MATHEMATICAL MODEL AND SIMULATOR OF ROTOR WITH VIBRATING BLADES

MODEL MATEMATYCZNY I SYMULATOR STOPNIA WIRNIKOWEGO Z DRGAJĄCYMI ŁOPATKAMI

Michał Wachlaczko, Radosław Przysowa, Mariusz Żokowski

Instytut Techniczny Wojsk Lotniczych
michal.wachlaczko@itwl.pl, radoslaw.przysowa@itwl.pl,
mariusz.zokowski@itwl.pl

Abstract: *The paper presents description of rotating bladed disk mathematical model. Correctly defined mathematical model of rotor allows creation of numerical simulation model which can be used to generate tip-timing data. First of all, the model is necessary to conduct a research on blade response due to input force in form of changing rotational speed. This enables the possibility to determine turbojet engine terminal operating conditions causing its failure.*

Keywords: *numerical simulation, engine model, turbojet engine, bladed disk*

Streszczenie: *Tematem publikacji jest opis modelu matematycznego ułopatkowanej tarczy stopnia wirującego silnika odrzutowego. Poprawnie stworzony model matematyczny wirnika pozwala na stworzenie modelu symulacyjnego, który może posłużyć do generowania danych tip-timing. Przede wszystkim jest on potrzebny do badania odpowiedzi łopatek wieńca wirnikowego na wymuszenie w postaci zmian prędkości obrotowej silnika. Pozwala to na określenie warunków pracy silnika odrzutowego, dla których mogło by nastąpić jego uszkodzenie.*

Słowa kluczowe: *symulacja numeryczna, model silnika, silnik turbinowy, ułopatkowana tarcza*

1. Introduction

Blade vibration measurements of rotary fluid-flow machines using “Blade Tip-Timing” method provides many pieces of information about technical state of bladed disk as well as rotor unbalance and control system operation. Correct identification of blade vibratory modes and frequencies during machine operation is only possible if we possess vibratory forms and frequencies obtained from experimental data and/or numerical methods (modal analysis utilizing dedicated CAD/CAM/CAE software).

When we know the needed blade structural and material data rotor, mathematical models may be formulated in order to simulate real rotor operation. This enables further the verification of blade working condition in the whole range of rotational speeds and allows to compare it with experimental data and model tune-up. Confirmed simulation results make it possible to predict and simulate working conditions, which in reality would cause machine failure and gather data about terminal and destructing operations.

Bladed disk simulator (Fig. 1) is a computer program oriented on dynamic object analysis, which means obtaining model response for given input acting on rotating blades. In turbojet/turbofan engine force input is precisely connected with constant changes of rotational speed.

Fig. 1 Bladed disk simulator

2. Equations of motion

Considerations regarding rotor simulation model have to begin with definition of motion equations. Usually motion equation is a 2nd order ordinary differential equation, so initial and boundary conditions have to be defined. Rotor system consists of “*i*” blades in disk and is simplified to **Single Degree Of Freedom** (SDOF) system with structural damping in elastic foundation. Every single blade is fixed in rotor disk with initial stiffness and damping in blade root and there are certain blade-to-blade links with inter-blade stiffness and damping as well.

The motion equation of rotor model can be written as:

$$M\ddot{x} + C\dot{x} + Kx = F(t) \quad (1)$$

where the matrices are defined as:

$$M = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & \frac{m_2}{m_1} & 0 & \dots & 0 \\ 0 & 0 & \frac{m_3}{m_1} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \frac{m_i}{m_1} \end{bmatrix} \quad \text{- mass matrix (2)}$$

$$C = 2\zeta\omega_n \begin{bmatrix} \frac{c_{l,1}+c_1+c_{1,2}}{c_1} & -\frac{c_{1,2}}{c_1} & 0 & \dots & -\frac{c_{l,1}}{c_1} \\ -\frac{c_{1,2}}{c_1} & \frac{c_{1,2}+c_2+c_{2,3}}{c_1} & -\frac{c_{2,3}}{c_1} & \dots & 0 \\ 0 & -\frac{c_{2,3}}{c_1} & \frac{c_{2,3}+c_3+c_{3,4}}{c_1} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & -\frac{c_{i-1,i}}{c_1} \\ -\frac{c_{i,1}}{c_1} & 0 & 0 & -\frac{c_{i-1,i}}{c_1} & \frac{c_{i-1,i}+c_i+c_{i,1}}{c_1} \end{bmatrix} \quad \text{- damping matrix (3)}$$

$$K = \omega_n^2 \begin{bmatrix} \frac{k_{l,1}+k_1+k_{1,2}}{k_1} & -\frac{k_{1,2}}{k_1} & 0 & \dots & -\frac{k_{l,1}}{k_1} \\ -\frac{k_{1,2}}{k_1} & \frac{k_{1,2}+k_2+k_{2,3}}{k_1} & -\frac{k_{2,3}}{k_1} & \dots & 0 \\ 0 & -\frac{k_{2,3}}{k_1} & \frac{k_{2,3}+k_3+k_{3,4}}{k_1} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & -\frac{k_{i-1,i}}{k_1} \\ -\frac{k_{i,1}}{k_1} & 0 & 0 & -\frac{k_{i-1,i}}{k_1} & \frac{k_{i-1,i}+k_i+k_{i,1}}{k_1} \end{bmatrix} \quad \text{- stiffness matrix (4)}$$

$$c = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_i \end{bmatrix} \quad \begin{array}{l} \text{- foundation damping} \\ \text{vector (in blades' } \\ \text{roots)} \end{array} \quad (5)$$

$$c_{int} = \begin{bmatrix} c_{12} \\ c_{23} \\ c_{34} \\ \vdots \\ c_{i1} \end{bmatrix} \quad \begin{array}{l} \text{- inter-blade} \\ \text{damping vector (6)} \end{array}$$

$$\mathbf{k} = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ \vdots \\ k_i \end{bmatrix} \quad \begin{array}{l} \text{- foundation stiffness} \\ \text{vector (in blades'} \\ \text{roots)} \end{array} \quad (7)$$

$$\mathbf{k}_{int} = \begin{bmatrix} k_{12} \\ k_{23} \\ k_{34} \\ \vdots \\ k_{i1} \end{bmatrix} \quad \begin{array}{l} \text{- inter-blade stiffness} \\ \text{vector} \end{array} \quad (8)$$

- ζ - damping coefficient (9);
 ω_n - blade natural frequency (10);
 i - number of blades (11);
 $\ddot{\mathbf{X}}$ - blade-tip acceleration vector (12);
 $\dot{\mathbf{X}}$ - blade-tip velocity vector (13);
 $\mathbf{X}$ - blade-tip displacement vector (14);
 $\mathbf{F}$ - force input vector (15).

If all elements in $\mathbf{M}$ vector, $\mathbf{c}$ vector, $\mathbf{c}_{int}$ vector, $\mathbf{k}$ vector and $\mathbf{k}_{int}$ vector are equal, we get an “ideal” rotor accurately tuned. In reality, this situation never occurs because of two reasons. First of all, each blade has mechanical distortions considering e.g. slight dimensional differences. Second of all, the ideal rotor tune-up would cause bladed disk vibration sync leading eventually to synchronous blade vibration in the whole blade row resulting in self-excited vibrations with increasing amplitudes like flutter.

3. Simulation model of bladed disk

Knowing motion equation (1) and expressions (2÷15), we may divide initial values to known and determined. Matrices $\mathbf{M}$, $\mathbf{c}$, $\mathbf{c}_{int}$, $\mathbf{k}$ and $\mathbf{k}_{int}$ define blade and inter-blade mass, damping and stiffness properties. Therefore, matrices $\mathbf{C}$ and $\mathbf{K}$ have to be numerically derived.

In addition to simulation needs, determination of gain, viscous damping and spring stiffness, coefficients matrices can be carried out by simple math operations:

$$\begin{aligned} \mathbf{WZM} &= \frac{1}{\mathbf{M}} \\ \mathbf{WTL} &= \frac{\mathbf{c}}{\mathbf{M}} \\ \mathbf{WTS} &= \frac{\mathbf{K}}{\mathbf{M}} \end{aligned} \quad (16)$$

Initial conditions defined as start-up vibration displacement, velocity and acceleration have to be established. The right-hand side of equation (1) is defined as the external force input vector $\mathbf{F}$.

Force input control in the form of rotational speed changes can be introduced manually (during simulation model operations) or loaded from “.txt” file automatically.

In every program loop, in order to determine displacement, velocity and acceleration matrices of “*i*” blade, some mathematical operations must be derived on the basis of numerical integration of motion equation (1). The rotor simulation model carries out current blade-tip displacement of vibrating blade which allows to find time difference between real and expected blade-tip position under the blade observer.

4. Simulation assumptions

In order to build a rotor simulation model, National Instruments LabVIEW software was used.

The rotor model consists of six blades fixed in rotor disk and is reduced to SDOF system with damping in elastic foundation. Every blade is fixed in rotor with initial and specified damping and stiffness (in root) and there are blade-to-blade connections with inter-blade damping and stiffness. It is assumed that there are no blades’ stiffness changes with changing rotor speeds. Natural frequency of I mode blade vibration (bending vibrations) is equal to 350 [Hz].

Angular positions of blade-tip observers along the circumference of the casing must be defined as well.

5. Simulation results

Rotor model simulations were conducted for the conditions described below:

- condition “1” – steady increase of the rotor speed 0÷30000 [revs/min], blades stiffness and damping parameters are equal and constant, inter-blade stiffness and damping parameters are equal and constant, uniformly positioned blade observers, 1 EO force input
- condition “2” – steady increase of the rotor speed 0÷30000 [revs/min], blades stiffness and damping parameters are equal and constant, inter-blade stiffness and damping parameters are different and constant, uniformly positioned blade observers, 1 EO force input
- condition “3” – steady increase of the rotor speed 0÷30000 [revs/min], blades stiffness and damping parameters are different and constant, inter-blade stiffness and damping parameters are equal and constant, uniformly positioned blade observers, 1÷4 EO force input

Below presented figures demonstrate the rotor model simulation results. Figures 2a, 3a and 4a show blade-tip displacement during simulation of rotor operation and figures 2b, 3b and 4b show time difference between observed and expected blade time of arrival (TOA).

Mathematical model and simulator of rotor with vibrating blades
Model matematyczny i symulator stopnia wirnikowego z drgającymi łopatkami

Fig. 2a Blade-tip displacement – condition „1”

Fig. 2b Time difference between real and expected blade-tip time of arrival – condition „1”

Fig. 3a Blade-tip displacement – condition „2”

Fig. 3b Time difference between real and expected blade-tip time of arrival – condition „2”

Mathematical model and simulator of rotor with vibrating blades
Model matematyczny i symulator stopnia wirnikowego z drgającymi łopatkami

Fig. 4a Blade-tip displacement – condition „3”

Fig. 4b Time difference between real and expected blade-tip time of arrival – condition „3”

6. Summary

This paper presents a method of modelling a bladed axial flow rotor stage. The created model may be used as a generator of blade tip timing data. Implemented approach enables research on synchronous and asynchronous rotor mistuning and bladed disk vibrations in form of travelling wave.

Simulations show that demonstrated solution constitutes a very reliable way to determine vibratory research on bladed rotor stage. Results confirm theoretical considerations on SDOF system vibrations. Angular layout of the observers has practically no influence on simulation results.

Introduced simulation model may be easily applied to rotor with more than 6 blades (as shown in this paper), but still one has to remember that extended model will need more time to calculate simulation results.

7. References

- [1] Boyce M.P.: Gas Turbine Engineering Handbook, Gulf Professional Publishing, 2002
- [2] Dimitriadis G., Carrington I.B., Wright J.R., Cooper J.E.: Blade Tip-Timing Measurement of Synchronous Vibrations of Rotating Bladed Assemblies, Mechanical Systems and Signal Processing, 2002
- [3] Gallego-Garrido J., Dimitriadis G., Wright J.R.: Development of a Multiple Modes Simulator of Rotating Bladed Assemblies for Tip-Timing Data Analysis, Proceedings of the 2002 International Conference on Noise and Vibration Engineering, 2002
- [4] LabVIEW Manual: LabVIEW Fundamentals, National Instruments Corporation, 2005
- [5] Moon T.K., Stirling W.C.: Mathematical Methods and Algorithms for Signal Processing, Prentice Hall, Inc., 2000
- [6] Witoś M.: Theoretical Foundations Of Tip Timing Measurements, 2007

Projekt został sfinansowany ze środków Narodowego Centrum Nauki przyznanych na podstawie decyzji numer DEC-2011/01/D/ST8/07612.

mjr dr inż. Michał Wachłaczko – absolwent Wojskowej Akademii Technicznej (2003) – specjalność samoloty i śmigłowce (płatewiec i silnik), prace badawcze związane z numeryczną symulacją ruchu samolotów oraz systemy diagnostyczne turbinowych silników lotniczych. Żołnierz zawodowy. Od 2008 r. pracownik ITWL, obecnie na stanowisku kierownika pracowni w Zakładzie Silników Lotniczych.

mjr dr inż. Radosław Przysowa - pracownik Zakładu Silników Lotniczych ITWL od 2002 roku. Zajmuje się modelowaniem silników, rozwojem metod przetwarzania sygnałów oraz opracowaniem i testowaniem oprogramowania systemów pomiarowych. Opracował m.in. specjalizowany system pomiarowy do drgań łopatek wykorzystywany podczas prób silników i turbin, który był wielokrotnie wykorzystywany na rzeczywistych obiektach.

mjr dr inż. Mariusz Żokowski – absolwent Wojskowej Akademii Technicznej (2004) – specjalność osprzęt samolotów i śmigłowców, absolwent Politechniki Warszawskiej (2004) – specjalność elektrotechnika, absolwent studiów doktoranckich Politechniki Koszalińskiej (2008). Żołnierz zawodowy. Od 2007 r. pracownik ITWL, obecnie na stanowisku kierownika pracowni w Zakładzie Silników Lotniczych.