VERIFICATION OF DIAGNOSTIC THRESHOLDS WITH ELIMINATION OF AMBIENT FACTORS

WERYFIKACJA PROGÓW DIAGNOSTYCZNYCH Z ELIMINACJĄ OTOCZENIA

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Abstract: Measurements of signals received from ambient environment is a difficult task for technical diagnostics and becomes even infeasible in a series of cases. It is why the issues related to elimination of ambient factors from the identification process of diagnostic thresholds are truly justified. The method for elimination of environment impact is disclosed in this paper and consists in displacement of a specific set of measured diagnostic parameters (the initial set) in relation to the subsequent set of diagnostic parameters (the target set). Then the square of amplitude gain and the phase shift are calculated for these two mutually shifted data sets, which makes it possible to evaluate "cohesion" density of results obtained for diagnostic signals and thus their suitability for calculation of diagnostic thresholds. The method is characterized by the peculiarity that it takes account for ambient "environment" with no need to measure its parameters [6, 7, 8, 12, 13, 14].

Keywords: technical diagnostics, diagnostic signals

Streszczenie: Trudnym do rozwiązania problemem diagnostyki technicznej jest pomiar sygnałów otoczenia. W wielu przypadkach pomiar ten jest niemożliwy. Stąd podjęcie problemu eliminacji otoczenia z procesu identyfikacji progów diagnostycznych jest zasadne. Przedstawiony sposób eliminacji otoczenia polegający na przesuwaniu danego zbioru pomiarów parametrów diagnostycznych (zbioru początkowego) względem kolejnego zbioru pomiarów diagnostycznych (zbioru następnego), a następnie wyznaczenie kwadratu wzmocnienia amplitudowego i przesunięcia fazowego dla tak przesuniętych zbiorów, pozwala ocenić „skupienie” pomiarów sygnałów diagnostycznych i stąd ich przydatność do obliczenia progów diagnostycznych. Metoda znamienna jest tą okolicznością, że uwzględnia ona „otoczenie” bez konieczności jego pomiaru [6, 7, 8, 12, 13, 14].

Słowa kluczowe: diagnostyka techniczna, sygnały diagnostyczne

1. Introduction

Diagnostics of technical facilities is associated with substantial obstacles due to difficulties with description of their environment. So far, the ambient environment has been defined by total hours of flying time or, even better, by means of the vibroacoustic signal A/s^2 . An example for elimination of ambient impact to the diagnostic process is the method that is used to diagnose machine blades during operation of rotating machinery, when a blade is monitored during a very short time T_{02} when the blade is in the close vicinity of a sensor [6, 7, 10, 11, 12, 14].

The current practice demonstrates that the problems associated with diagnostic of turbine blades during operation of rotating machinery are really sophisticated when merely one measurable signal $y(t)$ is available for diagnostics and that signal is subjected to interferences, which makes the situation even worse. In addition, a $x(t)$ signal from the ambient environment is available as well but is it so weak that is virtually non-measurable. It is why the concept appeared to eliminate impact of ambient environment from the diagnostic processes of technical facilities and the way to resolve the issue has already been found.

Similar issues are encountered when the diagnostic process of antifriction bearings involves tribologic methods. Only one tribologic signal is measurable with high accuracy whilst signals from ambient environment (besides the total flying time) are hardly identifiable in fact.

2. Analysis of opportunities to eliminate effect of ambient environment onto technical facilities

The method adopted for the analysis is based on the presumption that signals $x(t)$ received from the ambient environment and the diagnostic signals $y(t)$ are time dependent waveforms of stochastic nature and subjected to interferences. In such a case it is reasonable to substitute the time domain t for $x(t)$ and $y(t)$ signals with the time variable of τ that occurs in the correlation functions $R_{xx}(\tau)$, $R_{yy}(\tau)$, $R_{xy}(\tau)$ [2, 3, 4, 8].

Such an approach enables the following benefits:

- elimination of signal interferences and possibility to amplify measured signals;
- possibility to express the $R_{xx}(\tau)$, $R_{yy}(\tau)$, $R_{xy}(\tau)$ signals in a simple form as analytic functions, which offers broad opportunities for further conversion of these functions into other ones (with specific and peculiar properties) to be analyzed in the frequency domain (ω). These are the functions of power self-density $S_{xx}(\omega)$ and $S_{yy}(\omega)$ as well as power mutual density $S_{xy}(\omega)$. The additional finding was also made that the observation time T_{02} for the blade tip can be easily split into two subintervals – the subinterval T_{01} when the blade is approaching the sensor and the subinterval T_{12} associated with the blade movement away of the sensor. The T_I moment occurs when the blade is exactly under the sensor.

Conversion of the $x(t)$ and $y(t)$ functions into their corresponding transforms $S_{xx}(\omega)$, $S_{yy}(\omega)$, $S_{xy}(\omega)$ makes it possible to easily consider relationships between the diagnostic signals $y(t)$ and the ambient signals $x(t)$ for these two mutually adjacent periods of the signal observation: T_{01} when a blade is approaching the sensor the sensor and T_{12} after the blades has passed the sensor and is moving away. The moment “1” takes place when the blade is right under the sensor.

Therefore the following formulas are valid: [3, 5, 7, 8, 14, 17]

$$A_{T01}^2 = \frac{S_{yy}^{T01}}{S_{xx}^{T01}} \quad (1)$$

$$A_{T12}^2 = \frac{S_{yy}^{T12}}{S_{xx}^{T12}} \quad (2)$$

$$A_{T12}^2 = \frac{S_{yy}^{T12}}{S_{xx}^{T12}} \quad (3)$$

$$A_{T12}^2 = \frac{S_{yy}^{T12}}{S_{xx}^{T12}} \quad (4)$$

where:

A_{T01}^2 , A_{T12}^2 – kwadrat wzmocnienia amplitudowego sygnałów x i y w czasie zbliżania się łopatkki do czujnika T_{01} i oddalanie się łopatkki od czujnika T_{12}

φ_{T01} , φ_{T21} – przesunięcie fazowe sygnałów x i y w czasie zbliżania się łopatkki od czujnika T_{01} i oddalanie się łopatkki od czujnika T_{12} .

In addition, the assumptions can be made that the T_{12} time assigned for observation of the x and y signals occurs shortly (in ms) after the time period T_{01} when observation of these signals is also possible

If so, the following assumption is possible:

$$S_{xx}^{T12} = S_{xx}^{T01} \quad (5)$$

Thus, based on equations 1, 2 and 5 one can obtain a new, abstractive parameter (although having a physical sense) that adopts the form of product for amplitude gains A_{T01}^2 i A_{T12}^2 [12÷14]:

$$A_{T12,T01}^2 = \frac{A_{T12}^2}{A_{T01}^2} = \frac{\frac{S_{yy}^{T12}}{S_{xx}^{T12}}}{\frac{S_{yy}^{T01}}{S_{xx}^{T01}}} \xrightarrow{S_{xx}^{T12}=S_{xx}^{T01}} \frac{S_{yy}^{T12}}{S_{yy}^{T01}} \quad (6)$$

Furthermore, formulas 3, 4 and 5 enable definition of a subsequent new abstractive parameter (although having a physical sense) that adopts the form of product for difference for phase shift values φ_{T12T01} [5.9].

$$\begin{aligned}\varphi_{T_{12},T_{01}} &= \varphi_{T_{12}} - \varphi_{T_{01}} = \operatorname{Arg} \frac{\frac{S_{xy}^{T_{01}}}{S_{xx}^{T_{01}}}}{\frac{S_{xy}^{T_{12}}}{S_{xx}^{T_{12}}}} = \operatorname{Arg} \frac{A_{T_{12}} e^{-j\varphi_{T_{12}}}}{A_{T_{01}} e^{-j\varphi_{T_{01}}}} = \\ &= \operatorname{Arg} A_{T_{12},T_{01}} e^{-j(\varphi_{T_{12}} - \varphi_{T_{01}})} \xrightarrow{S_{xx}^{T_{12}} = S_{xx}^{T_{01}}} \operatorname{Arg} \frac{S_{xy}^{T_{12}}}{S_{xy}^{T_{01}}} = \varphi_{T_{12}} - \varphi_{T_{01}} = \varphi_{T_{12},T_{01}} \quad (7)\end{aligned}$$

The expression $A_{T_{12},T_{01}}^2$ (6) links the diagnostic signals $y(t)$ to signals $x(t)$ received from the ambient environment, thus it represents a specific diagnostic model. The characteristic feature of that model is the fact that it is established merely from the measurable diagnostic signal $y(t)$ during two subsequent observation intervals T_{01} and T_{12} following shortly one after another and, what is even the most important, that the model takes account of the ambient signals $x(t)$ with no need to measure ambient parameters and the $y(t)$ signal is sufficiently filtered from interferences [1, 5, 8, 11, 16].

The expression $\varphi_{T_{12},T_{01}}$ (7) links the diagnostic signals $y(t)$ to signals $x(t)$ received from the ambient environment, so it represents a subsequent diagnostic model. Similarly to the previous model $A_{T_{12},T_{01}}^2$ this one is also established with no need to measure signals $x(t)$ received from the ambient environment. To determine the $S_{xy}^{T_{12}}$, and $S_{xy}^{T_{01}}$ signal one has to apply the distribution law in the form of the $\delta(t, \hat{t})$, function since it is easy to demonstrate that the product of mutual power density for the y signal and the x signal is insensitive to signals x arriving from the ambient environment, thus it sufficiently eliminates impact of the real surroundings for the $\varphi_{T_{12},T_{01}}$ model [6, 7, 10, 11, 12, 17].

Such course of proceeding can be easily adopted to analysis of data sets collected from measurements of tribologic diagnostic signals that are used to establish diagnostic thresholds [15, 20]. Thus:

- T_{01} observation time becomes the time for taking subsequent measurements, e.g. 1, 2, 3;
- T_{12} observation time becomes the time for taking shifted measurements, e.g. 2, 3, 4.

Mutual displacement (shift) of data sets enables observation of interrelationships between signals that follow one after another with the delay of short time intervals. That time delay can be considered as very short compared to the ‘lifetime’ of the bearing system. Hence, also for tribologic measurements the formula (6) makes it possible to develop squared amplitude gain for shifted data sets from such measurements:

$$A_{(1-3);(2-4)}^2 = \frac{S_{yy}^{2-4}}{S_{yy}^{1-3}} \quad (8)$$

where: $S_{yy}^{1-3}, S_{yy}^{2-4}$ – discrete power of measured signals defined by the Cauchy product for these measurements [4];
 $2 \div 4$ – measurements taken for the sequence 2 ÷ 4;
 $1 \div 3$ – measurements taken for the sequence 1 ÷ 3.

Dispersion (non-uniformity of the data set distribution) for subsequent measurements are expressed by the M factor [1, 3, 4, 17, 19]:

$$M_i = \frac{\frac{A_i}{(1+A_i)}}{\frac{A_{i0}}{(1+A_{i0})}} \quad (9)$$

where: A_i – amplitude gain for data sets obtained from measurements and doomed for investigations.

The data sets from measurements shall not be dispersed when the M factor is low:

$M = 1,1 \div 1,3$ – excellent;

$M = 1,4 \div 1,5$ – good;

$M = 1,6 \div 1,8$ – satisfying;

$M = 1,85 \div 2,05$ – doubtful.

Assessment of the M_i factor must be correlated with observations for the phase shifts a_i [1, 3, 4, 8, 17, 19] for displaced data sets.

Thus, the formula (7) makes it possible to find the difference of phase shift values for data sets from tribologic measurements.

$$a_i = \varphi_{(2-4)} - \varphi_{(1-3)} = \varphi_{(1-3);(2-4)} = \text{Arg} \frac{s_{xy}^{2-4}}{s_{xy}^{1-3}} \quad (10)$$

where: a_i – parameters of arguments for data sets obtained from measurements and dedicated for investigations

The signal received from ambient environment and occurring in the formula (7) expresses the random distribution $\delta(t, \hat{t})$, where $\hat{t}$ stands for delay (displacement) of one signal after another [5, 16]. Hence, the data set of x signals received from the ambient environment and occurring in the formula (10) can be expressed by any other set, e.g. $\{1, 1, 1\}$, the same for the sets $2 \div 4$, i.e. $\{1, 1, 1\}$.

Data sets from test measurements shall not be excessively dispersed when the a coefficient is low [1, 3, 8, 15, 17, 19]:

$a = 0,00 \div 0,80$ – excellent (0° - 45°);

$a = 0,80 \div 0,90$ – good (45° - 52°);

$a = 0,90 \div 1,00$ – satisfying (52° - 57°);

$a = 1,00 \div 1,15$ – doubtful (57° - 66°).

3. Verification of diagnostic thresholds with elimination of signals from ambient environment in the frequency domain

To move to the frequency domain to carry out verification of diagnostic thresholds with elimination of signals from ambient environment it is first necessary to compute the Cauchy products for subsequent sets of measurement data, which is the process equivalent to the discrete Fourier transform [5, 9]. Computation the Cauchy product is easy, which is shown in Fig. 1.

Fig. 1 Diagram for computation of a convolution for data sequences representing signals from diagnostic tests (the sets $\{2, 2, 3, 3, 4\}$ and $\{1, 1, 2\}$).

Fig. 1 illustrates convolution for two data sequences, namely $\{2, 2, 3, 3, 4\} * \{1, 1, 2\}$. In this case the $\{1, 1, 2\}$ sequence is written vertically from bottom to top and then shifted downwards until subsequent pairs of both the first and second sequences appears. Here, similarly to the standard procedure usually applied to the signal analysis, the measure for interrelationships between the signals can be established, namely the coherence and amplitude gain between the signals x received from ambient environment and diagnostic signals y . Self-convolutions of the x and y signals as well mutual convolutions (equivalent to the Fourier transform) can be considered as the power densities for the following signals: self-convolutions S_{xx} and S_{yy} as well as mutual convolutions S_{xy} .

Measurement of copper (Cu) concentration taken by means of the XRF (X-Ray Fluorescence) method for the engine #1 and the engine #2 are summarized in the Table 1 [15, 20].

Computations of cardinality (power) for the data sets have been carried out with application of the Cauchy product (Fig. 1) for the interrelationships between the following mutually shifted measurement results from Table 1:

- 1÷3 and 2÷4;
- 1÷4 and 2÷5;
- 1÷5 and 2÷6;
- 1÷6 and 2÷7;
- 1÷7 and 2÷8.

The examples provided herein are far away from exhausting of all options for investigation of relationships between mutually shifted data sets with results for signal measurements.

Results of computations for the engine #1 and the engine #2 are summarized in Tables 2 ÷ 11, whilst the graphs for the dispersion coefficient M_i are revealed in Fig. 2 and 3.

Table 1. Results from XRF measurements for content of Cu, Fe, and MOA measurements for content of Cu, and Fe for engines #1 and #2.

Engine #1					
Measurement No.	Total flying hours	XRF Cu	XRF Fe	MOA Cu	MOA Fe
1	1666	0.19	0.088	0.0	0.0
2	1693	0.11	0.082	0.0	0.0
3	1745	0.39	0.069	0.0	0.3
4	1755	0.51	0.022	0.1	1.5
5	1805	0.41	0.013	0.4	2.3
6	1862	0.44	0.018	0.2	2.7
7	1900	0.37	0.014	0.3	4.2
8	1920	0.35	0.002	0.1	1.1
9	1980	0.37	0.001	0.1	1.0
10	2000	0.48	0.117	0.3	5.0
11	2043	0.46	0.046	0.2	6.3
12	2096	0.39	0.068	0.3	9.5
13	2150	0.27	0.054	0.3	7.0
14	2167	0.39	0.089	0.2	5.6
15	2214	0.36	0.071	0.2	4.0
16	2267	0.32	0.020	0.2	4.2
17	2297	0.35	0.051	0.1	2.0
18	2343	0.37	0.106	0.1	4.0
Engine #2					
Measurement No.	Total flying hours	XRF Cu	XRF Fe	MOA Cu	MOA Fe
1	1666	0.16	0.083	0.0	0.4
2	1693	0.17	0.153	0.0	0.9
3	1745	0.28	0.094	0.0	0.0
4	1755	0.49	0.024	0.0	0.4
5	1805	0.31	0.002	0.2	1.4
6	1862	0.25	0.003	0.1	1.5
7	1900	0.32	0.001	0.1	1.4
8	1920	0.38	0.000	0.2	2.3
9	1980	0.53	0.043	0.3	5.8
10	2000	0.50	0.008	0.1	1.1
11	2043	0.47	0.016	0.1	1.8
12	2096	0.39	0.005	0.2	2.9
13	2150	0.27	0.015	0.1	2.7
14	2167	0.38	0.010	0.1	2.3
15	2214	0.39	0.007	0.1	1.6
16	2267	0.27	0.000	0.1	1.7
17	2343	0.36	0.023	0.1	1.6
18	2400	0.30	0.002	0.1	1.3

Table 2. Computations of the M_i coefficients for data sets 1÷3 and 2÷4 for XRF measurements of Cu concentration (engine #1).

$S_{yyi}^{(2-4)}$	$S_{yyi}^{(1-3)}$	$S_{yyi}^{(2-4)}/S_{yyi}^{(1-3)}$	A_i	$A_i/1+A_i$	$M_i = \frac{\frac{A_i}{(1+A_i)}}{\frac{A_{i0}}{(1+A_{i0})}}$
0.0120	0.0362	0.3308	0.575	0.365	1.000
0.0863	0.0416	2.0726	1.440	0.590	1.616
0.2667	0.1619	1.6472	1.283	0.562	1.539
0.4011	0.0863	4.6508	2.157	0.683	1.871
0.2589	0.1554	1.6659	1.291	0.563	1.543

Table 3. Computations of the M_i coefficients for data sets 1÷4 and 2÷5 for XRF measurements of Cu concentration (engine #1).

$S_{yyi}^{(2-5)}$	$S_{yyi}^{(1-4)}$	$S_{yyi}^{(2-5)}/S_{yyi}^{(1-4)}$	A_i	$A_i/1+A_i$	$M_i = \frac{\frac{A_i}{(1+A_i)}}{\frac{A_{i0}}{(1+A_{i0})}}$
0.0120	0.0362	0.3308	0.575	0.365	1.000
0.0863	0.0416	2.0726	1.440	0.590	1.616
0.2667	0.1619	1.6472	1.283	0.562	1.539
0.4906	0.2798	1.7535	1.324	0.570	1.560
0.5813	0.2667	2.1796	1.476	0.596	1.633
0.4162	0.4011	1.0375	1.019	0.505	1.382
0.1673	0.2589	0.6462	0.804	0.446	1.220

Table 4. Computations of the M_i coefficients for data sets 1÷5 and 2÷6 for XRF measurements of Cu concentration (engine #1).

$S_{yyi}^{(2-6)}$	$S_{yyi}^{(1-5)}$	$S_{yyi}^{(2-6)}/S_{yyi}^{(1-5)}$	A_i	$A_i/1+A_i$	$M_i = \frac{\frac{A_i}{(1+A_i)}}{\frac{A_{i0}}{(1+A_{i0})}}$
0,0120	0,0362	0,3308	0,575	0,365	1,000
0,0863	0,0416	2,0726	1,440	0,590	1,616
0,2667	0,1619	1,6472	1,283	0,562	1,539
0,4906	0,2798	1,7535	1,324	0,570	1,560
0,6766	0,4223	1,6021	1,266	0,559	1,530
0,7594	0,4906	1,5478	1,244	0,554	1,518
0,6102	0,5813	1,0497	1,025	0,506	1,386
0,3561	0,4162	0,8555	0,925	0,481	1,316
0,1895	0,1673	1,1327	1,064	0,516	1,412

Table 5. Computations of the M_i coefficients for data sets 1÷6 and 2÷7 for XRF measurements of Cu concentration (engine #1).

$S_{yyi}^{(2-7)}$	$S_{yyi}^{(1-6)}$	$S_{yyi}^{(2-7)}/S_{yyi}^{(1-6)}$	A_i	$A_i/1+A_i$	$M_i = \frac{\frac{A_i}{(1+A_i)}}{\frac{A_{i0}}{(1+A_{i0})}}$
0.0120	0.0362	0.3308	0.575	0.365	1.000
0.0863	0.0416	2.0726	1.440	0.590	1.616
0.2667	0.1619	1.6472	1.283	0.562	1.539
0.4906	0.2798	1.7535	1.324	0.570	1.560
0.6766	0.4223	1.6021	1.266	0.559	1.530
0.8410	0.6562	1.2817	1.132	0.531	1.454
0.9045	0.6766	1.3368	1.156	0.536	1.468
0.7358	0.7594	0.9690	0.984	0.496	1.359
0.4948	0.6102	0.8108	0.900	0.474	1.298
0.3249	0.3561	0.9125	0.955	0.489	1.338
0.1393	0.1895	0.7350	0.857	0.462	1.264

Table 6. Computations of the M_i coefficients for data sets 1÷7 and 2÷8 for XRF measurements of Cu concentration (engine #1).

$S_{yyi}^{(2-4)}$	$S_{yyi}^{(1-3)}$	$S_{yyi}^{(2-4)}/S_{yyi}^{(1-3)}$	A_i	$A_i/1+A_i$	$M_i = \frac{\frac{A_i}{(1+A_i)}}{\frac{A_{i0}}{(1+A_{i0})}}$
0.0120	0.0362	0.3308	0.575	0.365	1.000
0.0863	0.0416	2.0726	1.440	0.590	1.616
0.2667	0.1619	1.6472	1.283	0.562	1.539
0.4906	0.2798	1.7535	1.324	0.570	1.560
0.6766	0.4223	1.6021	1.266	0.559	1.530
0.8410	0.6562	1.2817	1.132	0.531	1.454
0.9806	0.8185	1.1980	1.095	0.523	1.431
1.0101	0.8410	1.2010	1.096	0.523	1.432
0.8488	0.9045	0.9384	0.969	0.492	1.348
0.6095	0.7358	0.8283	0.910	0.476	1.305
0.4422	0.4948	0.8937	0.945	0.486	1.331
0.2597	0.3249	0.7992	0.894	0.472	1.293
0.1210	0.1393	0.8690	0.932	0.482	1.321

Table 7. Computations of the M_i coefficients for data sets 1÷3 and 2÷4 for XRF measurements of Cu concentration (engine #2).

$S_{yyi}^{(2-4)}$	$S_{yyi}^{(2-3)}$	$S_{yyi}^{(2-4)}/S_{yyi}^{(1-3)}$	A_i	$A_i/1+A_i$	$M_i = \frac{\frac{A_i}{(1+A_i)}}{\frac{A_{i0}}{(1+A_{i0})}}$
0.0295	0.0271	1.0907	1.044	0.511	1.000
0.0961	0.0566	1.6974	1.303	0.566	1.107
0.2471	0.1215	2.0331	1.426	0.588	1.151
0.2747	0.0961	2.8598	1.691	0.628	1.230
0.2417	0.0781	3.0958	1.759	0.638	1.248

Table 8. Computations of the M_i coefficients for data sets 1÷4 and 2÷5 for XRF measurements of Cu concentration (engine #2).

$S_{yyi}^{(2-5)}$	$S_{yyi}^{(1-4)}$	$S_{yyi}^{(2-5)}/S_{yyi}^{(1-4)}$	A_i	$A_i/1+A_i$	$M_i = \frac{\frac{A_i}{(1+A_i)}}{\frac{A_{i0}}{(1+A_{i0})}}$
0.0295	0.0271	1.0907	1.044	0.511	1.000
0.0961	0.0566	1.6974	1.303	0.566	1.107
0.2471	0.1215	2.0331	1.426	0.588	1.151
0.3812	0.2579	1.4781	1.216	0.549	1.074
0.4147	0.2471	1.6786	1.296	0.564	1.105
0.3045	0.2747	1.1084	1.053	0.513	1.004
0.0959	0.2417	0.3969	0.630	0.386	0.757

Table 9. Computations of the M_i coefficients for data sets 1÷5 and 2÷6 for XRF measurements of Cu concentration (engine #2).

$S_{yyi}^{(2-6)}$	$S_{yyi}^{(1-5)}$	$S_{yyi}^{(2-6)}/S_{yyi}^{(1-5)}$	A_i	$A_i/1+A_i$	$M_i = \frac{\frac{A_i}{(1+A_i)}}{\frac{A_{i0}}{(1+A_{i0})}}$
0.0295	0.0271	1.0907	1.044	0.511	1.000
0.0961	0.0566	1.6974	1.303	0.566	1.107
0.2471	0.1215	2.0331	1.426	0.588	1.151
0.3812	0.2579	1.4781	1.216	0.549	1.074
0.5016	0.3490	1.4372	1.199	0.545	1.067
0.4457	0.3812	1.1693	1.081	0.520	1.017
0.3444	0.4147	0.8303	0.911	0.477	0.933
0.1565	0.3045	0.5140	0.717	0.418	0.817
0.0639	0.0959	0.6658	0.816	0.449	0.880

Table 10. Computations of the M_i coefficients for data sets 1÷6 and 2÷7 for XRF measurements of Cu concentration (engine #2).

$S_{yyi}^{(2-7)}$	$S_{yyi}^{(1-6)}$	$S_{yyi}^{(2-7)}/S_{yyi}^{(1-6)}$	A_i	$A_i/1+A_i$	$M_i = \frac{\frac{A_i}{(1+A_i)}}{\frac{A_{i0}}{(1+A_{i0})}}$
0.0295	0.0271	1.0907	1.044	0.511	1.000
0.0961	0.0566	1.6974	1.303	0.566	1.107
0.2471	0.1215	2.0331	1.426	0.588	1.151
0.3812	0.2579	1.4781	1.216	0.549	1.074
0.5016	0.3490	1.4372	1.199	0.545	1.067
0.5543	0.4644	1.1938	1.093	0.522	1.022
0.5209	0.5016	1.0386	1.019	0.505	0.988
0.4672	0.4457	1.0483	1.024	0.506	0.990
0.2596	0.3444	0.7538	0.868	0.465	0.910
0.1597	0.1565	1.0203	1.010	0.503	0.984
0.0999	0.0639	1.5637	1.250	0.556	1.088

Table 11. Computations of the M_i coefficients for data sets 1÷7 and 2÷8 for XRF measurements of Cu concentration (engine #2).

$S_{yyi}^{(2-4)}$	$S_{yyi}^{(1-3)}$	$S_{yyi}^{(2-4)}/S_{yyi}^{(1-3)}$	A_i	$A_i/1+A_i$	$M_i = \frac{\frac{A_i}{(1+A_i)}}{\frac{A_{i0}}{(1+A_{i0})}}$
0.0295	0.0271	1.0907	1.044	0.511	1.000
0.0961	0.0566	1.6974	1.303	0.566	1.107
0.2471	0.1215	2.0331	1.426	0.588	1.151
0.3812	0.2579	1.4781	1.216	0.549	1.074
0.5016	0.3490	1.4372	1.199	0.545	1.067
0.5543	0.4644	1.1938	1.093	0.522	1.022
0.6510	0.6056	1.0749	1.037	0.509	0.996
0.6786	0.5543	1.2242	1.106	0.525	1.028
0.6315	0.5209	1.2123	1.101	0.524	1.026
0.3940	0.4672	0.8434	0.918	0.479	0.937
0.2910	0.2596	1.1212	1.059	0.514	1.007
0.2391	0.1597	1.4970	1.224	0.550	1.077
0.1431	0.0999	1.4332	1.197	0.545	1.067

Graphs for the dispersion coefficient M_i for measurements of copper concentration carried out by means of the XRF method (engine #1)

Fig. 2 Graphs for the dispersion coefficient M_i for measurements of copper concentration carried out by means of the XRF method (engine #1)

Graphs for the dispersion coefficient M_i for measurements of copper concentration carried out by means of the XRF method (engine #2)

Fig. 3 Graphs for the dispersion coefficient M_i for measurements of copper concentration carried out by means of the XRF method (engine #2)

Next, the phase shift of data sets was investigated. Computation results for the engine #1 and engine #2 are summarized in Tables 12÷21 whilst graphs for the coefficient of phase shift a_i are shown in Fig. 4 and 5.

Table 12. XRF measurements for Cu concentration, engine #1, computation for 3 measurement results

$S_{xyi}^{(2-4)}$	$S_{xyi}^{(1-3)}$	$\alpha_i = S_{xyi}^{(2-4)} / S_{xyi}^{(1-3)}$	$a_i = \alpha_0 - \alpha_i$ [rad]	a_i [°]
0.1094	0.1902	0.5752	0.0000	0
0.5036	0.2996	1.6809	-1.1057	-63
1.0124	0.6938	1.4592	-0.8840	-51
0.9030	0.5036	1.7931	-1.2179	-70
0.5088	0.3942	1.2907	-0.7155	-41

Table 13. XRF measurements for Cu concentration, engine #1, computation for 4 measurement results

$S_{xyi}^{(2-5)}$	$S_{xyi}^{(1-4)}$	$\alpha_i = S_{xyi}^{(2-5)} / S_{xyi}^{(1-4)}$	$a_i = \alpha_0 - \alpha_i$ [rad]	a_i [°]
0.1094	0.1902	0.5752	0.0000	0
0.5036	0.2996	1.6809	-1.1057	-63
1.0124	0.6938	1.4592	-0.8840	-51
1.4214	1.2026	1.1819	-0.6068	-35
1.3120	1.0124	1.2959	-0.7207	-41
0.9178	0.9030	1.0164	-0.4412	-25
0.4090	0.5088	0.8039	-0.2287	-13

Table 14. XRF measurements for Cu concentration, engine #1, computation for 5 measurement results

$S_{xyi}^{(2-6)}$	$S_{xyi}^{(1-5)}$	$\alpha_i = S_{xyi}^{(2-6)} / S_{xyi}^{(1-5)}$	$a_i = \alpha_0 - \alpha_i$ [rad]	a_i [°]
0.1094	0.1902	0.5752	0.0000	0
0.5036	0.2996	1.6809	-1.1057	-63
1.0124	0.6938	1.4592	-0.8840	-51
1.4214	1.2026	1.1819	-0.6068	-35
1.8567	1.6116	1.1521	-0.5769	-33
1.7473	1.4214	1.2293	-0.6541	-37
1.3531	1.3120	1.0313	-0.4561	-26
0.8443	0.9178	0.9199	-0.3447	-20
0.4353	0.4090	1.0643	-0.4891	-28

Table 15. XRF measurements for Cu concentration, engine #1, computation for 6 measurement results

$S_{xyi}^{(2-7)}$	$S_{xyi}^{(1-5)}$	$\alpha_i = S_{xyi}^{(2-4)}/S_{xyi}^{(1-3)}$	$a_i = \alpha_0 - \alpha_i$ [rad]	a_i [°]
0.1094	0.1902	0.5752	0.0000	0
0.5036	0.2996	1.6809	-1.1057	-63
1.0124	0.6938	1.4592	-0.8840	-51
1.4214	1.2026	1.1819	-0.6068	-35
1.8567	1.6116	1.1521	-0.5769	-33
2.2299	2.0469	1.0894	-0.5142	-29
2.1205	1.8567	1.1421	-0.5669	-32
1.7263	1.7473	0.9880	-0.4128	-24
1.2175	1.3531	0.8998	-0.3246	-19
0.8085	0.8443	0.9576	-0.3824	-22
0.3732	0.4353	0.8573	-0.2822	-16

Table 16. XRF measurements for Cu concentration, engine #1, computation for 7 measurement results

$S_{xyi}^{(2-8)}$	$S_{xyi}^{(1-7)}$	$\alpha_i = S_{xyi}^{(2-4)}/S_{xyi}^{(1-3)}$	$a_i = \alpha_0 - \alpha_i$ [rad]	a_i [°]
0.1094	0.1902	0.5752	0.0000	0
0.5036	0.2996	1.6809	-1.1057	-63
1.0124	0.6938	1.4592	-0.8840	-51
1.4214	1.2026	1.1819	-0.6068	-35
1.8567	1.6116	1.1521	-0.5769	-33
2.2299	2.0469	1.0894	-0.5142	-29
2.5778	2.4201	1.0652	-0.4900	-28
2.4684	2.2299	1.1070	-0.5318	-30
2.0742	2.1205	0.9782	-0.4030	-23
1.5654	1.7263	0.9068	-0.3316	-19
1.1564	1.2175	0.9498	-0.3746	-21
0.7211	0.8085	0.8919	-0.3167	-18
0.3479	0.3732	0.9322	-0.3570	-20

Table 17. XRF measurements for Cu concentration, engine #2, computation for 3 measurement results

$S_{xyi}^{(2-4)}$	$S_{xyi}^{(1-3)}$	$\alpha_i = S_{xyi}^{(2-4)}/S_{xyi}^{(1-3)}$	$a_i = \alpha_0 - \alpha_i$ [rad]	a_i [°]
0.1719	0.1646	1.0443	0.0000	0
0.4513	0.3365	1.3412	-0.2968	-17
0.9429	0.6159	1.5309	-0.4866	-28
0.7710	0.4513	1.7084	-0.6640	-38
0.4916	0.2794	1.7595	-0.7151	-41

Table 18. XRF measurements for Cu concentration, engine #2, computation for 4 measurement results

$S_{xyi}^{(2-5)}$	$S_{xyi}^{(1-4)}$	$\alpha_i = S_{xyi}^{(2-4)} / S_{xyi}^{(1-3)}$	$a_i = \alpha_0 - \alpha_i$ [rad]	a_i [°]
0.1719	0.1646	1.0443	0.0000	0
0.4513	0.3365	1.3412	-0.2968	-17
0.9429	0.6159	1.5309	-0.4866	-28
1.2526	1.1075	1.1310	-0.0867	-5
1.0807	0.9429	1.1461	-0.1018	-6
0.8013	0.7710	1.0393	0.0051	0
0.3097	0.4916	0.6300	0.4144	24

Table 19. XRF measurements for Cu concentration, engine #2, computation for 5 measurement results

$S_{xyi}^{(2-6)}$	$S_{xyi}^{(1-5)}$	$\alpha_i = S_{xyi}^{(2-4)} / S_{xyi}^{(1-3)}$	$a_i = \alpha_0 - \alpha_i$ [rad]	a_i [°]
0.1719	0.1646	1.0443	0.0000	0
0.4513	0.3365	1.3412	-0.2968	-17
0.9429	0.6159	1.5309	-0.4866	-28
1.2526	1.1075	1.1310	-0.0867	-5
1.5053	1.4172	1.0622	-0.0178	-1
1.3334	1.2526	1.0645	-0.0202	-1
1.0540	1.0807	0.9753	0.0691	4
0.5624	0.8013	0.7019	0.3425	20
0.2527	0.3097	0.8160	0.2284	13

Table 20. XRF measurements for Cu concentration, engine #2, computation for 6 measurement results

$S_{xyi}^{(2-7)}$	$S_{xyi}^{(1-5)}$	$\alpha_i = S_{xyi}^{(2-4)} / S_{xyi}^{(1-3)}$	$a_i = \alpha_0 - \alpha_i$ [rad]	a_i [°]
0.1719	0.1646	1.0443	0.0000	0
0.4513	0.3365	1.3412	-0.2968	-17
0.9429	0.6159	1.5309	-0.4866	-28
1.2526	1.1075	1.1310	-0.0867	-5
1.5053	1.4172	1.0622	-0.0178	-1
1.8213	1.6699	1.0907	-0.0463	-3
1.6494	1.5053	1.0957	-0.0514	-3
1.3700	1.3334	1.0274	0.0169	1
0.8784	1.0540	0.8334	0.2110	12
0.5687	0.5624	1.0112	0.0331	2
0.3160	0.2527	1.2505	-0.2061	-12

Table 21. XRF measurements for Cu concentration, engine #2, computation for 7 measurement results

$S_{xyi}^{(2-8)}$	$S_{xyi}^{(1-7)}$	$\alpha_i = S_{xyi}^{(2-4)}/S_{xyi}^{(1-3)}$	$a_i = \alpha_0 - \alpha_i [\text{rad}]$	$a_i [^\circ]$
0.1719	0.1646	1.0443	0.0000	0
0.4513	0.3365	1.3412	-0.2968	-17
0.9429	0.6159	1.5309	-0.4866	-28
1.2526	1.1075	1.1310	-0.0867	-5
1.5053	1.4172	1.0622	-0.0178	-1
1.8213	1.6699	1.0907	-0.0463	-3
2.1996	1.9859	1.1076	-0.0633	-4
2.0277	1.8213	1.1133	-0.0690	-4
1.7483	1.6494	1.0600	-0.0156	-1
1.2567	1.3700	0.9173	0.1271	7
0.9470	0.8784	1.0781	-0.0337	-2
0.6943	0.5687	1.2209	-0.1765	-10
0.3783	0.3160	1.1972	-0.1528	-9

Phase shift angles detected for the engine #1

Fig. 4 Phase shift angles detected for the engine #1

Phase shift angles detected for the engine #2

Fig. 5 Phase shift angles detected for the engine #2

Then evaluation is carried out for graphs from Fig. 2 ÷ 5.

Any experienced expert can easily come to the conclusions that thresholds established for the engine #2 are dependable.

Some doubts may only arise with regard to the threshold established from 3 measurements.

4. Recapitulation and concusions

To eliminate impact of ambient environment factors from the process dedicated to determination of diagnostic thresholds $\mu+\sigma$; $\mu+2\sigma$; $\mu+3\sigma$ one has to take an additional measurement $i+1$ (e.g. for 3 measurements that normally serve for determination of diagnostic thresholds it will be the additional measurement #4 and such a measurement can be taken beside regular maintenance checks). Then the test is made how the displaced (shifted) set of measurement data is behaved, e.g. 2÷4, in relation to the set of primary measurements (Tables 1 and 2). The measures for interrelationships between subsequent data set acquired from measurements and subjected to examinations are dispersions (overshot/overreguation) factors M_i for adjacent data sets: 1÷3 and 2÷4, then 1÷4 and 2÷5, etc. as well as its phase shift angle α_i .

Further analysis of the M_i parameter values disclosed in Fig. 2 and 3 makes it possible to come to the conclusions that the engines #1 and #2 are totally different devices. The engine #2 is more vulnerable to diagnostic operations than the engine

#1 since the measurement results collected for the engine #1 (Table 2) cannot serve as the basis to compute diagnostic thresholds: $M_i=1.871$.

Then, comparison between Tables 3 and 4 reveals that 4 measurements (plus the fifth one) are sufficient as a good ground to determine diagnostic thresholds for engines #1 and #2.

Furthermore, analysis of the a_i parameter on its graphs as shown in Fig. 4 and 5 confirms the conclusions that engines #1 and #2 are different in terms of diagnostic features. The engine #2 is more vulnerable to diagnostic operations than the engine #1 since the measurement results collected for the engine #1 (Table 11) cannot serve as the basis to compute diagnostic thresholds: $\alpha_i=-1.2179$ [rad]. Then, comparison between Tables 13 and 14 reveals that 4 measurements (plus the fifth one) are already sufficient as a good ground to determine diagnostic thresholds for engines #1 and #2.

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