DIFFERENT ANALYSIS PROCEDURES FOR COMPUTING THE RELIABILITY OF REPAIRABLE SYSTEMS

RÓŻNE PROCEDURY ANALIZY DLA OBLICZANIA NIEZAWODNOŚCI SYSTEMÓW PODLEGAJĄCYCH NAPRAWIE

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Abstract: This paper will review the different analysis scenarios that can be used when examining the reliability of repairable systems. Five different methods will be reviewed, specifically:

- 1) Using the mean value of the times between failure (TBFs) at the system level.
- 2) Using the distributions of the components of the system and creating a reliability block diagram (RBD).
- 3) Using the distribution of the times between failure (TBFs) at the system level.
- 4) Using the system level failure data and fitting the non-homogeneous Poisson process (NHPP) model.
- 5) Using the system level failure data and fitting the general renewal process (GRP) model.

In the paper, the five different methods will be discussed (brief background), and conclusions about the applicability and accuracy of each method will be given. In addition, this paper can be used to further understand the assumptions behind each analysis method, the data required and the type of results that can be obtained, thus assisting analysts in choosing the most appropriate method.

Streszczenie: Niniejszy artykuł omawia różne scenariusze analizy, które można zastosować podczas badania niezawodności systemów podlegających naprawie. Omówionych zostanie pięć różnych metod:

- 1) Użycie średniej wartości czasu między usterkami (TBFs) na poziomie systemu.
- 2) Użycie rozkładu elementów systemu i utworzenie schematu blokowego niezawodności (RBD).
- 3) Użycie rozkładu czasów między usterkami (TBFs) na poziomie systemu.
- 4) Użycie danych dotyczących usterek na poziomie systemu i dopasowanie niehomogenicznego modelu procesu Poissona (NHPP).
- 5) Użycie danych dotyczących usterek na poziomie systemu i dopasowanie modelu procesu ogólnego odnawiania (GRP).

W artykule omówionych zostanie pięć różnych metod (krótkie wprowadzenie) i przedstawione zostaną wnioski dotyczące możliwości zastosowania i dokładności każdej z nich. Poza tym artykuł ten może zostać wykorzystany do dokładniejszego zrozumienia założeń każdej metody analizy, wymaganych danych oraz rodzaju wyników które można porównać, co czyni go pomocnym w doborze najbardziej odpowiedniej metody.

1. Introduction

This paper reviews the different analysis scenarios that can be used when examining repairable systems. Five different methods will be reviewed, using a hypothetical system, specifically:

- 1) Using the mean value of the times-between failures at the system level.
- 2) Using the distribution of the times-between failures at the system level.
- 3) Using the distributions of the components of the system and creating a Reliability Block Diagram (RBD).
- 4) Using the system level failure data and fitting the Non-Homogeneous Poisson Process (NHPP) model.
- 5) Using the system level failure data and fitting the General Renewal Process (GRP) model.

The following procedure will be used to create failure data and compare results:

- A. A hypothetical system will be created using a RBD and with known component failure distributions. This will represent a system where the True reliability and expected failures are known.
- B. The repair duration will be considered negligible for the purpose of this comparison.
- C. A simulation will be performed on this system for a defined mission duration, and the events (failures) during the simulation, and the associated responsible part, will be extracted.
- D. Two more such simulations will be performed (total of 3, with different mission durations), which in a way would represent 3 fielded systems of different operational times/ages.
- E. The results from these three simulations will then be analyzed with above mentioned five different analysis methods.

2. Simulation

For simplicity, we chose a Racing Car for our hypothetical system. The system is broken down to 2 major subsystems: Front Assembly and Rear Assembly.

The Front Assembly is comprised of:

- Front Brakes
- Front Suspension

The Rear Assembly is comprised of:

- Rear Brakes
- Rear Suspension
- Engine
- Transmission

The RBDs for the system are:

The following failure distributions and corresponding parameters were set to represent the “True” reliability behavior of these components:

Table 1: True failure distributions for the different components of the system

Component	Distribution	Parameter 1	Parameter 2
Brakes Front	Weibull	5,8	474
Front Suspension	Lognormal	7,215	0,563
Brakes Rear	Weibull	5,8	556
Engine	Weibull	2,6	1230
Rear Suspension	Weibull	2,14	1606
Transmission	Lognormal	6,91	0,5281

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In Table 1, for the Weibull distribution, Parameter 1 is Beta and Parameter 2 is Eta. For the Lognormal distribution, Parameter 1 is the LnMean, and Parameter 2 is the LnStd.

Simulation Assumptions

Three simulations were performed, based on the above RBD and the distributions of Table 1, the first for a mission time of 2500Km, the second for 1976Km and the third for 800Km. In addition, and to make the simulation more realistic, Preventive Maintenance was performed on the brakes every 305Km. The idea here is that the RBD represents an actual system that operates in the field, and since we run three simulations, each simulation represents one fielded system. The failure distributions that will generate the failures are known, but we'll pretend that we don't know them, and try to estimate the reliability using the five methods mentioned in the introduction of this report. The Preventive Maintenance Policy was added in order to replicate a more realistic scenario of systems operating in the field.

Simulation Results

After the three simulations were run, the following events were obtained:

Table 2: Simulation Results

System1 Age=2500Km		System2 Age=1976Km		System3 Age=800Km	
Time-to-Event	Component	Time-to-Event	Component	Time-to-Event	Component
249.8556746	engine	305	PM Brakes	305	PM Brakes
305.00001	PM Brakes	610.0001	PM Brakes	453.8681512	rear suspension
584.2324998	front suspension	872.4111452	engine	610.00011	PM Brakes
610.00012	PM Brakes	899.7569744	right front brake	743.4900418	transmission
915.00022	PM Brakes	899.7569844	PM Brakes		
971.9650444	engine	1204.757074	PM Brakes		
1220.00033	PM Brakes	1371.701827	right front brake		
1525.00043	PM Brakes	1371.701837	PM Brakes		
1830.00053	PM Brakes	1470.438019	engine		
1861.689518	front suspension	1572.565067	rear suspension		
1994.56836	rear suspension	1676.701947	PM Brakes		
2127.083186	transmission	1754.884827	transmission		
2134.282155	right rear brake				
2134.282165	PM Brakes				
2186.903602	engine				
2439.282265	PM Brakes				

As seen from Table 2, for each simulation the event time was recorded, as well as the corresponding component that initiated an event. We will now assume that this is all we know about the design, and that each simulation represents a system in operation, where the first system operated for 2500Km so far, system 2 for 1976Km and system 3 for 800Km. Note that the event times are in a cumulative scale.

3. Analyses and Individual Results

Using the Mean Value of the System's Times-Between-Failures

In this approach the objective is to estimate the MTBF of the Design (system), and based on this estimation possibly make predictions about future events. Under this model, we only look at the Times-Between-Failures for each system, or,

Table 3: Time-Between-Failures (TBF) for each System

System1 Age=2500Km		System2 Age=1976Km		System3 Age=800Km	
Time-to-Event	Time-Between-Failures	Time-to-Event	Time-Between-Failures	Time-to-Event	Time-Between-Failures
249.8556746	249.8556746	872.4111452	872.4111452	453.8681512	453.8681512
584.2324998	334.3768252	899.7569744	27.3458292	743.4900418	289.6218906
971.9650444	387.7325446	1371.701827	471.9448526		
1861.689518	889.7244736	1470.438019	98.736192		
1994.56836	132.878842	1572.565067	102.127048		
2127.083186	132.514826	1754.884827	182.31976		
2134.282155	7.198969				
2186.903602	52.621447				

Notice that in Table 3 all the Preventive Maintenance events have been removed, since they don't represent failures.

There are two ways to utilize the TBF data of Table 3 in order to obtain an MTBF. The first is to simply sum all the TBFs and divide by the total number of events, or,

$$MTBF = \frac{\sum_{i=1}^N t_i}{N} \quad (1)$$

where:

t_i = TBF from each system

N = Total number of events from all systems

The MTBF calculated using this approach could be very misleading, since it assumes a random failure behavior (i.e. constant Rate of Occurrence of Events, aka Failure Rate). If the system exhibits an aging pattern (wear-out), or an infant mortality pattern, this equation will average out all the TBFs and the Mean could be overestimated in the first case, or underestimated in the latter.

Due to this, an improved estimator is to obtain the MTBF from the distribution of the TBFs. Under this approach a distribution is fitted to the TBFs which will represent the behavior of the TBFs. In addition, the time between the last failure of each system and the "current age" of each respective system can be included in the model as "Suspensions¹."

¹ Note that the ability to include/consider the Suspensions is a main difference between this approach and the simplified method given by Eqn. 1.

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For this particular example, the data is entered in Weibull++, and the following model is obtained:

Notice, that for each system there is one Suspension which is the time between the last event and the current age of the system.

Under this analysis, and for this example, the Best-fit distribution is the Weibull, with beta=1.11 and eta=337Km. Based on this distribution, the MTBF can be calculated, or,

Therefore, in this example, $MTBF=324.5Km$. This estimate is preferred over the one using Eqn. 1, because the behavior of the TBFs is considered, and it's based on a best-fit model rather than assuming a constant rate of occurrence of failures (ROCOF). (Note that a constant ROCOF is similar to assuming an exponential distribution, or more correctly a Homogeneous Poisson Process).

However, caution must be taken when using this approach, because even though this analysis represents a better estimate than the one given by Eqn. 1, it is still an average, and as it will be shown later, averages work well when sufficient data is present AND when systems have reached a "Steady State" AND when predicting future events.

A second caution is regarding the misuse of this approach. In many cases, analysts misinterpret this model as being the Failure Distribution of the System, and perform additional estimations, such as Reliability, BX calculations, etc. These types of results are incorrect, since the model simply describes the behavior of the TBFs, and in essence it's a model of the MTBF. From this model, and in addition to the MTBF, we can obtain, for example, what percentage of the TBFs is within a given time range, BUT this does NOT represent a Reliability/Unreliability calculation. For example, if we compute $P(t=200km)$, for this model we get 43%. This does NOT mean that the probability of failure of the system is 43%, rather that 43% of the TBFs were in the order of 200Km or less. Obviously, this is very different than a Reliability/Unreliability statement. Also notice, that this statement is invariant of the chronological order of the TBFs, and the 43% of the TBFs that are less or equal to 200Km could have occurred at the beginning of the life of the system, at the latter stages, or just randomly...all we can tell from this model is that we expect 43% of time between events to be less or equal to 200Km. The following graphic explains this point. In this graphic a chronological order of failure events of a system is depicted. In the graphic, T_i represents the cumulative time to event, and t_i represents the time-between events. In addition, the vertical red line represents the time when the system has accumulated 200Km of operation, and all the times between events that are less or equal to 200Km are contained within the circle. If we were to estimate the Reliability at 200Km, then it is defined as the probability that the system will operate for 200Km without a failure. It can be easily seen from the graph, that this is different than percentage of TBFs whose order of magnitude is less or equal to 200Km (circled events). The percentage of events included in this circle is what was calculated above to be 43%.

Therefore, it is concluded that with this model Reliability predictions are NOT valid.² However, the model can be used in order to predict Expected Number of Failures (ENOF) over time, as described in the next subsection.

Results of the “MTBF” Model

In the previous section we determined from the distribution of the TBFs that the MTBF is 324.5Km. This estimate can be used to compute the expected Number of Failures over the Age of the System, or,

$$ENOF = \frac{Age}{MTBF}$$

The following Table provides the estimated number of failures based on the above equation, at different system ages and based on the calculated MTBF of 324.5Km. The estimate is compared to the Actual (True) number of failures which are determined from the original distributions and RBD.

Table 4: Expected Failures for Different System Ages, using MTBF

Age(Km)	True	Age/mean	Percent Difference
250	0,053	0,77	1353,62
500	0,32	1,54	381,51
1500	3,22	4,62	43,56
2000	4,8	6,16	28,40
3000	8,2	9,24	12,74

From Table 4 we can see that the difference between the estimate and the true value improves at higher ages. This is expected since “Averages” are more suited as system age approaches infinity.

Using the Distribution of the System’s Time-Between-Failures

This approach is based on the approach used previously, with the exception of using the actual distribution of the TBFs instead of the MTBF. This can be simply done using BlockSim. A single block is created in BlockSim, with a failure

² If the Reliability at 200Km was sought, a distribution to the first failure event from each system could be used.

distribution obtained from the TBFs. In this example, we obtained a Weibull distribution with $\beta=1.11$, and $\eta=337\text{Km}$ (see previous subsection). A Repair Distribution is also needed in BlockSim (since this is a repairable system), and since in this example we ignore the downtime, the Corrective Maintenance duration is set to Zero in BlockSim.

Results of using the TBF Distribution

Under these settings we run multiple BlockSim simulations for different system ages, and we record the results, in this case ENOF.

Table 5: Expected Failures for Different System Ages, using the TBF Distribution in BlockSim

Age(Km)	True	TBF Distribution	Percent Difference
250	0,053	0,721	1260,38
500	0,32	1,46	356,25
1500	3,22	4,47	38,82
2000	4,8	6,08	26,67
3000	8,2	9,18	11,95

It can be seen that little improvement is achieved using this approach. However, it is a useful approach in case we need to model large-scale systems, comprised of multiple repairable subsystems.

Using Component Distributions and RBDs

In this case we determine the individual failure distributions of the components from the data. This is done by obtaining the Time-Between-Failure of each individual component (e.g., time-between engine failures from each system). For example, for the engine, the following are the TBFs for each system:

System1 Age=2500Km		System2 Age=1976Km		System3 Age=800Km	
System Age	Component Age	System Age	Component Age	System Age	Component Age
249.8556746	249.8556746	872.4111452	872.4111452		800
971.9650444	722.1093698	1470.438019	598.0268738		
2186.903602	1214.938558		505.561981		
	313.096398				

Based on this data, the failure distribution of the engine is determined:

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Similarly, the distributions of all the components can be determined, and are given in the following Table:

Table 6: Component Failure Distributions

Component	Distribution	Parameter 1	Parameter 2
Brakes Front L	WBL	3.222573874	716.1151497
Brakes Front R	WBL	3.222573874	716.1151497
Brakes Rear L	WBL	15.36052078	391.4134836
Brakes Rear R	WBL	15.36052078	391.4134836
Engine	WBL	2.817058658	905.7981868
Front Suspension	LGN	7.290293365	0.652096778
Rear Suspension	WBL	2.457312008	1564.3648
Transmission	WBL	3.142642565	1737.347234

It should be noted that for the Brakes, all the Preventive Maintenance actions were considered as Suspensions when building the model. Also, the data from all the Rear Brakes were considered as one dataset (independent of the side), and similarly for the Front Brakes.

It can be seen that this method requires sufficient information (failures) at the component level. If component failures are scarce, then it will be difficult and possibly inaccurate to implement this method.

Results of using Component Distributions

These distributions were entered in BlockSim and simulations were performed for different system ages. A Preventive Maintenance Policy for the Brakes was included in the model as well.

The results are given in the Table below:

Table 7: Expected Failures using Component Distribution in a RBD

Age(Km)	True	RDB Analysis	Percent Difference
250	0,053	0,079	49,06
500	0,32	0,366	14,38
1500	3,22	3,29	2,17
2000	4,8	5,2	8,33
3000	8,2	8,44	2,93

Using the NHPP Model

In this approach the individual cumulative times-to-event for each system are considered, and the NHPP model with a Power Intensity Function (aka Failure Rate) is fitted to the data. The model is given by the following Equation:

$$\Pr[N(T) = n] = \frac{[\Lambda(T)]^n e^{-\Lambda(T)}}{n!} \quad (2)$$

Where:

$$\Lambda(T) = \lambda \cdot T^\beta \quad (3)$$

and

$\Pr[N(t)=n]$ is the probability that n failures will be observed by time, T .

$\Lambda(t)$ is the Failure Intensity Function (Rate of Occurrence of Failures, or failure rate).

This model is an extension of the Homogeneous Poisson Process, in which the failure rate is assumed to be constant (i.e. exponential distribution). In this case however, (NHPP), the failure rate could be increasing, decreasing or constant (as in the Weibull distribution), based on the value of beta in Eqn. (3). The assumption of this model is that after each failure, the system is restored to the same condition it was prior to the failure (aka As-Bad-As-Old). This assumption is sufficient when dealing with large systems; however, it becomes less applicable for smaller systems (less components) where a replacement has a significant impact on the system (renewal).

NHPP Model Results

The NHPP with a Power Law Intensity Function is fitted to the cumulative failure times (mileage in this example) of each system, using RGA:

The beta in this case is 1.65, which indicates an increasing ROCOF for this system/design (i.e. wear-out). In other words, as the systems age, more events are observed and the TBF intervals decrease.

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This is shown in the following plot:

Based on the model, the Expected Number of Failures at different ages can be computed, as shown in the next plot:

Selected results are given in the next Table:

Table 8: Expected Failure Results based on the NHPP model

Age(Km)	True	NHPP	Percent Difference
250	0,053	0,196	269,81
500	0,32	0,61	90,63
1500	3,22	3,76	16,77
2000	4,8	6,05	26,04
3000	8,2	11,8	43,90

Using the GRP Model

An improvement to the NHPP model is the GRP model. As mentioned in the subsection on the NHPP model, the model assumes that after each failure the system is As-Bad-As-Old (i.e. at the same condition as it was prior to the failure). The GRP (General Renewal Process) model relaxes this assumption, by including an additional parameter, q , which is a measure of the degree of restoration

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(renewal), and it's determined from the data. The data used is the same as those used in the NHPP model, i.e., cumulative times-to-event of each system.

GRP Model Results

The GRP model is fitted to the data using Weibull++, as shown next:

The ENOF results from this model and the corresponding difference from the true values are given in the next table:

Table 9: Expected Failures prediction based on the GRP Model

Age(Km)	True	GRP	Percent Difference
250	0,053	0,128	141,51
500	0,32	0,49	53,13
1500	3,22	3,8	18,01
2000	4,8	5,9	22,92
3000	8,2	10,34	26,10

4. Conclusions

Five different models were used to model the failure behavior of a repairable system. The data were generated using simulation, based on predefined failure distributions. The Expected Number of Failures was used as a metric for comparing to the true behavior of the system (which is known, since the generating

failure distributions are known). The following Table and Plot compare the results of the five different methods:

Age(Km)	True	Age/mean	TBF Distribution	RDB Analysis	NHPP	GRP
250	0,053	0,77	0,721	0,079	0,196	0,128
500	0,32	1,54	1,46	0,366	0,61	0,49
1500	3,22	4,62	4,47	3,29	3,76	3,8
2000	4,8	6,16	6,08	5,2	6,05	5,9
3000	8,2	9,24	9,18	8,44	11,8	10,34

It can be seen that the RBD Simulation approach offers the more realistic estimates in this example. Of course the estimation always depends on the number of observed events, and this why the analysis method should be chosen based on the available information. If, for example, very few failure events have been observed, the RBD simulation approach based on component distributions, would be very hard to adopt, and most likely a bad estimator. The simulation using the system's TBF distribution could offer a better estimator when dealing with few failures at the component level, but it typically becomes a good predictor when extrapolating to longer system ages. In addition, this method cannot be used for Reliability/Unreliability calculations. The MTBF method is recommended only for quick, back of the envelope calculations, since the simulation based on the TBF distribution is similar and slightly more accurate. Finally, the GRP model is

typically more accurate than the NHPP model, but more complicated. Nevertheless, it is recommended over the NHPP model (the NHPP model is actually a special case of the GRP model).

Therefore, the following are recommended:

- 1) When sufficient data is available, the RBD simulation based on the component distributions method should be preferred.
- 2) The GRP and the simulation based on the system's TBFs distribution should be the next options. The choice between the two is dependent on the desired metrics to be computed. If for example Reliability calculations are required, then the TBF distribution method is not suitable. The advantage of the GRP model is that more metrics can be computed, as well as confidence bounds.

In addition to the above recommendations, this report can be used to further understand the assumptions behind each analysis, the required data needed, and the type of results that can be obtained.