdot p_6^* \quad (21)$$

$$p_{12}^* = b \cdot p_6^* \quad (22)$$

where:

$$a = \frac{\lambda_{66} - \frac{\lambda_{B6} \cdot \lambda_{6B}}{\lambda_{BB}}}{\lambda_{86} + \frac{\lambda_{89} \cdot \lambda_{96}}{\lambda_{99}}}; b = \frac{\frac{a \cdot \lambda_{77} \cdot \lambda_{88}}{\lambda_{78}} - \lambda_{67}}{\lambda_{127} + \frac{\lambda_{1211} \cdot \lambda_{117}}{\lambda_{111}}}; c = \frac{a}{\lambda_{1010}} \cdot \left(\frac{\lambda_{710} \cdot \lambda_{88}}{\lambda_{78}} + \frac{\lambda_{89} \cdot \lambda_{910}}{\lambda_{99}} + \lambda_{810} \right) \quad (23)$$

3.3. Values of Boundary Probabilities p_i^*

Values of boundary probabilities p_i^* of being in states S_i of Markov process $X(t)$ were determined for data obtained from investigations of operation and maintenance carried out in a real transport system. Results are demonstrated in Tables 1 and 2.

Table 1. Values of boundary probabilities p_i^* for the operation process

p_1^*	p_2^*	p_3^*	p_4^*	p_5^*	p_6^*	p_A^*
0,533041	0,000739	0,001623	0,000820	0,001483	0,380063	0,082231

Table 2. Values of boundary probabilities p_i^* for the ensuring efficiency

p_B^*	p_6^*	p_7^*	p_8^*	p_9^*	p_{10}^*	p_{11}^*	p_{12}^*
0,537706	0,380063	0,004512	0,004370	0,003693	0,001956	0,002400	0,065300

3.4. Determination of Probabilities $p_i(t)$

Then in order to set the probabilities

$$p_i(t) = P\{X(t) = i\} \quad (24)$$

that at the moment t the process $X(t)$ is in the i -th state, there were built, the system of differential equations by A. N. Kolmogorov, respectively:

- on the basis of intensity matrix of transitions A_1 (2), for the process of operation

$$p_j'(t) = \sum_i \lambda_{ij} \cdot p_i(t), \quad i, j = 1, 2, \dots, 6, A \quad (25)$$

- on the basis of intensity matrix of transitions A_2 (13), for the process of ensuring efficiency

$$p_j'(t) = \sum_i \lambda_{ij} \cdot p_i(t), \quad i, j = B, 6, 7, \dots, 12 \quad (26)$$

In order to set $p_i(t)$ for arbitrary λ_{ij} a computer programme had been developed enabling to solve the system of equations (25) and (26) by means of the numerical methods of the linear algebra. For the data obtained from investigating the operation and maintenance process being performed in a real transport system, the probabilities $p_i(t)$ of staying in the process states $X(t)$ were set, as presented in graphic form in the Figure 4 and 5.

Figure 4. Probabilities $p_i(t)$ of being in states S_i of process $X(t)$ (model of the means of transport operation process)

Figure 5. Probabilities of $p_i(t)$ of being in states S_i of process $X(t)$ (model of ensuring means of transport efficiency process)

4. Conclusions

Analyzing the determined values of probabilities p_i^* and $p_i(t)$ for the process of means of transport operation it can be said that the states of the process in which a statistical technical object remains for the longest time are states S_1 (task accomplishment) and S_6 (stay on a halting place) – total over 91,7% of the operation time. The remaining time of operation and maintenance concerns efficiency ensuring processes of technical objects (about 8,2% of operation time), however, it is significant due to the possibility of accomplishment of a transport system task and the related costs. Total share of time for ensuring efficiency of technical objects by emergency service ($p_2^* + p_3^* + p_4^* + p_5^*$) in the total operation and maintenance time is merely 0,5%.

From the obtained values of probabilities p_i^* and $p_i(t)$ determined for the model ensuring efficiency of technical objects process it results that the time when the statistical object remains in a real subsystem amounts almost to 53,8%, whereas, total time of activities of service-repair character realized in service stations is 8,2%. In the process of ensuring efficiency the repair time amounts to 79,4%, service 9,8%, refuel time 5,5%, diagnosis 5,3%. The remaining 38% is connected with waiting for the start of the task realization (efficient objects) or for the start of service-repair processes (inefficient objects).

Analyzing probability charts $p_i(t)$ it can be noticed that for the majority of states the time in which probabilities reach boundary values is contained within 20 ÷ 30 hours. However, total stabilization of the process parameters takes place no sooner than after 40 hours.

References

1. Csenki A.: *Dependability for systems with a partitioned state space Markov and semi-Markov theory and computational implementation*. Springer Verlag, New York, 1994.
2. Fleming W. H., Soner H. M.: *Controlled Markov processes and viscosity solutions*. Springer Verlag, New York, 1993.
3. Iosifescu M.: *Skończone procesy Markowa i ich zastosowanie*. PWN, Warszawa, 1988.
4. Jaźwiński J., Grabski F.: *Niektóre problemy modelowania systemów transportowych*. Instytut Technologii Eksploatacji, Warszawa-Radom, 2003.
5. Kulkarni V. G.: *Modeling and analysis of stochastic systems*. Chapman & Hall, New York, 1995.
6. Limnios N., Oprisan G.: *Semi-Markov processes and reliability*. Birkhauser, Boston, 2001.
7. Leszczyński J.: *Modelowanie systemów i procesów transportowych*. Wydawnictwo Politechniki Warszawskiej, Warszawa, 1994.
8. Woropay M., Żurek J., Migawa K.: *Model oceny i kształtowania gotowości operacyjnej podsystemu utrzymania ruchu w systemie transportowym*. Wydawnictwo i Zakład Poligrafii Instytutu Technologii Eksploatacji, Radom, 2003.

and doctoral theses.

Professor Maciej Woropay is the head of the Machine Maintenance Department at the Mechanical Engineering Faculty of the University of Technology and Life Sciences in Bydgoszcz. In his research he deals with problems connected with theory of systems, theory of reliability and safety, and maintenance process control in complex biotechnical systems, especially with control of these processes in real transport systems. He is the author and co-author of over 140 scientific papers published in Poland and abroad, as well as textbooks and academic scripts; he has been a promoter of 150 Master's and Bachelor's theses

Dr eng. Klaudiusz MIGAWA is the adjunct in the Machine Maintenance Department at the Mechanical Engineering Faculty of the University of Technology and Life Sciences in Bydgoszcz. In his research he deals with problems relating to modelling processes and the systems of exploitation and steering the processes of exploitation in folded systems. He guides investigations relating to the questions of the opinion and formation of readiness and the reliability of the systems of the exploitation of the means of municipal transportation.

Dr eng. Mirosław SZUBARTOWSKI is cooperating with the Machine Maintenance Department of the University of Technology and Life Sciences in Bydgoszcz. In his research he deals with problems connected with reliability and availability in the transport systems. He is realizing the scientific research urban traffic system. Author and co-author of several scientific papers.