STOCHASTIC MODELLING OF THE REPAIRABLE SYSTEM

STOCHASTYCZNE MODELOWANIE NAPRAWIALNEGO SYSTEMU

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Abstract: All reliability models consisting of random time factors form stochastic processes. In this paper we recall the definitions of the most common point processes which are used for modelling of repairable systems. Particularly this paper presents stochastic processes as examples of reliability systems for the support of the maintenance related decisions. We consider the simplest one-unit system with a negligible repair or replacement time, i.e., the unit is operating and is repaired or replaced at failure, where the time required for repair and replacement is negligible. When the repair or replacement is completed, the unit becomes as good as new and resumes operation. The stochastic modelling of recoverable systems constitutes an excellent method of supporting maintenance related decision-making processes and enables their more rational use.

Keywords: repairable system, point process, stochastic modelling, Poisson process

Streszczenie: Wszelkie modele niezawodności uwzględniające czynniki z losowym czasem przyjmują postać różnych procesów stochastycznych. W tej publikacji przywołujemy definicje podstawowych procesów stosowanych do modelowania systemów naprawialnych. W szczególności przedstawione są procesy stochastyczne wspomagające podejmowanie decyzji dotyczących utrzymania systemu technicznego. Rozważamy najprostszy jednoelementowy system z zaniedbywalnym czasem naprawy lub wymiany, tj. jednostka jest użytkowana a po uszkodzeniu jest naprawiana lub wymieniana w na tyle krótkim czasie, że można go pominąć. Po naprawie lub wymianie jednostka jest tak dobra jak nowa i dalej jest użytkowana. Stochastyczne modelowanie systemów naprawialnych stanowi doskonałą metodę wspomagania procesów decyzyjnych w ich utrzymaniu i pozwala na bardziej racjonalną ich eksploatację.

Słowa kluczowe: system naprawialny, proces punktowy, modelowanie stochastyczne, proces Poissona

1. Introduction

In maintenance modelling of a technical object, mathematics find applications in work sampling, inventory control analysis, failure data analysis, establishing optimum preventive maintenance policies, maintenance cost analysis, and project management control. Some of the areas of mathematics used in maintenance include set theory, probability, calculus, differential equations, stochastic processes, and Laplace transforms. A wide knowledge of probabilities, statistics, and stochastic processes is needed for learning the reliability theory mathematically. In the reliability theory, stochastic processes are the most powerful mathematical tools for analyzing reliability models. High system reliability can be achieved by maintenance. A classical problem is to determine how reliability can be improved by using mathematical models. For this reason this paper presents essential mathematical concepts of stochastic models for the repairable system. In order to minimize failures in engineering systems, the designer must understand “why” and “how” failures occur. This helps them prevent failures. In order to maximize system performance, it is also important to know how often such failures may occur. This involves predicting the occurrence of failures. The knowledge of stochastic processes and mathematical tools is indispensable for engineers, managers and researchers in reliability and maintenance. In this area of knowledge a lot of interesting articles and books have already been written [2], [3], [5], [6], [7], [8], [12], [14], [16], [17], [18]. Reliability techniques have to be developed and expanded as objective models become more complex and large-scale [11].

2. Maintenance and failure taxonomies

This section presents some terms and definitions directly or indirectly used in engineering maintenance. Definitions of basic notions are quoted as follows [15], [3], [10]:

- repairable system – a system which, after failing to perform one or more of its functions satisfactorily, can be restored to fully satisfactory performance by any method, other than replacement of the entire system;
- repair – a restoration wherein a failed system (device) is returned to operable condition;
- perfect repair – a repair under which a failed system is replaced with a new identical one;
- minimal repair – a repair of limited effort wherein the device is returned to the operable state it was in just before failure;
- degraded failure – the component is not in the state which the manufacturer intended for performing its function, but the system function is being fulfilled;
- critical failure – the component is unable to perform its function due to critical-degradation of its state. Critically-degraded components are always repaired or renewed, and the repair typically begins as soon as possible;

- total failure – a component not only fails to meet specifications, but fails to meet specifications to any degree. Non-total critical failure occurs when the component fails to meet specifications but still has some residual functionality;
- maintenance – all actions appropriate for retaining an item/part/equipment in, or restoring it to a given condition;
- maintenance engineering – the activity of equipment/item maintenance that develops concepts, criteria, and technical requirements in conceptual and acquisition phases to be used and maintained in a current status during the operating phase in order to assure effective maintenance support of equipment;
- preventive maintenance (PM) – all actions carried out on a planned, periodic, and specific schedule to keep an item/equipment in stated working condition through the process of checking and reconditioning. These actions are precautionary steps undertaken to forestall or lower the probability of failures or an unacceptable level of degradation in later service, rather than correcting them after they occur;
- corrective maintenance (CM) – the unscheduled maintenance or repair in order to return items/equipment to a defined state, carried out because maintenance persons or users perceived deficiencies or failures;
- predictive maintenance – the use of modern measurement and signal processing methods to accurately diagnose item/equipment condition during operation;
- maintenance concept – a statement of the overall concept of the item/product specification or policy that controls the type of maintenance action to be employed for the item under consideration;
- maintenance plan – a document that outlines the management and technical procedures to be employed in order to maintain an item; usually describes facilities, tools, schedules, and resources;
- reliability – the probability that an item will perform its stated function satisfactorily for the desired period when used under specified conditions;
- maintainability – the probability that a failed item will be restored to an adequate working condition;
- mean time to repair (MTTR) – a figure of merit depending on item maintainability equal to the mean item repair time. In the case of exponentially distributed times to repair, MTTR is the reciprocal of the repair rate;
- overhaul – a comprehensive inspection and restoration of an item or a piece of equipment to an acceptable level at a durability time or usage limit;
- quality – the degree to which an item, function, or process satisfies requirements of the customer and user;
- maintenance person – an individual who conducts preventive maintenance and responds to a user's service call to a repair facility, and performs corrective maintenance on an item. Also called custom engineer, service person, technician, field engineer, mechanic, repair person, etc.;
- inspection – the qualitative observation of an item's performance or condition.

Maintenance of units after failure may be costly, and sometimes requires a long time. Thus, the most important problem is to determine when and how to preventively maintain units before failure. However, it is not wise to maintain units with unnecessary frequency. From such viewpoints, the commonly considered maintenance policies are preventive replacement for units without repair, and preventive maintenance for units with repair on a specific schedule. Classifying into three large groups, planned maintenance of units is carried out at a certain age, after a certain period of time, or after a specified number of occurrences. Consequently, the object of optimization problems is to determine the frequency and timing of several kinds of maintenance and replacement policies in accordance with maintenance costs and effects.

3. Poisson process as a model of a repairable system

If failures occur exponentially, i.e., the unit fails constantly during a time interval $(t, t + dt]$ irrespective of time t , the system forms a Poisson process [9], [14]. Roughly speaking, failures occur randomly in $(t, t + dt]$ with probability λdt for constant $\lambda > 0$; and interarrival times between failures have an exponential distribution $(1 - e^{-\lambda t})$. Then, it is said that failures occur in a Poisson process with rate λ .

We consider a Poisson process with only one state in which the unit is operating. When we consider a one-unit system whose repair time is non-negligible, the process forms an alternating renewal process with two states that repeats up and down alternately.

Let $N(t)$ denote the number of failures (events) in the time interval $(0, t]$ and let T_i be the time of the i th failure. The times T_i are called failure times or event times. Define $T_0 = 0$ and denote $X_i = T_i - T_{i-1}$, $i = 1, 2, \dots$, – the time between failure number $i - 1$ and failure number i . The times X_i are called working times or waiting times and also inter-arrival times. The observed sequence $\{T_i, i = 1, 2, \dots\}$ of failure times $T_1, T_2, \dots$ forms a point process, and $\{N(t), t \geq 0\}$ is the corresponding counting process. In the context of failure-repair models it is assumed here that all repair times are equal to 0. In practice this corresponds to the situation, when repair actions are conducted immediately or the repair times can be neglected with comparison to the working times X_i .

The process $N(t), t \geq 0$ is said to be a (homogeneous) Poisson process with rate (or intensity) $\lambda > 0$ if

- (i) $N(0) = 0$; i.e. there are no events at time 0;
- (ii) the number of events $N(s_2) - N(s_1)$ and $N(t_2) - N(t_1)$ in disjoint time intervals $(s_1, s_2]$ and $(t_1, t_2]$ are independent random variables (independent increments);
- (iii) the distribution of the number of events in a certain interval depends only on the length of the interval and not on its position (stationary increments);
- (iv) there exists a constant λ such that $\lim_{\Delta t \rightarrow 0} \frac{\mathbb{P}(N(\Delta t)=1)}{\Delta t} = \lambda$;
- (v) $\lim_{\Delta t \rightarrow 0} \frac{\mathbb{P}(N(\Delta t) \geq 2)}{\Delta t} = 0$.

The process defined above is denoted by HPP(λ). Note that if the random variables $X_1, X_2, \dots$ (waiting times) are independent and exponentially distributed $\text{Exp}(\frac{1}{\lambda})$, the counting process $\{N(t), t \geq 0\}$ is the HPP(λ). The corresponding sequence $\{T_i, i = 1, 2, \dots\}$ is also called the HPP(λ). Let us note that in the HPP(λ) the number of events in an interval of length t is a random variable having the Poisson distribution with the parameter λt , and that the time of the n -th event is a random variable having Erlang distribution with the parameters n and $1/\lambda$. Farther practical properties of the Poisson process are formulated.

Superposition of Poisson process.

Let $N_1(t)$ be the number of failed units from a certain population in $(0, t]$ and $N_2(t)$ be the number of failed units from the other. If the arrival times from two populations are independent and have the respective Poisson processes with rates λ_1 and λ_2 , the total number $N(t)$ of failed units arriving at a repair shop has the probability

$$\mathbb{P}(N(t) = n) = \frac{((\lambda_1 + \lambda_2)t)^n}{n!} e^{-(\lambda_1 + \lambda_2)t}$$

Thus, the process $\{N(t), t > 0\}$ is also a Poisson process with rate $\lambda_1 + \lambda_2$.

Decomposition of Poisson process.

Let $N(t)$ be the number of failed units occurring in $(0, t]$ and be a Poisson process with rate λ . Classifying into two large groups of failed units, the number $N_1(t)$ of minor ones occurs with probability p and the number $N_2(t)$ of major ones occurs with probability $q = 1 - p$, independent of $N_1(t)$. Then, because the number $N_1(t)$ has a binomial distribution with parameter p , given that n failures have occurred, the joint probability is

$$\mathbb{P}(N_1(t) = k, N_2(t) = n - k) = \frac{(p\lambda t)^k}{k!} e^{-p\lambda t} \frac{(pqt)^{n-k}}{(n-k)!} e^{-q\lambda t}$$

Thus, the two Poisson processes $(N_1(t), t > 0)$ and $(N_2(t), t > 0)$ are independent and have the Poisson processes with rates $p\lambda$ and $q\lambda$, respectively. In general, when the total number $N(t)$ of failed units with a Poisson process with rate λ is classified into k groups of $N_j(t)$, $j = 1, \dots, k$ with probability p_j where $\sum_{j=1}^k N_j(t) = N(t)$ and $\sum_{j=1}^k p_j = 1$, the joint probability is

$$\mathbb{P}(N_1(t) = n_1, \dots, N_k(t) = n_k) = \prod_{j=1}^k \frac{(p_j \lambda t)^{n_j}}{n_j!} e^{-p_j \lambda t}$$

that is called a multi-Poisson process [12].

Next, when events occur in a Poisson process, we obtain the distribution of the inter-arrival time X ; given that there was an event in $[0, t]$. This probability is given, for $u \leq t$,

$$\mathbb{P}(X \leq u | N(t) = 1) = \frac{\mathbb{P}(N(u) = 1, N(t) - N(u) = 0)}{\mathbb{P}(N(t) = 1)} = \frac{\lambda u e^{-\lambda u} e^{-\lambda(t-u)}}{\lambda t e^{-\lambda t}} = \frac{u}{t}$$

that is a uniform distribution over $[0, t]$. That is, when the event was detected at time t ; it occurs constantly over $[0, t]$.

Binomial distribution. Let $N(t)$ be a Poisson process with rate λ . Then, from conditional probability, for $0 \leq u \leq t$ and $k = 0, 1, \dots, n$

$$\begin{aligned} \mathbb{P}(N(u) = k | N(t) = n) &= \frac{\mathbb{P}(N(u) = k, N(t) = n)}{\mathbb{P}(N(t) = n)} \\ &= \frac{\mathbb{P}(N(u) = k) \mathbb{P}(N(t) - N(u) = n - k)}{\mathbb{P}(N(t) = n)} \\ &= \frac{((\lambda u)^k / k!) e^{-\lambda u} ((\lambda(t-u))^{n-k} / (n-k)!) e^{-\lambda(t-u)}}{((\lambda t)^n / n!) e^{-\lambda t}} \\ &= \binom{n}{k} \left(\frac{u}{t}\right)^k \left(1 - \frac{u}{t}\right)^{n-k} \end{aligned}$$

that is a binomial distribution with parameters n and u/t .

The process $\{N(t), t \geq 0\}$ is a renewal process if the random variables $X_1, X_2, \dots$ are independent and identically distributed with cumulative distribution function (cdf) F with $F(0) = 0$. The renewal process is denoted by $\text{RP}(F)$. The sequence $\{T_i, i = 1, 2, \dots\}$ is called the $\text{RP}(F)$ too. If F is the cdf of the exponential distribution with the parameter λ , then $\text{RP}(F)$ is $\text{HPP}(\lambda)$.

The major weaknesses of a HPP are the constant rate assumption and the fact that the distribution of the number of events in an interval depends only on the length of the interval and not on its position.

A process $\{N(t), t \geq 0\}$ is a nonhomogeneous Poisson process with intensity function $\lambda(t), t \geq 0$, if ([9], [12]):

- (i) $N(0) = 0$; i.e. there are no events at time 0;
- (ii) the number of events $N(s_2) - N(s_1)$ and $N(t_2) - N(t_1)$ in disjoint time intervals $(s_1, s_2]$ and $(t_1, t_2]$ are independent random variables (independent increments);
- (iii) there exists a function $\lambda(t)$ such that $\lim_{\Delta t \rightarrow 0} \frac{\mathbb{P}(N(t+\Delta t) - N(t) = 1)}{\Delta t} = \lambda(t)$;
- (iv) for each $t > 0$, $\lim_{\Delta t \rightarrow 0} \frac{\mathbb{P}(N(t+\Delta t) - N(t) \geq 2)}{\Delta t} = 0$.

The process defined above is denoted by $\text{NHPP}(\lambda(t))$. The important consequence of conditions (i)–(iv) is that the number of failures in the interval $(s, t]$, $t > s$, has the Poisson distribution with the parameter $\int_s^t \lambda(u) du$, i.e.,

$$\mathbb{P}(N(t) - N(s) = k) = \frac{\left(\int_s^t \lambda(u) du\right)^k}{k!} \exp\left(-\int_s^t \lambda(u) du\right)$$

The functions $\lambda(t)$ and $\Lambda(t) = \int_0^t \lambda(u)du$ are called the intensity function and the cumulative intensity function of the process, respectively. Of course, if $\lambda(t) = \lambda$, then the NHPP($\lambda(t)$) is the HPP(λ). Note that the definition of a NHPP relaxes the stationarity assumption of a HPP. The mean value function of an NHPP($\lambda(t)$) is $\mathbb{E}(N(t)) = \Lambda(t)$. This function gives the mean (expected) number of failures up to time t .

Let $F_i(t)$ denote the cdf of the occurrence times $T_i, i = 1, 2, \dots$, and f_i be the density function (pdf). We have

$$\begin{aligned} F_i(t) &= \mathbb{P}(T_i \leq t) = \mathbb{P}(N(t) \geq i) = 1 - \sum_{k=1}^{i-1} \mathbb{P}(N(t) = k) \\ &= 1 - \sum_{k=0}^{i-1} \frac{\left(\int_0^t \lambda(y)dy\right)^k}{k!} \exp\left(-\int_0^t \lambda(y)dy\right) \\ &= 1 - \sum_{k=0}^{i-1} \frac{(\Lambda(t))^k}{k!} e^{-\Lambda(t)} \end{aligned}$$

and

$$\begin{aligned} f_i(t) &= \frac{dF_i(t)}{dt} = - \sum_{k=0}^{i-1} \left(\frac{k(\Lambda(t))^{k-1} \Lambda'(t)}{k!} e^{-\Lambda(t)} + \frac{(\Lambda(t))^k}{k!} e^{-\Lambda(t)} (-\Lambda'(t)) \right) \\ &= \Lambda'(t) e^{-\Lambda(t)} \frac{(\Lambda(t))^{i-1}}{(i-1)!} = \lambda(t) e^{-\Lambda(t)} \frac{(\Lambda(t))^{i-1}}{(i-1)!} \end{aligned}$$

Using this definition, we derive the probability

$$\mathbb{P}(N(t+u) - N(t) = n) = \frac{(\Lambda(t+u) - \Lambda(t))^n}{n!} e^{-(\Lambda(t+u) - \Lambda(t))}, \quad n = 0, 1, \dots$$

for a fixed t . Clearly,

$$\begin{aligned} \mathbb{E}(N(t+u) - N(t)) &= \sum_{n=0}^{\infty} n \frac{(\Lambda(t+u) - \Lambda(t))^n}{n!} e^{-(\Lambda(t+u) - \Lambda(t))} \\ &= \Lambda(t+u) - \Lambda(t) \end{aligned}$$

Finally, the distribution of the inter-arrival time $X_1 = S_1$, given that there was an event in $[0, t]$, is [16, p. 78], for $u \leq t$,

$$\begin{aligned} \mathbb{P}(S_1 \leq u | N(t) = 1) &= \frac{\mathbb{P}(N(u) = 1, N(t) - N(u) = 0)}{\mathbb{P}(N(t) = 1)} \\ &= \frac{\mathbb{P}(N(u) = 1) \mathbb{P}(N(t) - N(u) = 0)}{\mathbb{P}(N(t) = 1)} = \frac{\Lambda(u) e^{-\Lambda(u)} e^{-(\Lambda(t) - \Lambda(u))}}{\Lambda(t) e^{-\Lambda(t)}} \\ &= \frac{\Lambda(u)}{\Lambda(t)}. \end{aligned}$$

The two point processes, the nonhomogeneous Poisson process $\{N(t), t \geq 0\}$ and the renewal process $\{T_i, i = 1, 2, \dots\}$, are widely investigated in the literature on reliability to model minimal repairs and perfect repairs.

When we consider two types of repair maintenance according to minor or major failures of a unit, the process forms a stochastic process with three states. In such ways, if the process transits among many states and has a Markov property in which the future behavior depends only on the present state and is independent of its past history, it forms a Markov process. If the duration times of the states are multiples of a time unit such as day, week, month, year, and a certain number of occurrences, then the process is called a discrete-time Markov chain, and if the duration times are distributed exponentially, it is called a continuous-time Markov chain.

4. Compound Poisson process as a model of the repairable system

Consider a standard cumulative damage model [13, p.16]: shocks occur in a Poisson Process with rate λ . A unit is subjected to shocks and suffers some damage due to these shocks. Let random variables X_k ($k = 1, 2, \dots$) denote a sequence of interarrival times between successive shocks, and random variables W_k ($k = 1, 2, \dots$) denote the damage produced by the k th shock, where $W_0 = 0$. It is assumed that the sequence of (W_k) is nonnegative, independently, and identically distributed according to $\mathbb{P}(W_k \leq x) = G(x)$, with finite mean, and furthermore, W_k is independent of X_j ($j \neq k$). Let $N(t)$ denote the total number of shocks up to time t . Then, define a random variable $Z(t)$ that represents the total damage at time t

$$Z(t) \stackrel{\text{def}}{=} \sum_{k=0}^{N(t)} W_k,$$

$Z(t)$ is called a compound Poisson Process. The distribution of $Z(t)$ is

$$\begin{aligned} \mathbb{P}(Z(t) \leq x) &= \mathbb{P}\left(\sum_{k=0}^{N(t)} W_k \leq x\right) = \sum_{k=0}^{\infty} \mathbb{P}\left(\sum_{j=0}^k W_j \leq x | N(t) = k\right) \mathbb{P}(N(t) = k) \\ &= \sum_{k=0}^{\infty} \mathbb{P}(W_0 + W_1 + \dots + W_k \leq x) \frac{(\lambda t)^k}{k!} e^{-\lambda t} \\ &= \sum_{k=0}^{\infty} G^{(k)}(x) \frac{(\lambda t)^k}{k!} e^{-\lambda t} \end{aligned}$$

where $G^{(k)}$ denotes k -fold convolution of the distributions G . Using a Laplace-Stieltjes transform we can find

$$\mathbb{E}Z(t) = \lambda t \mathbb{E}W, \mathbb{D}^2 Z(t) = \lambda t \mathbb{E}(W^2).$$

In the end suppose that shocks occur in a nonhomogeneous Poisson process with an intensity function $\lambda(t)$ and a mean value function $\Lambda(t)$, then

$$\mathbb{P}(Z(t) \leq x) = \sum_{k=0}^{\infty} G^{(k)}(x) \frac{(\Lambda(t))^k}{k!} e^{-\Lambda(t)}, \quad \mathbb{E}Z(t) = \Lambda(t)\mathbb{E}W.$$

Suppose that shocks occur in a stochastic process and each amount of damage due to shocks is additive. This process forms a cumulative process that is also called a jump process ([1], [4]) or doubly stochastic process ([8]). The analyses of such processes are more difficult than the already defined stochastic processes because the process is made up of two dependent stochastic processes.

5. Summary

In this publication a need of the stochastic modelling was justified as well as definitions applied in the maintenance engineering were quoted. Moreover the theoretical bases of a stochastic process were explained and its various properties were derived. In particular, for the modelling of the system maintenance process a homogeneous, a nonhomogeneous and a compound Poisson process were investigated. To apply processes to reliability models, we took up periodic replacements as well as shock and damage models. If failures occur generously as an extended process of a Poisson one, the system forms a renewal process. The demonstrated processes play a major role in the analysis of probability models with sums of independent nonnegative random variables, and are a useful tool in reliability theory because many reliability models have a renewal property due to replacement and maintenance. The results can be applied to a wide variety of both theoretical and practical probability problems not only in maintenance engineering but also in natural, economical, and social sciences.

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