MODEL OF SYSTEM SAFETY RELATED TO CLIMATE-WEATHER CHANGE PROCESS

MODEL BEZPIECZEŃSTWA SYSTEMU PODDANEGO PROCESOWI ZMIAN KLIMATYCZNO-POGODOWYCH

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Abstract: The article is devoted to the climate-weather change impact on the critical infrastructure safety. A general safety analytical model of the technical system related to the climate-weather change process in its operating area is proposed. It is the integrated model of the complex technical system safety, linking its multistate safety model and the model of the climate-weather change process at its operating area. The conditional safety functions at the climate-weather particular states and the unconditional safety function of the complex system at changing in time climate-weather conditions, the mean value and the variance of the system unconditional lifetime and other safety indicators are defined in general for any critical infrastructure.

Keywords: safety, critical infrastructure, climate-weather change process

Streszczenie: Artykuł jest poświęcony wpływowi zmian pogodowo-klimatycznych na bezpieczeństwo infrastruktur krytycznych. Zaproponowano ogólny analityczny model bezpieczeństwa złożonego systemu technicznego związanego z procesem zmian klimatycznych, który oddziałuje na jego obszar operacyjny. Jest to zintegrowany model bezpieczeństwa systemu technicznego, który łączy wielostanowy model bezpieczeństwa i model procesu zmian klimatyczno-pogodowych w obszarze działania systemu. Warunkowe funkcje bezpieczeństwa w poszczególnych stanach klimatyczno-pogodowych i bezwarunkowa funkcji bezpieczeństwa systemu przy zmieniających się w czasie warunkach klimatyczno-pogodowych, wartość średnia i wariancja bezwarunkowego czasu życia systemu oraz inne wskaźniki bezpieczeństwa są zdefiniowane dla dowolnej infrastruktury krytycznej.

Słowa kluczowe: bezpieczeństwo, infrastruktura krytyczna, proces zmian klimatyczno-pogodowych

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1. Introduction

Most real complex technical systems are strongly influenced by changing in time climate-weather conditions at their operating areas. The time dependent interactions between the climate-weather change process states varying at the system operating area and the system components safety parameters changing are evident features of most real technical systems including critical infrastructures. The common critical infrastructure safety and the climate-weather change at its operating area analysis is a great value in the industrial practice because of negative impacts of extreme weather hazards on the critical infrastructure safety. The convenient tools for analyzing this problem is the multistate critical infrastructures safety modelling used with the semi-Markov modeling of the climate-weather change processes at their operating areas, leading to the construction the joint general safety models of the critical infrastructures related to the climate-weather change processes at their operating areas.

2. Climate-weather change process at critical infrastructure operating area

The climate-weather change process model described below is proposed in [2].

States of climate-weather change process

Assume that there are $a, a \in \mathbb{N}$, parameters that describe the climate-weather states in fixed area. We mark their values by $w_1, \dots, w_a$, and assume that the possible values of the i -th parameter $w_i, i = 1, \dots, a$, belong to the interval $\langle b_i, d_i \rangle$. Each of the intervals $\langle b_i, d_i \rangle$ is divided into $n_i, n_i \in \mathbb{N}$, disjoint subintervals $\langle b_{i1}, d_{i1} \rangle, \langle b_{i2}, d_{i2} \rangle, \dots, \langle b_{in_i}, d_{in_i} \rangle$, such that

$$\langle b_{i1}, d_{i1} \rangle \cup \dots \cup \langle b_{in_i}, d_{in_i} \rangle = \langle b_i, d_i \rangle, d_{ij_i} = b_{ij_i+1}, j_i = 1, \dots, n_i - 1, i = 1, \dots, a. \quad (1)$$

The vector $(w_1, w_2, \dots, w_a)$ describing the climate-weather states can take values from the a dimensional space points set

$$\langle b_1, d_1 \rangle \times \langle b_2, d_2 \rangle \times \dots \times \langle b_a, d_a \rangle. \quad (2)$$

Above set is composed of the following a dimensional space domains

$$\langle b_{1j_1}, d_{1j_1} \rangle \times \langle b_{2j_2}, d_{2j_2} \rangle \times \dots \times \langle b_{aj_a}, d_{aj_a} \rangle, j_i = 1, 2, \dots, n_i, i = 1, 2, \dots, a. \quad (3)$$

The above domains are called the climate-weather states of the climate-weather change process. We numerate them from 1 up to the value $w = n_1 \cdot n_2 \cdot \dots \cdot n_a$ and mark by $c_1, \dots, c_w$. Each of the climate-weather change process state c_j , $j = 1, 2, \dots, w$, of the vector form $(w_1, w_2, \dots, w_a)$ is called the x th, $x = 0, 1, \dots, a$, category extreme weather hazard state of the climate-weather change process if x of weather parameters w_i , $i = 1, \dots, a$, are at the 1st category extreme weather hazard state.

Semi-Markov model of climate-weather change process

Assume that the climate-weather change process for the critical infrastructure operating area is taking w , $w \in \mathbb{N}$, different climate-weather states $c_1, \dots, c_w$ in this area. Then, we can define the climate-weather change process $C(t)$, $t \in (-\infty, \infty)$ with discrete operation states from the set $\{c_1, \dots, c_w\}$ assuming that it is a semi-Markov process. It can be described by:

- the vector $[q_b(0)]_{1 \times w}$ of the initial probabilities $q_b(0) = P(C(0) = c_b)$, $b = 1, \dots, w$, of the process $C(t)$ staying at particular states c_b at the moment $t = 0$;
- the matrix $[q_{bl}]_{w \times w}$ of the probabilities of transitions q_{bl} , $b, l = 1, \dots, w$, $b \neq l$, of the process $C(t)$ from the states c_b to c_l ;
- the matrix $[C_{bl}(t)]_{w \times w}$ of the conditional distribution functions $C_{bl}(t)$, $b, l = 1, \dots, w$, of the conditional sojourn times C_{bl} at the state c_b when its next state is c_l .

Basic characteristics of the climate-weather change process after assuming that we have identified the above parameters are as follows.

The unconditional distribution functions of the sojourn times C_b , $b = 1, \dots, w$, of the climate-weather change process $C(t)$ at the states c_b are given by

$$C_b(t) = \sum_{l=1}^w q_{bl} C_{bl}(t), \quad b = 1, 2, \dots, w. \quad (4)$$

The mean values $E[C_b]$, $b = 1, \dots, w$, of the process $C(t)$ unconditional sojourn times C_b at the climate-weather states are given by

$$N_b = E[C_b] = \sum_{l=1}^w q_{bl} N_{bl} = \sum_{l=1}^w q_{bl} E[C_{bl}] = \sum_{l=1}^w (q_{bl} \cdot \int_0^{\infty} t dC_{bl}(t)), \quad b = 1, \dots, w, \quad (5)$$

where N_{bl} are mean values of the conditional sojourn times C_{bl} .

The limit values of the process $C(t)$ transient probabilities at the particular operation states are given by

$$q_b = \lim_{t \rightarrow \infty} q_b(t) = \lim_{t \rightarrow \infty} P(C(t) = c_b) = \frac{\pi_b N_b}{\sum_{l=1}^w \pi_l N_l}, \quad b = 1, \dots, w, \quad (6)$$

where the steady probabilities π_b of the vector $[\pi_b]_{1 \times w}$ satisfy the equations

$$[\pi_b] = [\pi_b][q_{bl}], \quad \sum_{l=1}^w \pi_l = 1. \quad (7)$$

The limit transient probabilities q_b , $b = 1, \dots, w$, defined by (6), are the long term proportions of the climate-weather change process $C(t)$ sojourn times at the particular states c_b in the case of a periodic climate-weather change process.

3. System operation at climate-weather variable conditions

Climate-weather impact on critical infrastructure components safety

We assume that the changes of the climate-weather change process $C(t)$ states at the critical infrastructure system operating area have an influence on the critical infrastructure system multistate components safety.

We consider the critical infrastructure composed of the components $E_{ij}^{(v)}$, $i = 1, 2, \dots, k$, $j = 1, 2, \dots, l_i$, of subsystems S_v , $v = 1, \dots, m$. Their conditional lifetime in the safety state subset $\{u, u+1, \dots, z\}$ while the climate-weather change process $C(t)$ at the system operating area is at the state c_b , $b = 1, \dots, w$, are denoted by $[T_{ij}^{(v)}]^{(b)}(u)$. For simplicity, we consider only the case when components have the exponential safety functions at the climate-weather change process $C(t)$ at the system operating area states c_b , $b = 1, \dots, w$, i.e. the coordinates of the vector

$$[S_{ij}^{(v)}(t, \cdot)]^{(b)} = [1, [S_{ij}^{(v)}(t, 1)]^{(b)}, \dots, [S_{ij}^{(v)}(t, z)]^{(b)}], \quad t \in (-\infty, \infty), \quad i = 1, 2, \dots, k, \quad (8)$$
$$j = 1, 2, \dots, l_i, \quad v = 1, \dots, m, \quad b = 1, 2, \dots, w,$$

are given by

$$[S_{ij}^{(v)}(t, u)]^{(b)} = P([T_{ij}^{(v)}]^{(b)}(u) > t | C(t) = c_b) = \exp[-[\rho_{ij}^{(v)}(u)]^{(b)} \lambda_{ij}^{(v)}(u)t], \quad (9)$$
$$t \in (-\infty, \infty), \quad u = 1, 2, \dots, z, \quad i = 1, 2, \dots, k, \quad j = 1, 2, \dots, l_i, \quad v = 1, \dots, m, \quad b = 1, 2, \dots, w,$$

where $\lambda_{ij}^{(v)}(u)$, $u = 1, 2, \dots, z$, $i = 1, 2, \dots, k$, $j = 1, 2, \dots, l_i$, are the intensities of the components $E_{ij}^{(v)}$, $i = 1, 2, \dots, k$, $j = 1, 2, \dots, l_i$, of the subsystem S_v , $v = 1, \dots, m$, departure from the safety state subset $\{u, u+1, \dots, z\}$ without of the climate-weather change influence and $[\rho_{ij}^{(v)}(u)]^{(b)}$, $u = 1, 2, \dots, z$, $i = 1, 2, \dots, k$, $j = 1, 2, \dots, l_i$, $v = 1, \dots, m$, $b = 1, 2, \dots, w$, are the coefficients of the climate-weather impact on the components $E_{ij}^{(v)}$, $i = 1, 2, \dots, k$, $j = 1, 2, \dots, l_i$, of the subsystems S_v , $v = 1, \dots, m$, intensities of ageing at the climate-weather change process operating area states c_b , $b = 1, \dots, w$.

The system conditional lifetime in the safety state subset $\{u, u+1, \dots, z\}$ while the climate-weather change process $C(t)$ at the system operating area is at the state c_b , $b = 1, \dots, w$, is denoted by $T''^{(b)}(u)$ and the conditional system safety function is given by the vector

$$[S''(t, \cdot)]^{(b)} = [1, [S''(t, 1)]^{(b)}, \dots, [S''(t, z)]^{(b)}], \quad (10)$$

with the coordinates defined by

$$[S''(t, u)]^{(b)} = P(T''^{(b)}(u) > t | C(t) = c_b), \quad t \in (-\infty, \infty), \quad u = 1, 2, \dots, z, \quad b = 1, \dots, w. \quad (11)$$

Next, the unconditional system lifetime in the safety state subset $\{u, u+1, \dots, z\}$ is marked by $T''(u)$ and the unconditional system safety function by the vector

$$S''(t, \cdot) = [1, S''(t, 1), \dots, S''(t, z)], \quad (12)$$

with the coordinates defined by

$$S''(t, u) = P(T''(u) > t), \quad t \in (-\infty, \infty), \quad u = 1, 2, \dots, z. \quad (13)$$

Moreover, the coordinates (13) of the unconditional system safety function defined by (12) could be obtained from

$$S''(t, u) \cong \sum_{b=1}^w q_b [S''(t, u)]^{(b)}, \quad t \geq 0, \quad u = 1, 2, \dots, z, \quad (14)$$

when the system operation time θ is large enough and q_b , $b = 1, 2, \dots, w$, are the climate-weather change process $C(t)$ at the system operating area limit transient probabilities at the state c_b , $b = 1, 2, \dots, w$, given by (6).

Critical infrastructure safety indicators

The mean value and the variance of the system unconditional lifetime $T''(u)$ in the safety state subset $\{u, u+1, \dots, z\}$ are given by formulae [2]

$$\mu''(u) \cong \sum_{b=1}^w q_b \cdot \left(\int_0^{\infty} [S''(t, u)]^{(b)} dt \right), \quad u = 1, 2, \dots, z, \quad (15)$$

$$\sigma''^2(u) = 2 \int_0^{\infty} t S''(t, u) dt - [\mu''(u)]^2, \quad u = 1, 2, \dots, z, \quad (16)$$

where $S''(t, u)$, $u = 1, 2, \dots, z$, are given by (13)-(14).

Thus, the mean values of the unconditional system lifetimes in particular safety states are obtained from formulae

$$\bar{\mu}''(u) = \mu''(u) - \mu''(u+1), \quad u = 0, 1, \dots, z-1, \quad \bar{\mu}''(z) = \mu''(z), \quad (17)$$

where $\mu''(u)$, $u = 0, 1, \dots, z$, are given by (15).

Moreover, if r is the system critical safety state, then the system risk function

$$r''(t) = P(S''(t) < r \mid S''(0) = z) = P(T''(r) \leq t), \quad t \geq 0, \quad (18)$$

defined as a probability that the system is in the subset of safety states worse than the critical safety state r , $r \in \{1, \dots, z\}$ while it was in the safety state z at the moment $t = 0$ is given by

$$r''(t) = 1 - S''(t, r), \quad t \geq 0, \quad (19)$$

where $S''(t, r)$ is the coordinate of the system unconditional safety function given by (13) for $u = r$.

The moment τ the system risk function exceeds a permitted level δ is given by

$$\tau = (r'')^{-1}(\delta), \quad (20)$$

where $(r'')^{-1}(t)$, if it exists, is the inverse function of the risk function $r''(t)$ given by (19). Other critical infrastructure safety indices are:

- the intensities of ageing of the critical infrastructure related to the climate-weather change impact, i.e. the coordinates of the vector

$$\lambda''(t, \cdot) = [0, \lambda''(t, 1), \dots, \lambda''(t, z)], \quad t \geq 0, \quad (21)$$

where

$$\lambda''(t, u) = - \frac{dS''(t, u)}{dt}, \quad t \geq 0, \quad u = 1, 2, \dots, z, \quad (22)$$

- the coefficients of the climate-weather impact on the critical infrastructure intensities of ageing, i.e. the coordinates of the vector

$$\rho''(t, \cdot) = [0, \rho''(t, 1), \dots, \rho''(t, z)], \quad t \geq 0, \quad (23)$$

where

$$\lambda''(t, u) = \rho''(t, u) \cdot \lambda(t, u), \quad t \geq 0, \quad u = 1, 2, \dots, z, \quad (24)$$

and $\lambda(t, u)$ are the intensities of ageing of the critical infrastructure without of climate-weather impact, i.e. the coordinate of the vector

$$\lambda(t, \cdot) = [0, \lambda(t, 1), \dots, \lambda(t, z)], \quad t \geq 0, \quad (25)$$

- the coefficients of the climate-weather impact on the critical infrastructure without considering varying in time

$$\rho''(u) = \frac{1/\mu''(u)}{1/\mu(u)}, \quad u = 1, 2, \dots, z, \quad (26)$$

where $\mu''(u)$, $\mu(u)$, $u = 1, 2, \dots, z$, are respectively the mean values of the system unconditional lifetime in the safety state subset $\{u, u+1, \dots, z\}$ with and without considering the climate-weather impact on the safety of the critical infrastructure,

- the coefficient of the climate-weather resilience of the critical infrastructure

$$RI''(u) = 1/\rho''(u), \quad u = 1, 2, \dots, z, \quad (27)$$

where $\rho''(u)$, $u = 1, 2, \dots, z$, are the coefficients of the climate-weather impact on the critical infrastructure given by (26).

4. Conclusions

The proposed method of the critical infrastructure safety indicators determination with considering the climate-weather change process influence is a very practically approach. It is applied to the port oil piping transportation system and the maritime ferry technical system in [5], where their safety indicators are evaluated.

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